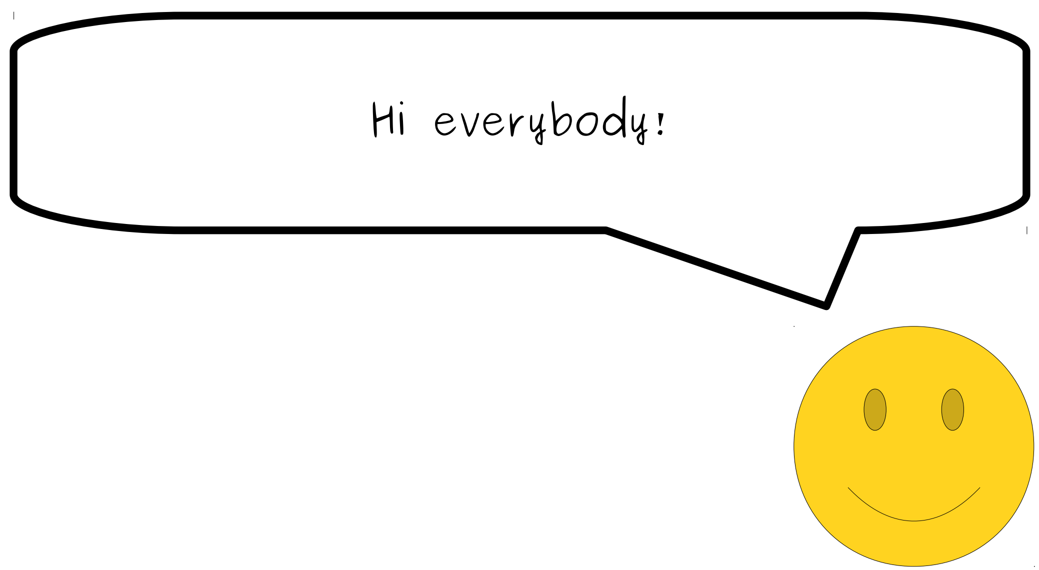
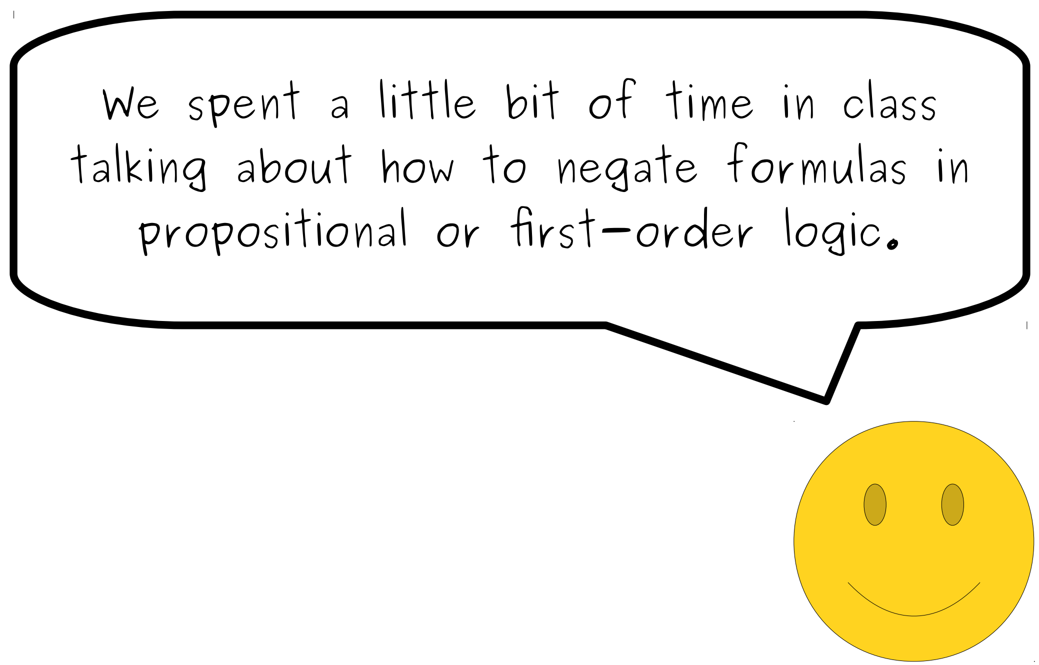
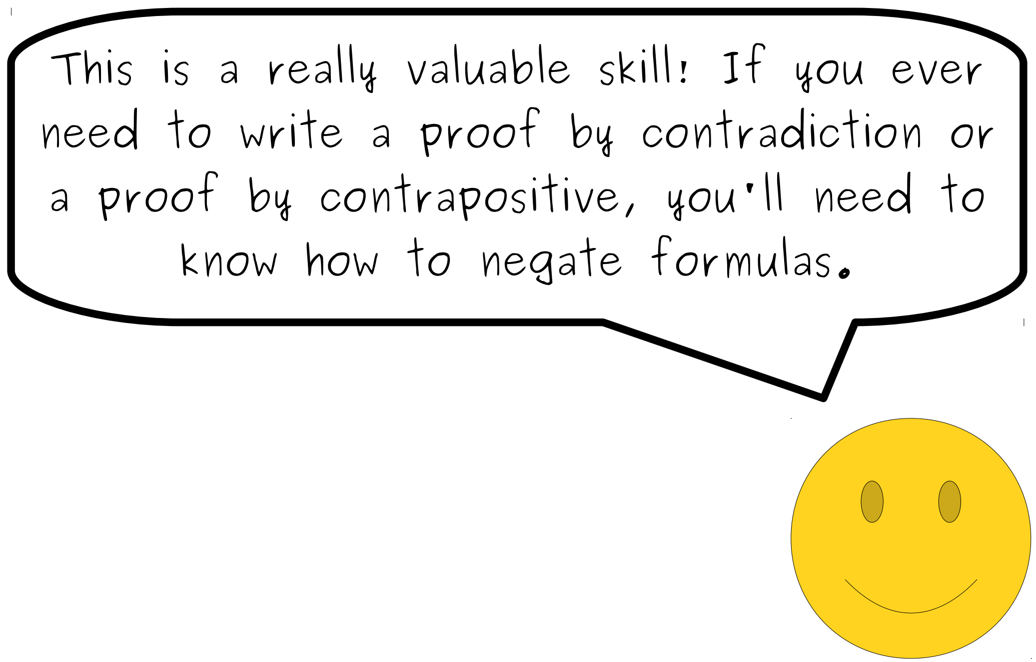
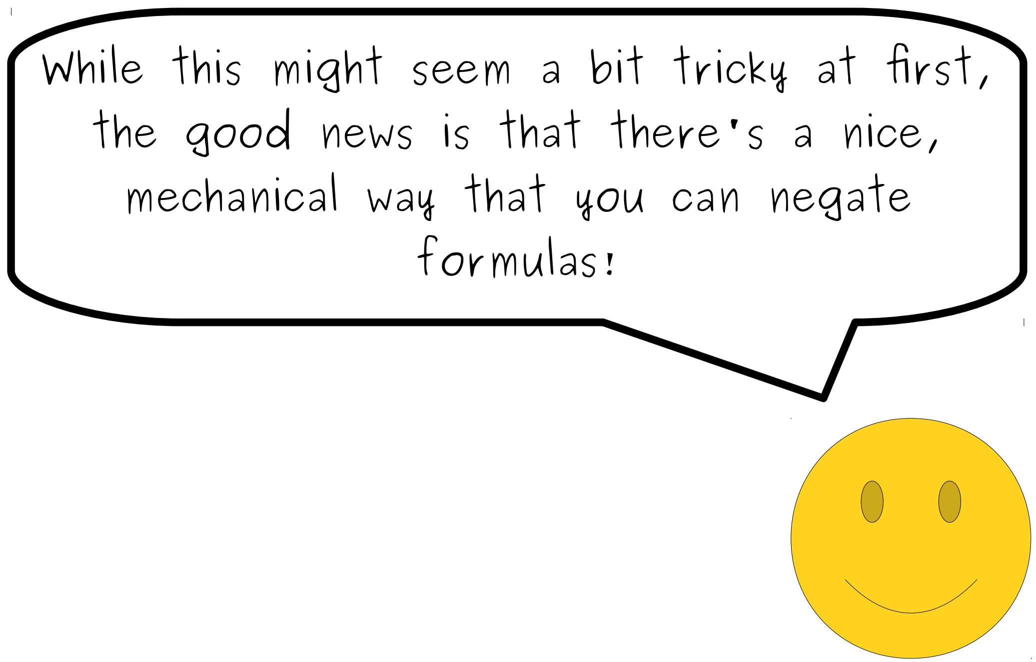
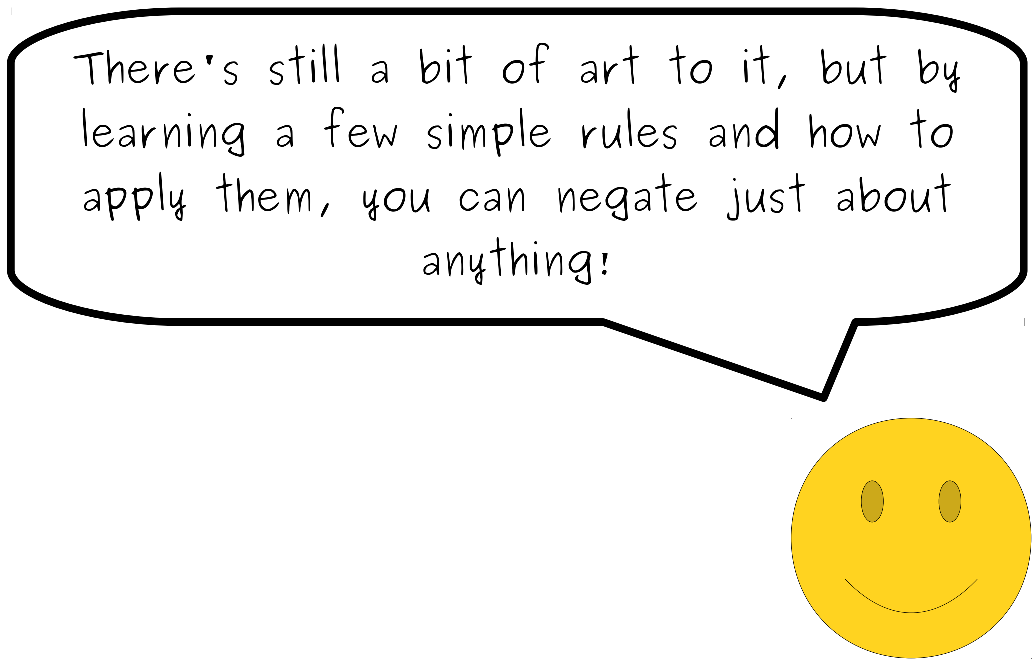
Guide to Negating Formulas

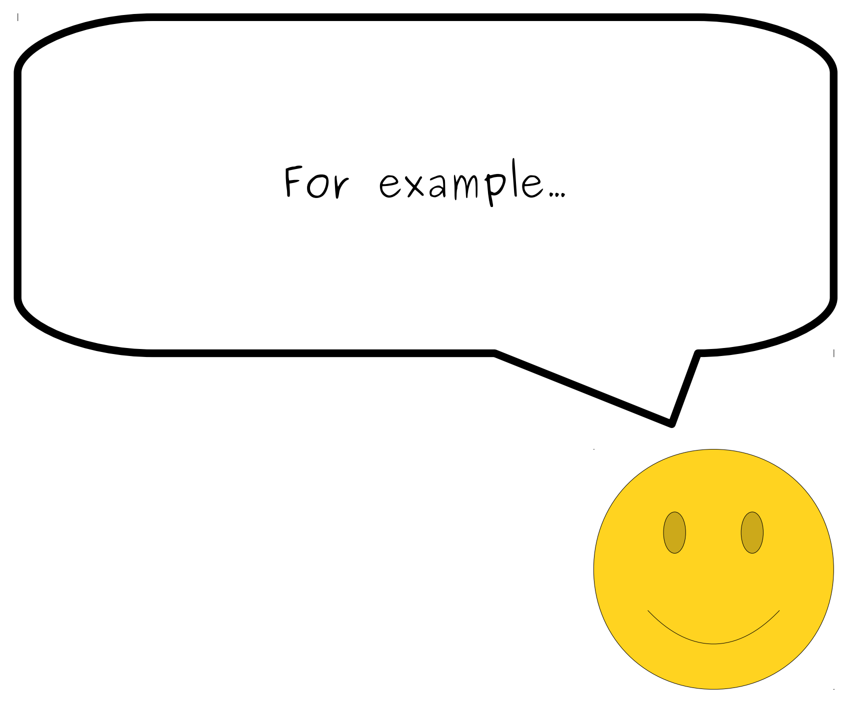

Hi everybody!


We spent a little bit of time in class talking about how to negate formulas in propositional or first-order logic.

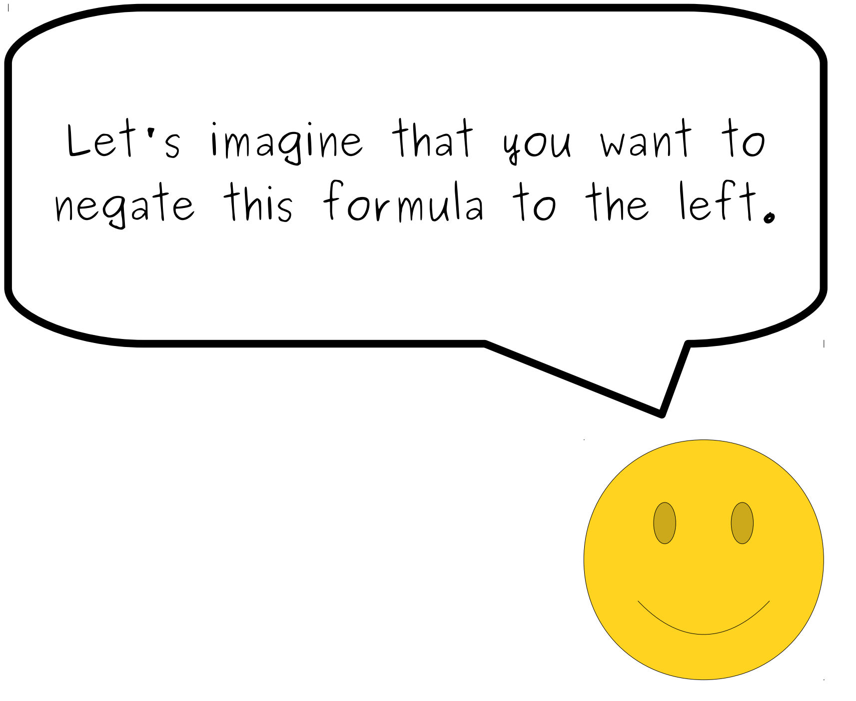

This is a really valuable skill! If you ever need to write a proof by contradiction or a proof by contrapositive, you'll need to know how to negate formulas.


While this might seem a bit tricky at first, the good news is that there's a nice, mechanical way that you can negate formulas!

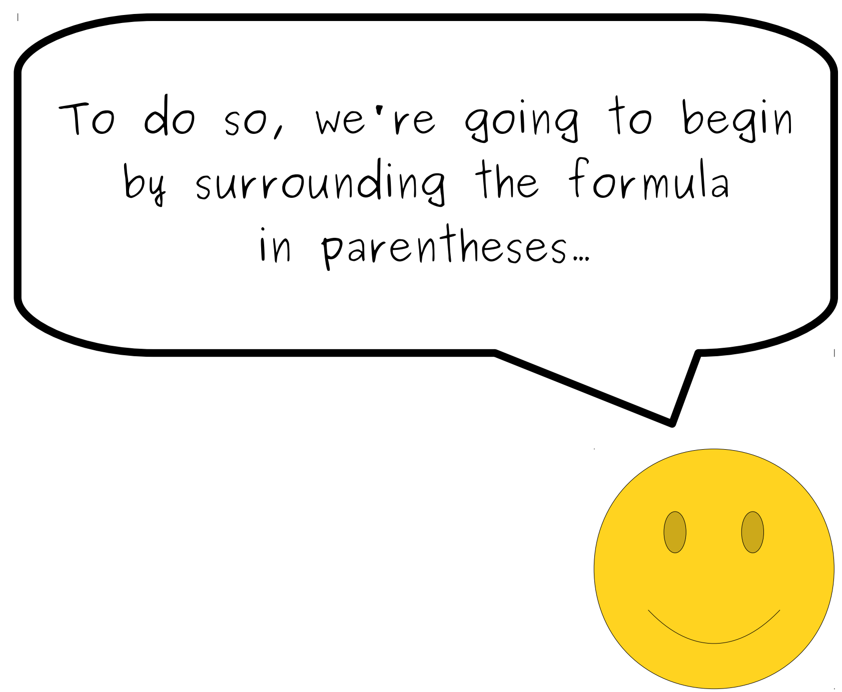

There's still a bit of art to it, but by learning a few simple rules and how to apply them, you can negate just about anything!


For example…

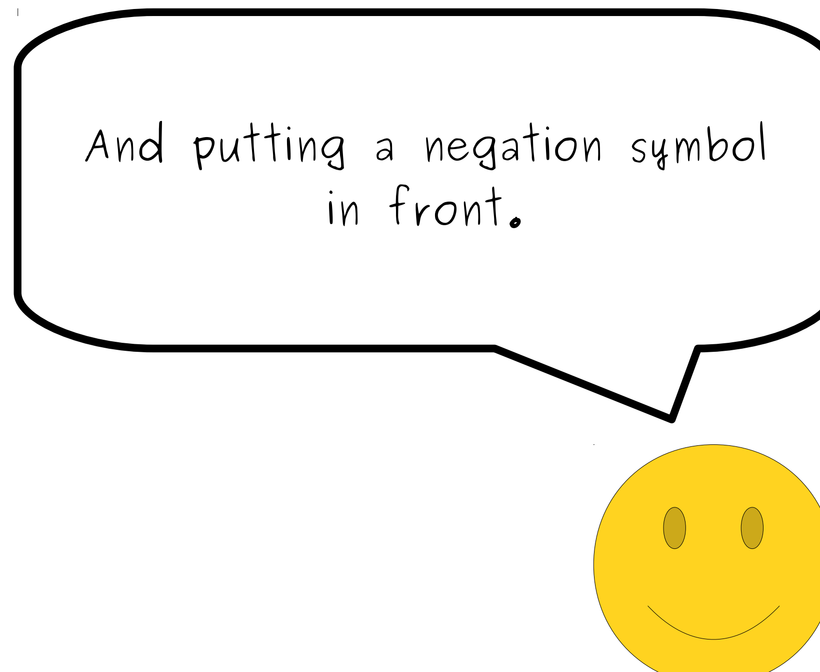
$$\neg p \wedge (q \vee r)$$



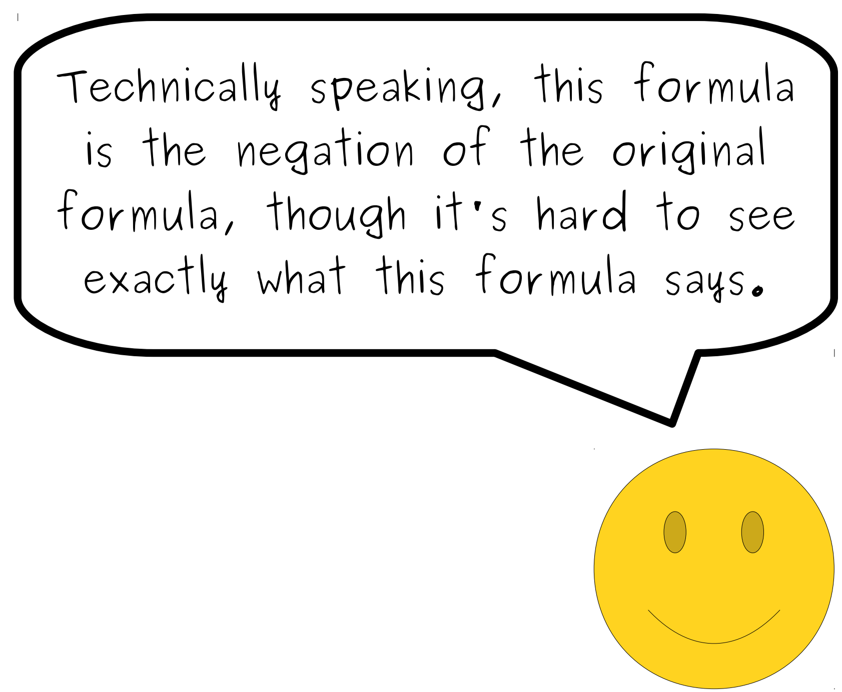
$$(\neg p \land (q \lor r))$$



$$\neg(\neg p \wedge (q \vee r))$$



$$\neg(\neg p \land (q \lor r))$$



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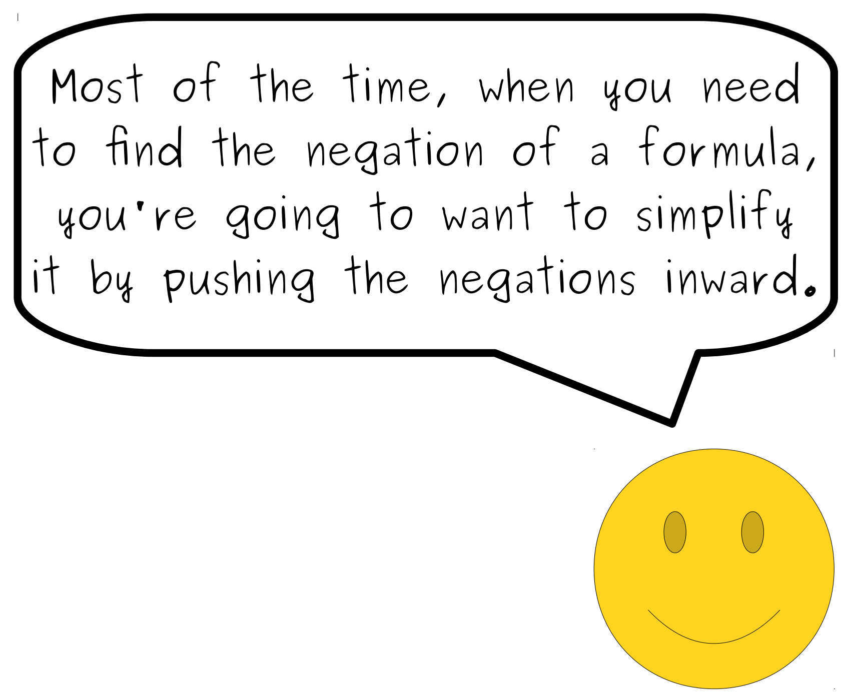

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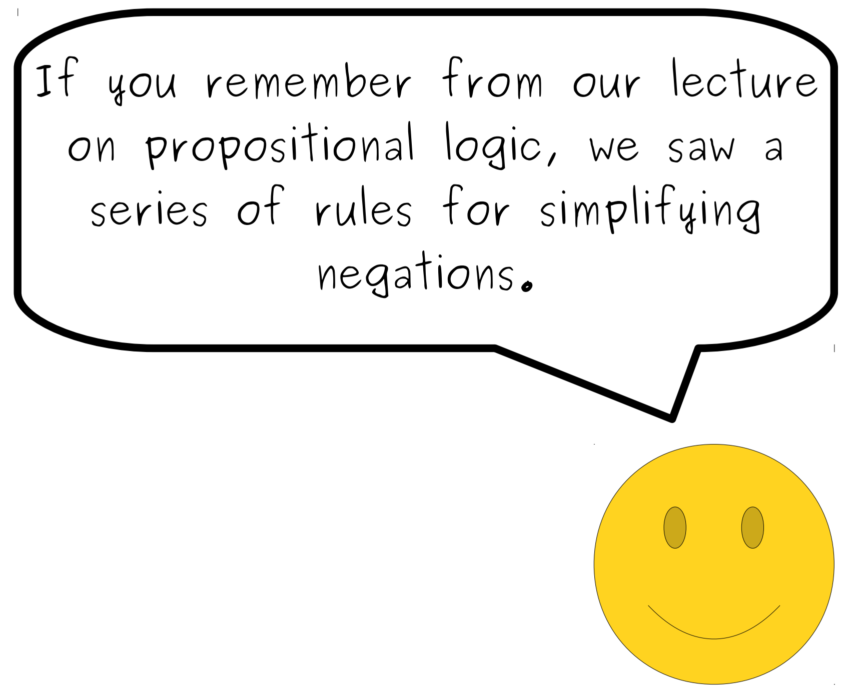


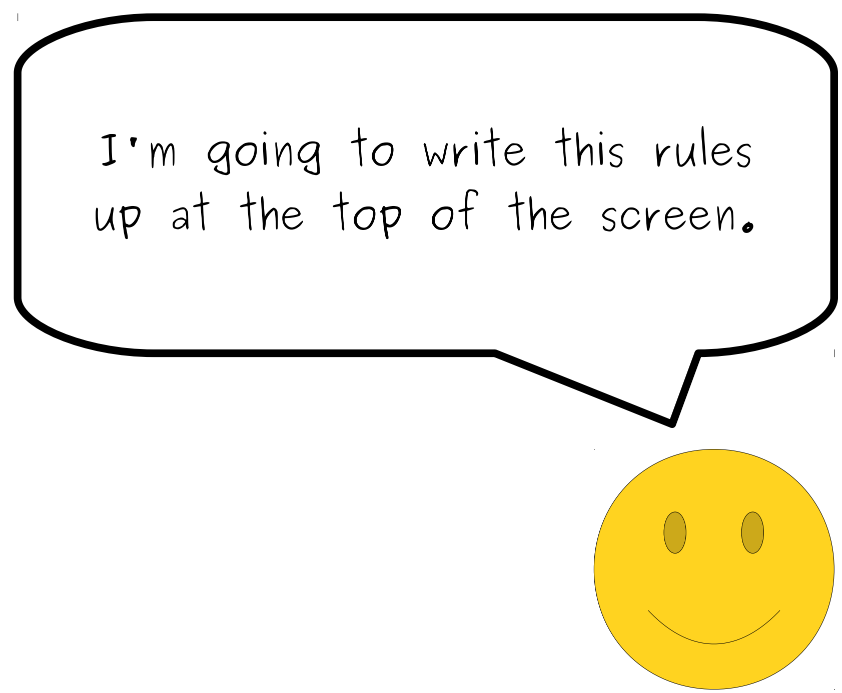
$$\neg(\neg p \land (q \lor r))$$

For example, we saw **de Morgan's laws**, which say that

$$\neg(A \land B) \equiv \neg A \lor \neg B$$
$$\neg(A \lor B) \equiv \neg A \land \neg B$$

$$\frac{\neg(\boldsymbol{A} \lor \boldsymbol{B})}{\neg \boldsymbol{A} \land \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\neg \boldsymbol{A} \lor \neg \boldsymbol{B}}$$

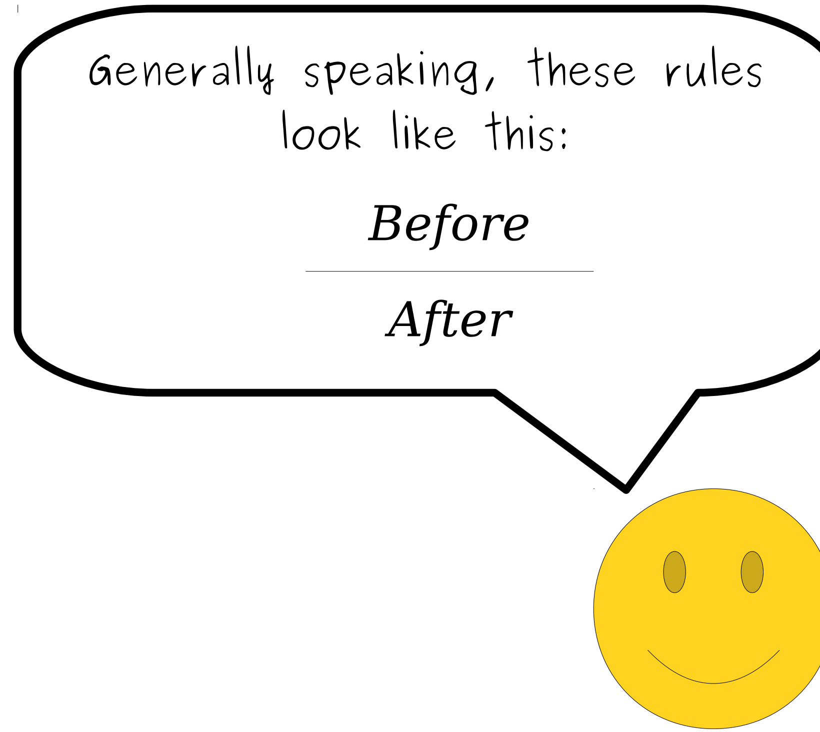
$$\neg(\neg p \land (q \lor r))$$



$$\frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

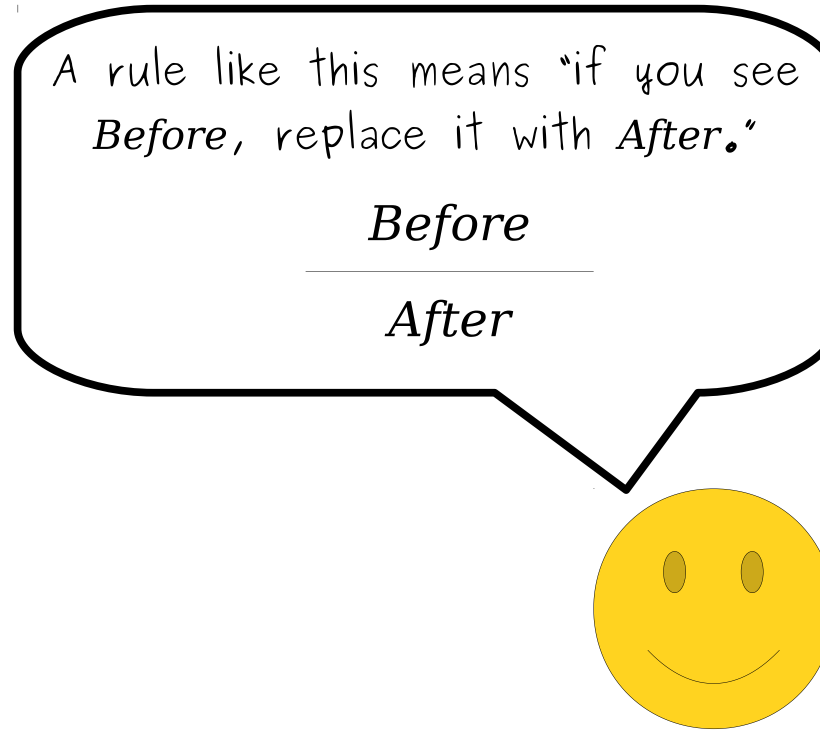
$$\neg(\neg p \land (q \lor r))$$

$$\frac{\neg(\boldsymbol{A} \lor \boldsymbol{B})}{\neg \boldsymbol{A} \land \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\neg \boldsymbol{A} \lor \neg \boldsymbol{B}}$$



$$\neg(\neg p \land (q \lor r))$$

$$\frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$



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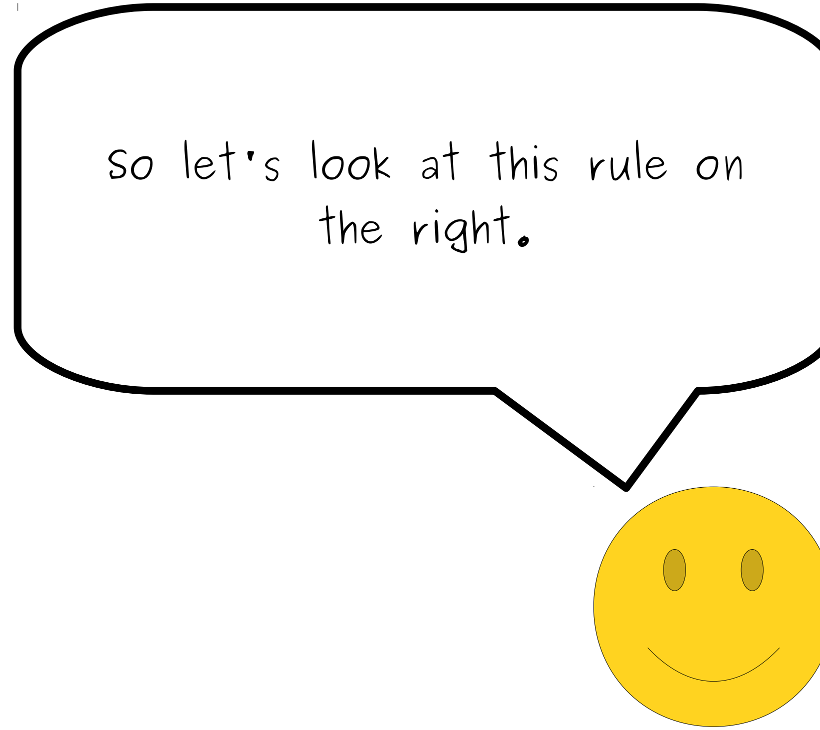


$$\neg(\neg p \land (q \lor r))$$

$$\frac{\neg(A \lor B)}{\neg A \land \neg B}$$

$$\frac{\neg(A \land B)}{\neg A \lor \neg B}$$

This rule says

"if you see something of the form

$\neg(A \land B)$

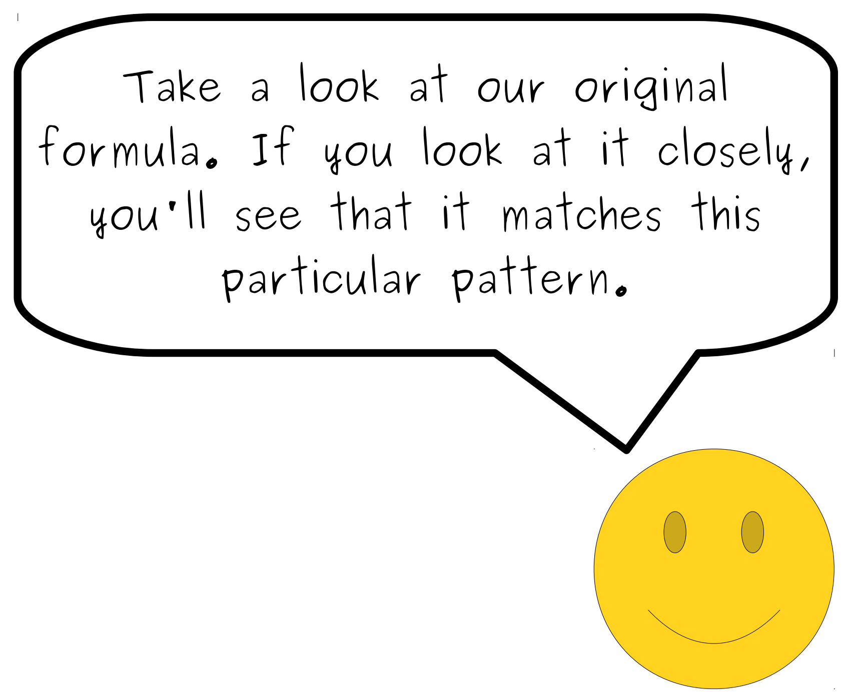
replace it with

$\neg A \lor \neg B$."

$$\neg(\neg p \land (q \lor r))$$

$$\frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\neg(\neg p \land (q \lor r))$$



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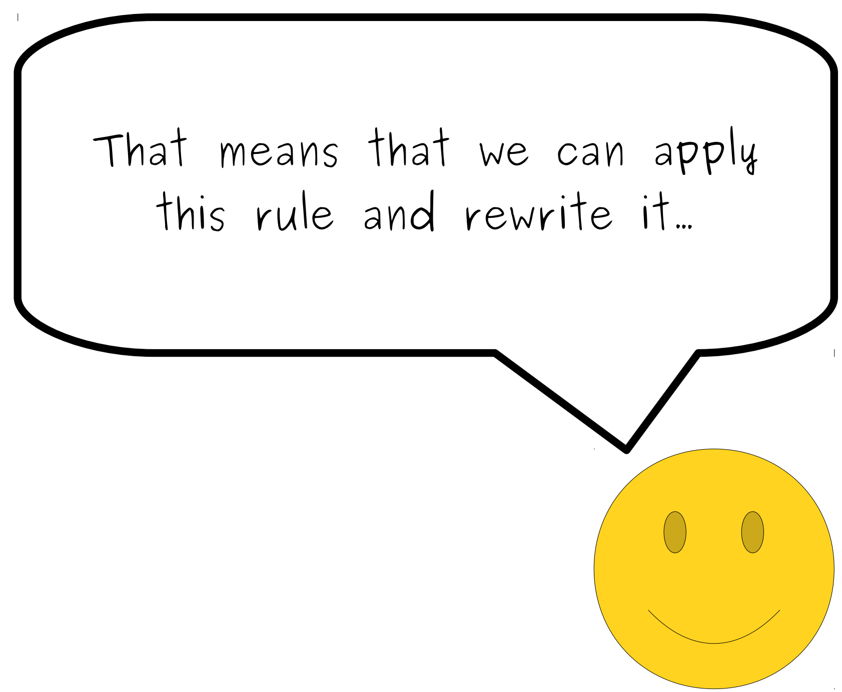
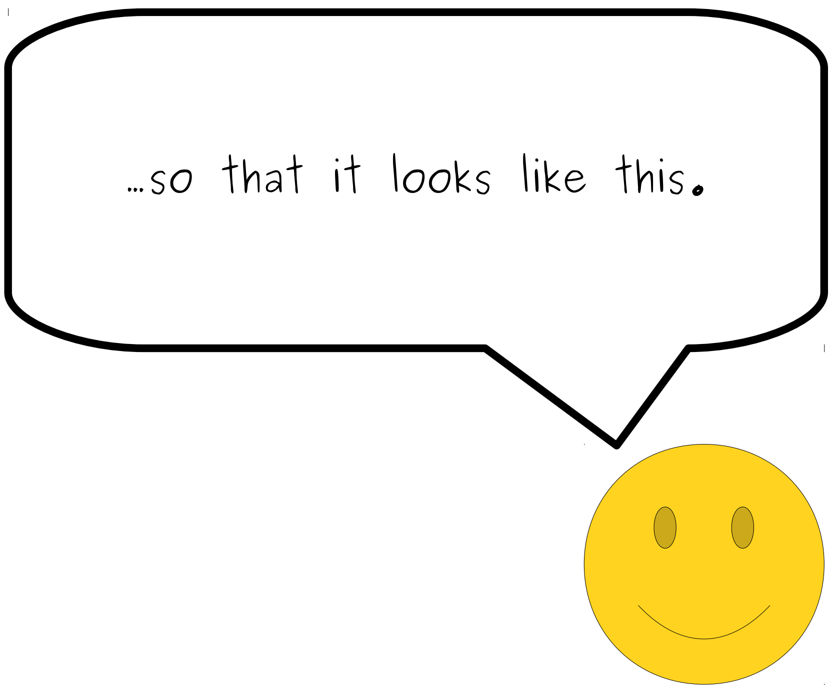


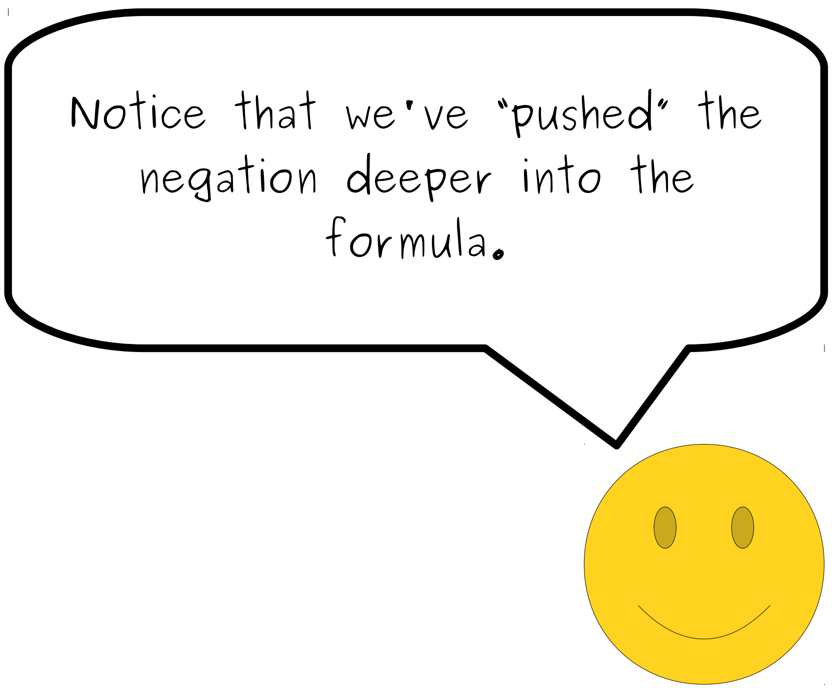
$$\frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B}$$

$$\neg\neg p \vee \neg(q \vee r)$$



$$\frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\neg\neg p \lor \neg(q \lor r)$$



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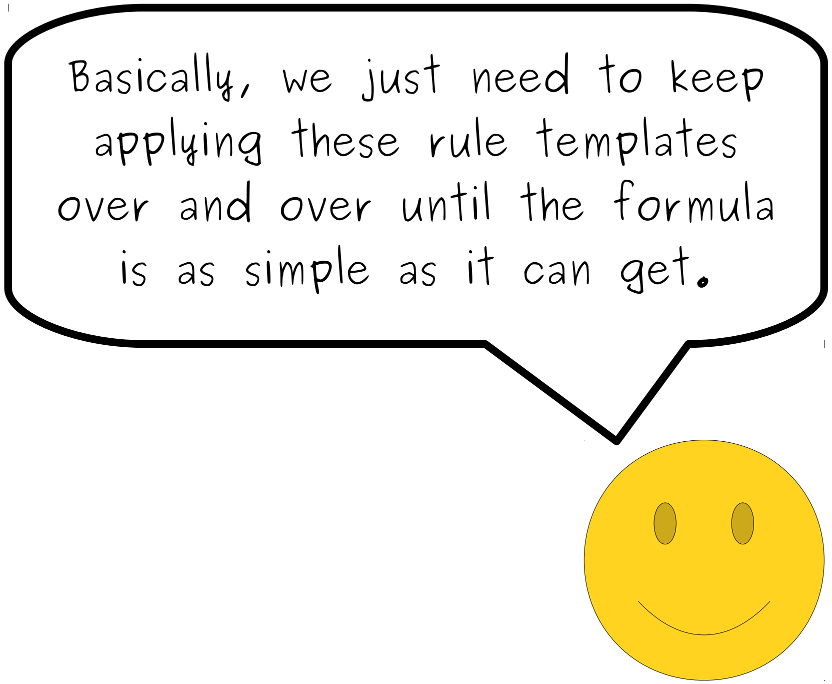
$$\neg\neg p \lor \neg(q \lor r)$$



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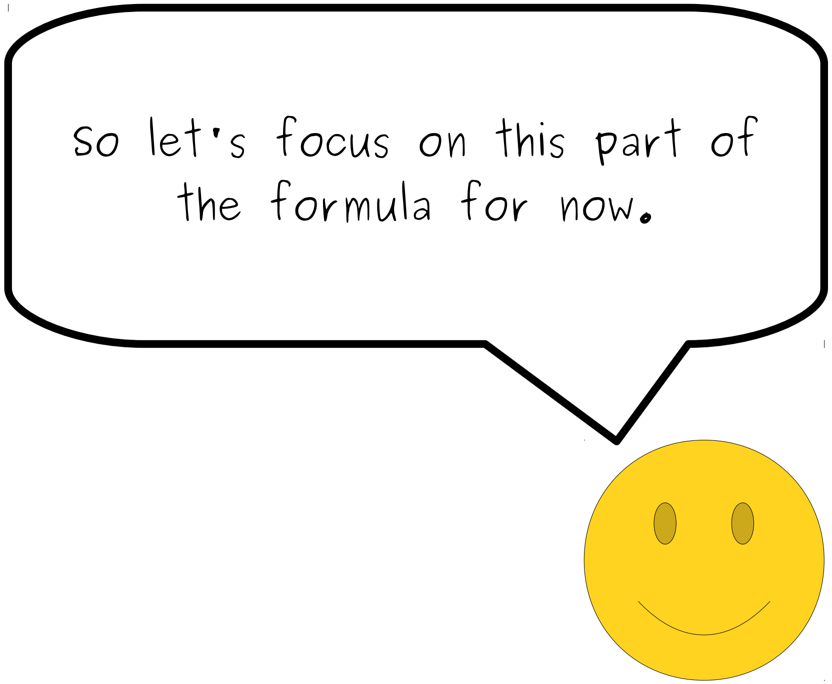

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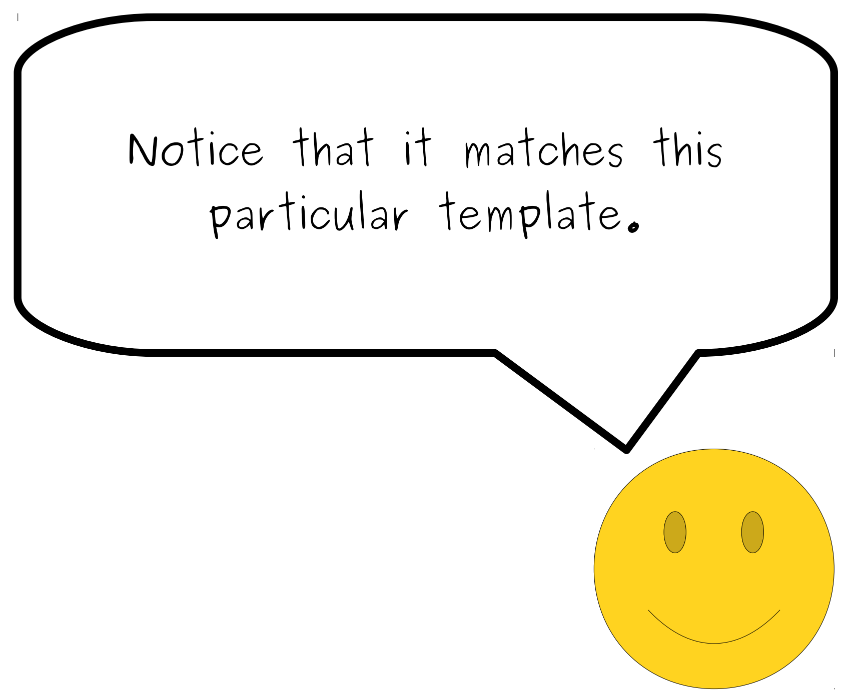

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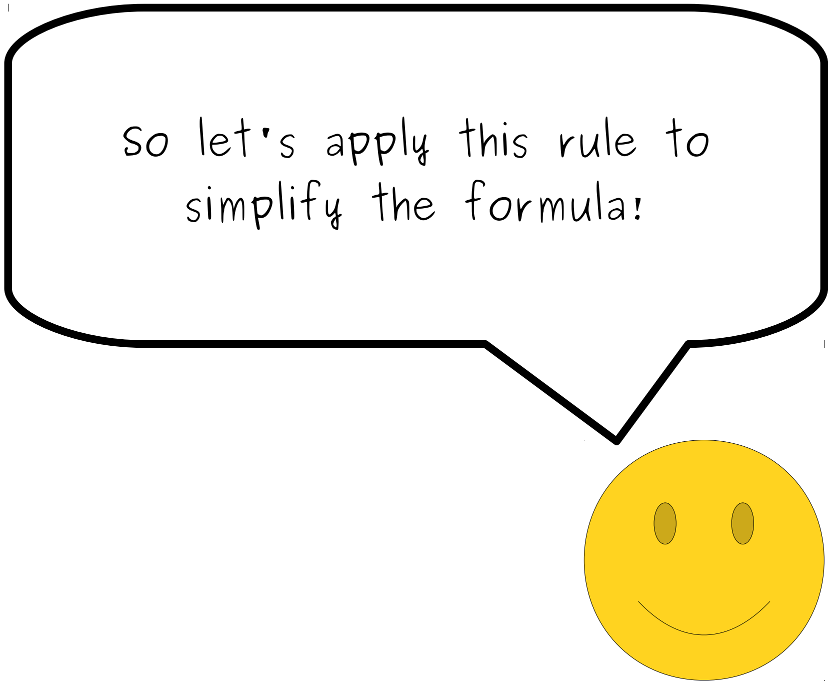
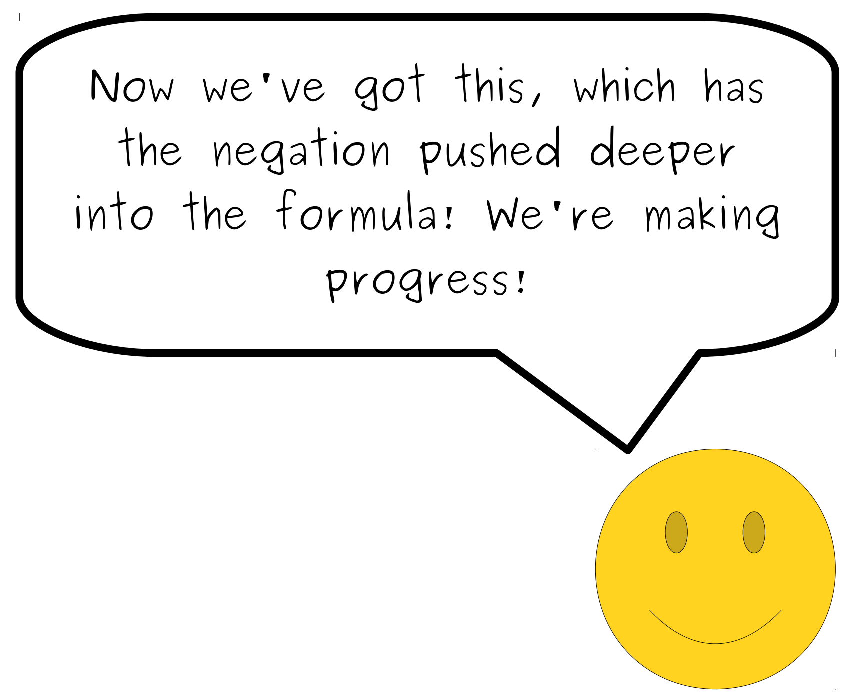


$$\frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\neg\neg p \lor (\neg q \land \neg r)$$



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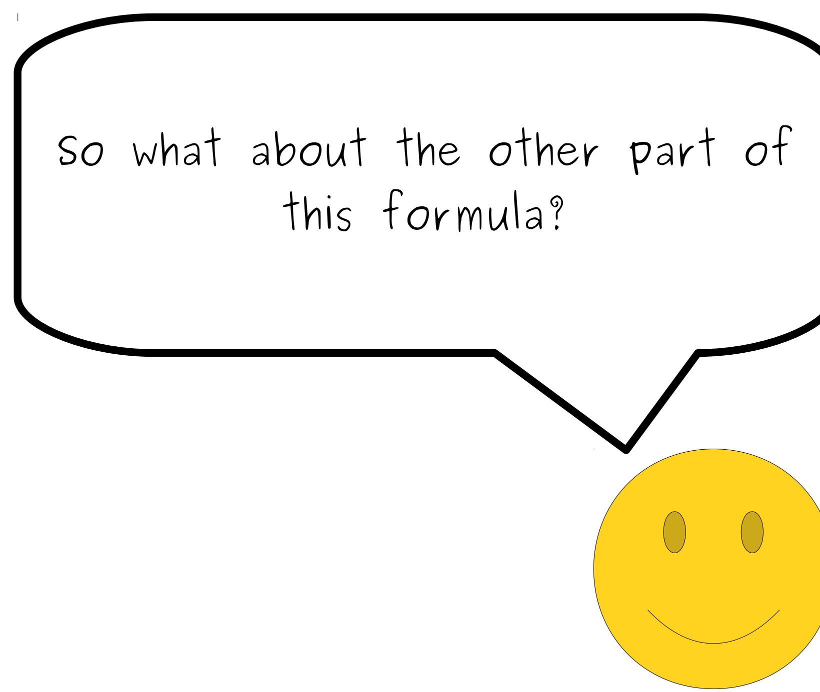

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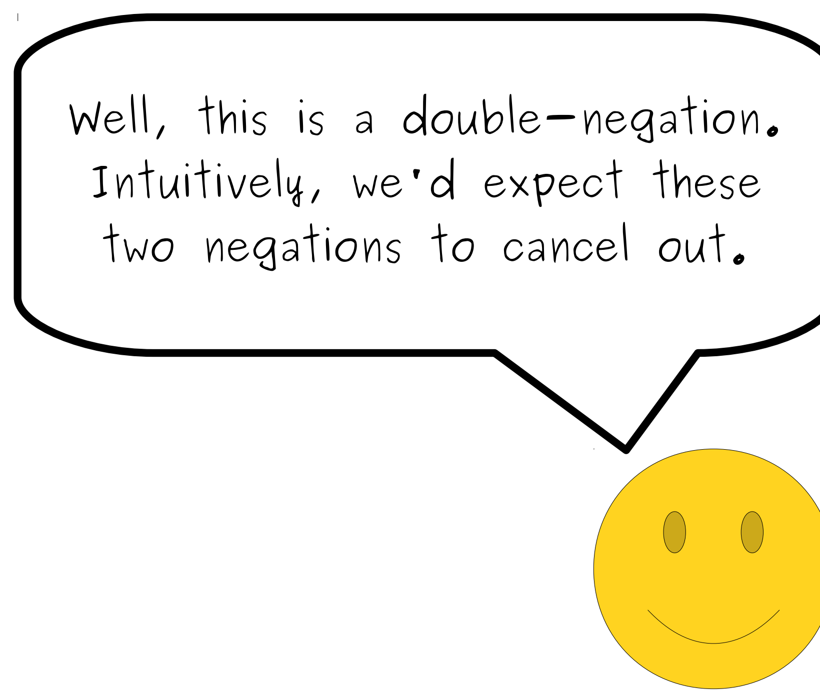

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B}$$

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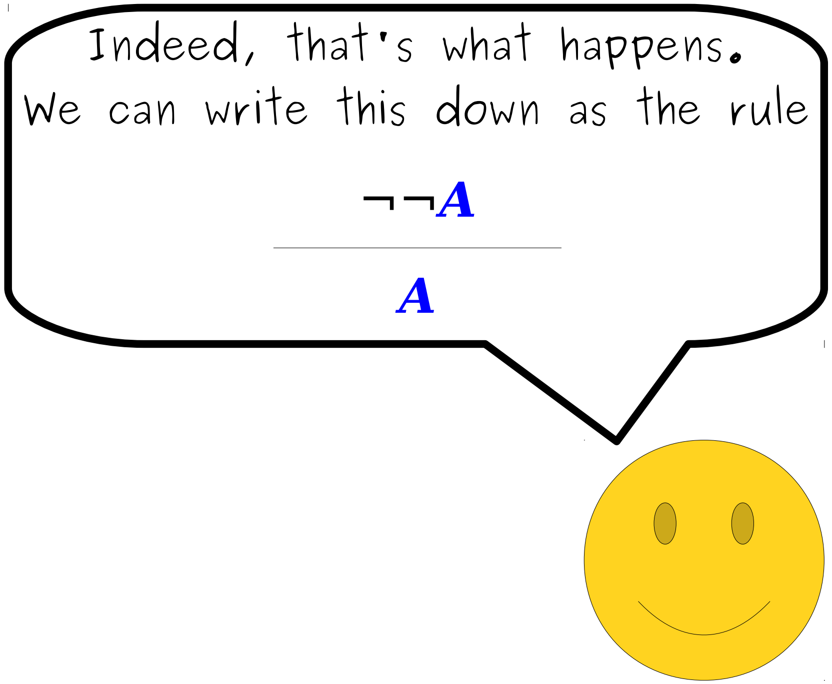

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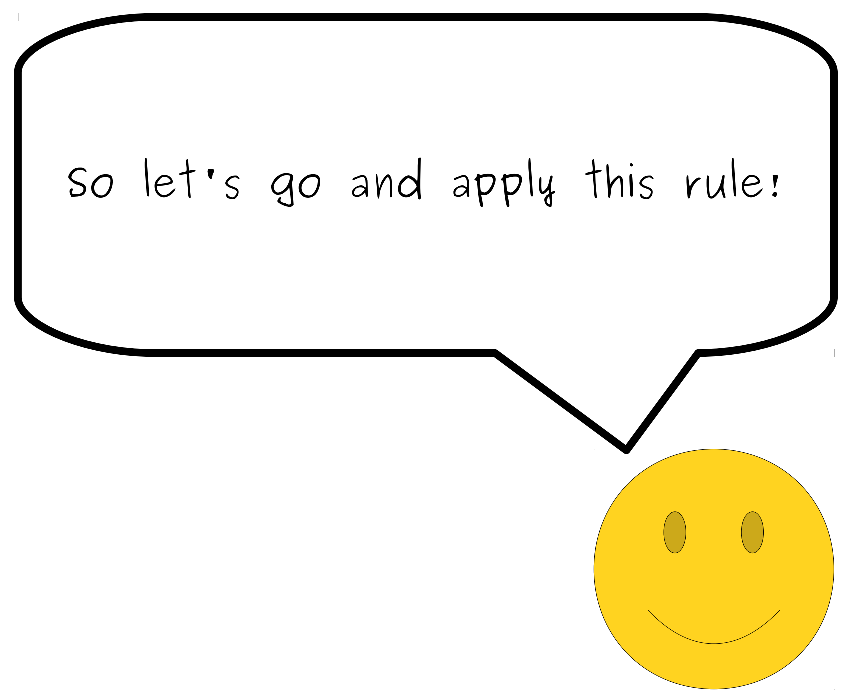

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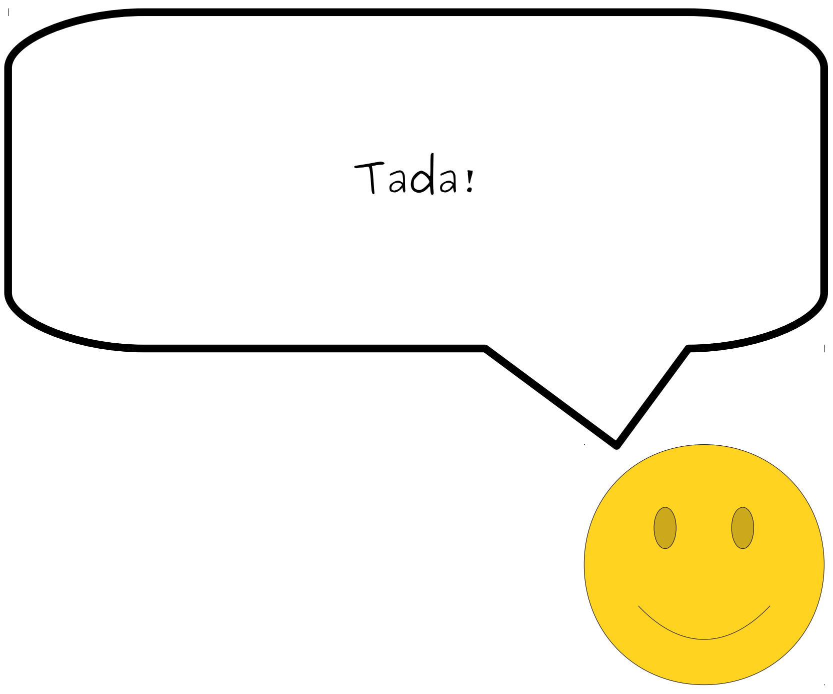

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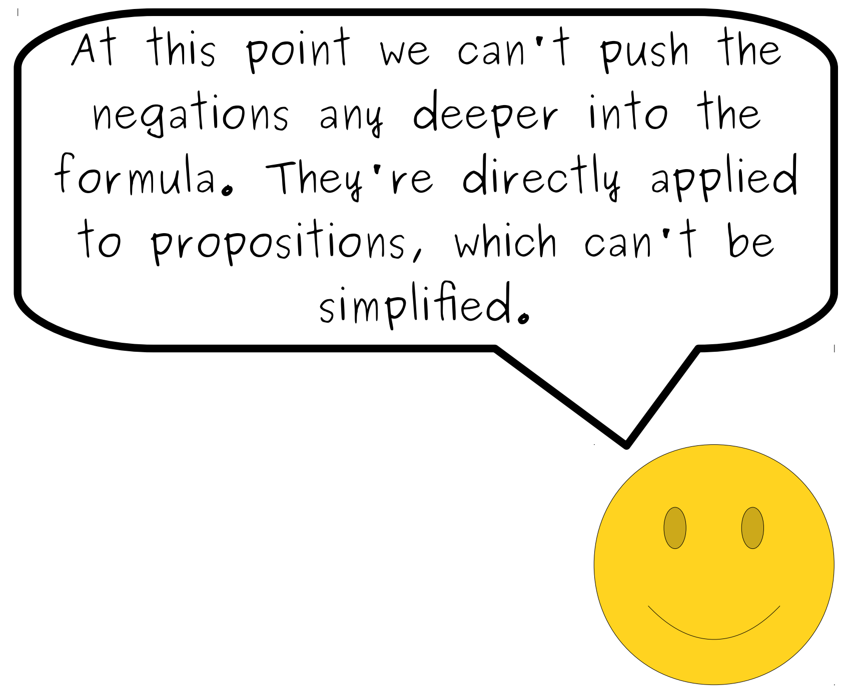

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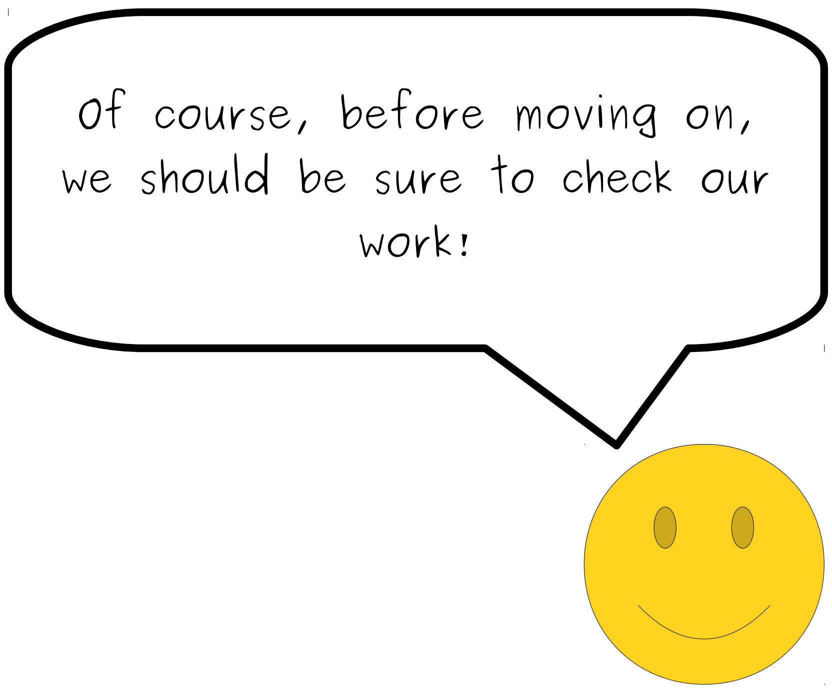

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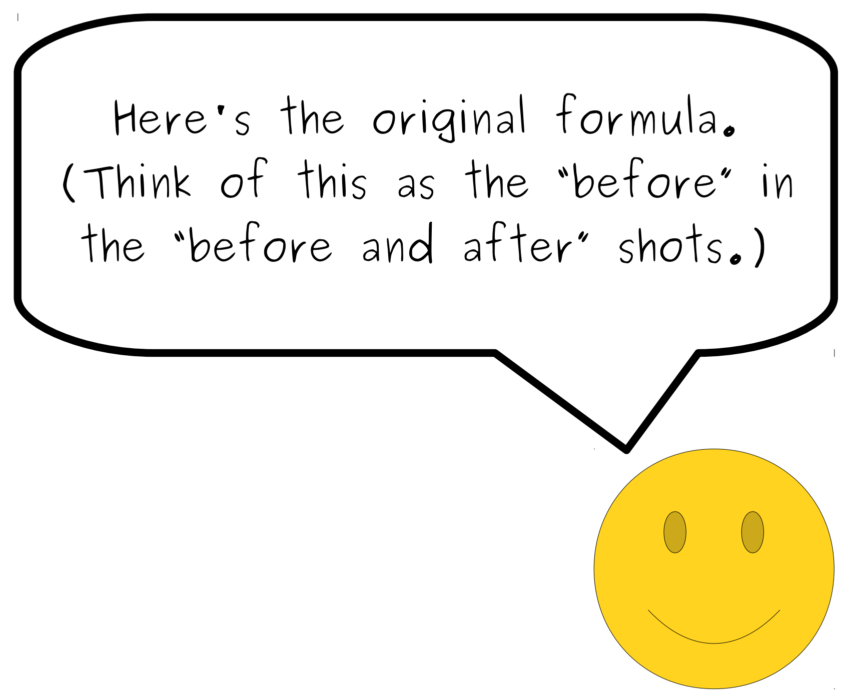

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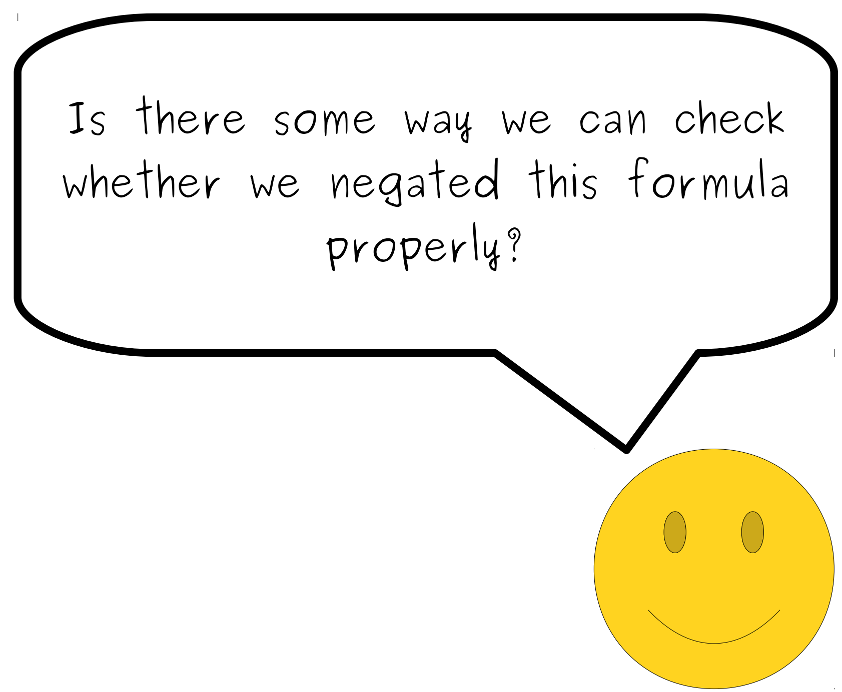

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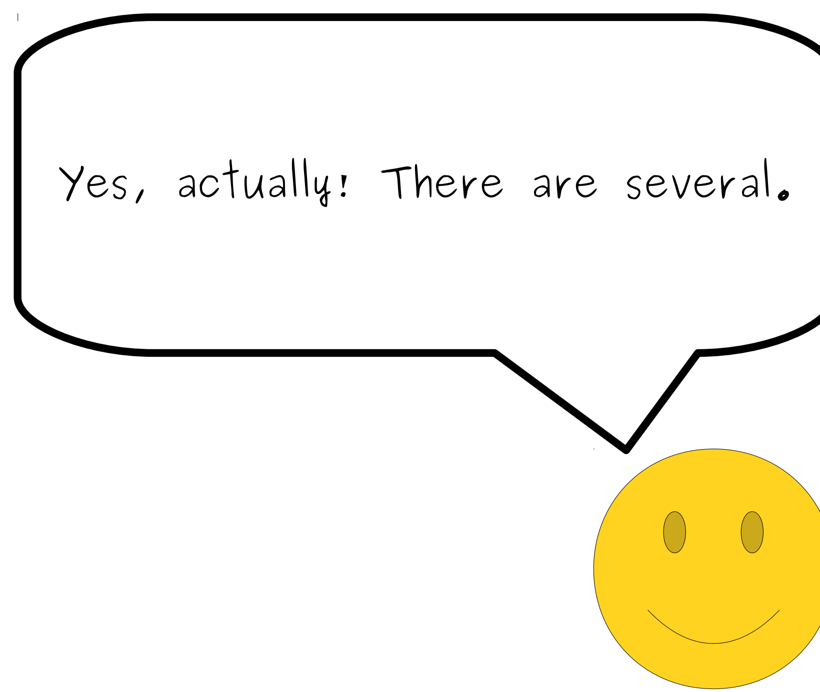
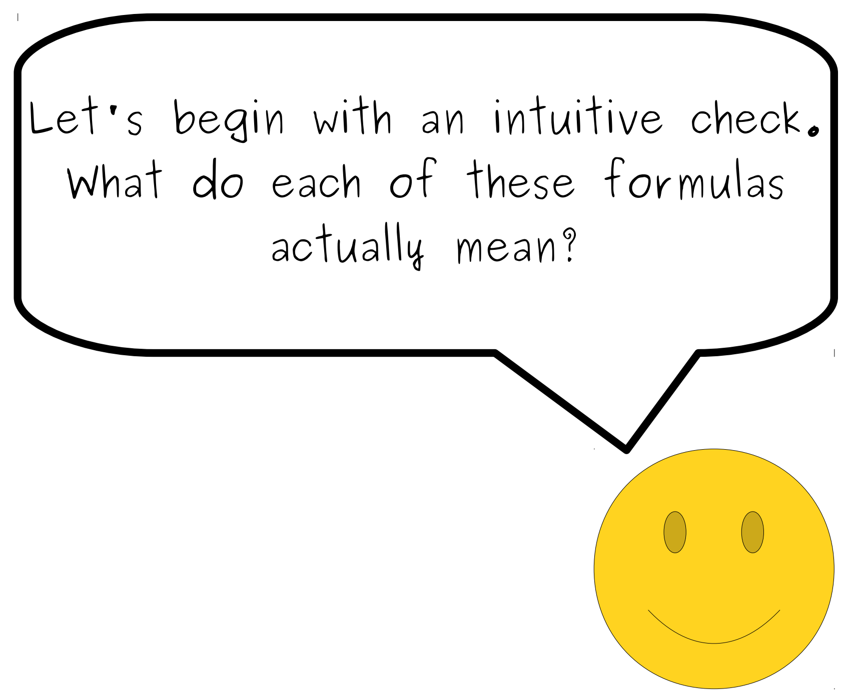


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B}$$

$$p \vee (\neg q \wedge \neg r)$$

$$\neg p \wedge (q \vee r)$$



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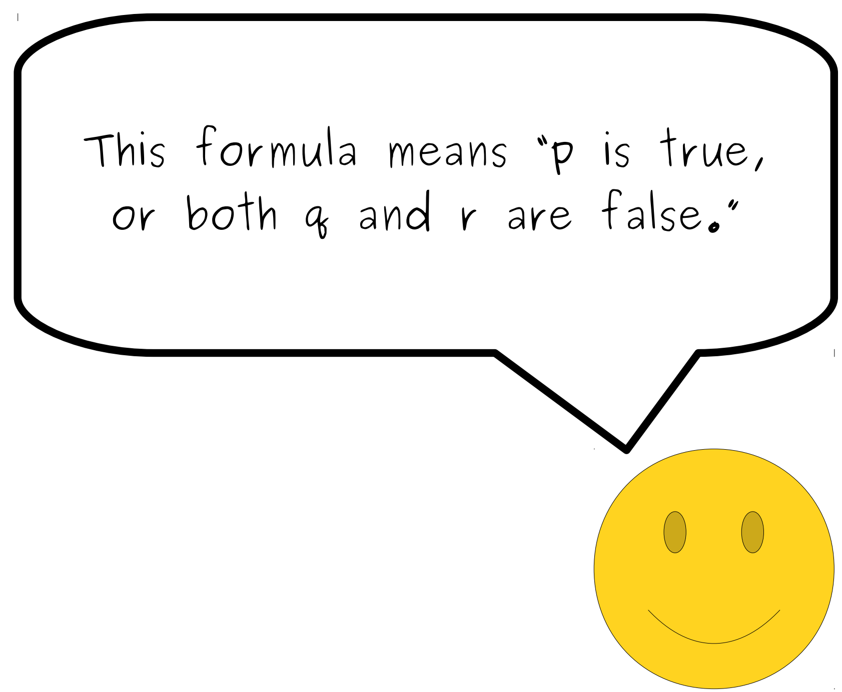

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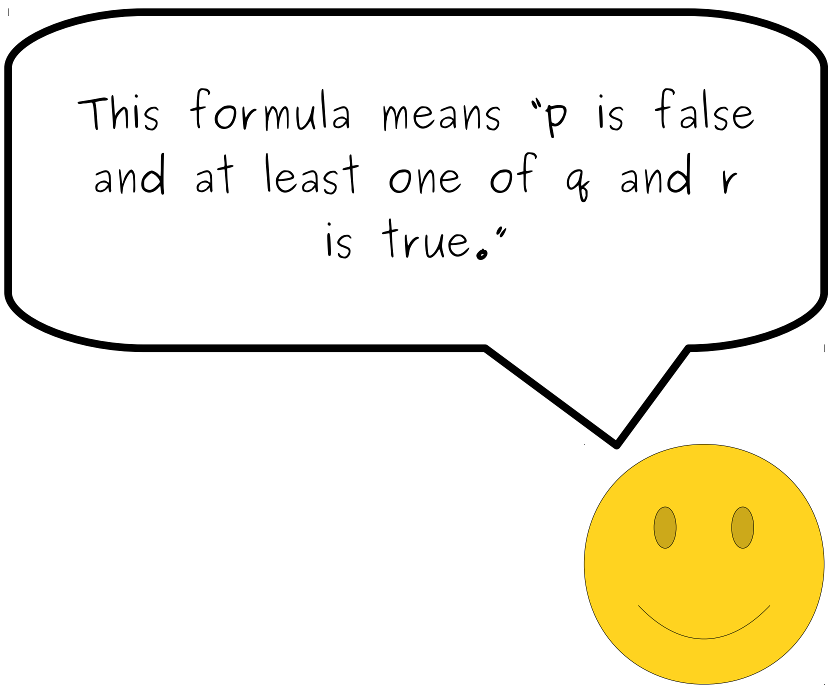

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$$\frac{\neg\neg \mathbf{A}}{\mathbf{A}} \qquad \frac{\neg(\mathbf{A} \vee \mathbf{B})}{\neg \mathbf{A} \wedge \neg \mathbf{B}} \qquad \frac{\neg(\mathbf{A} \wedge \mathbf{B})}{\neg \mathbf{A} \vee \neg \mathbf{B}}$$

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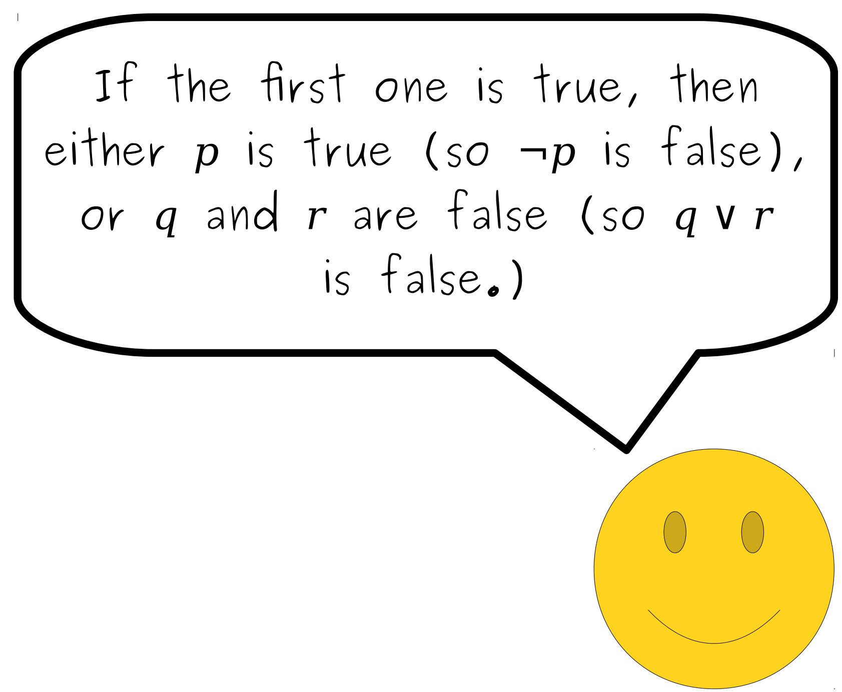

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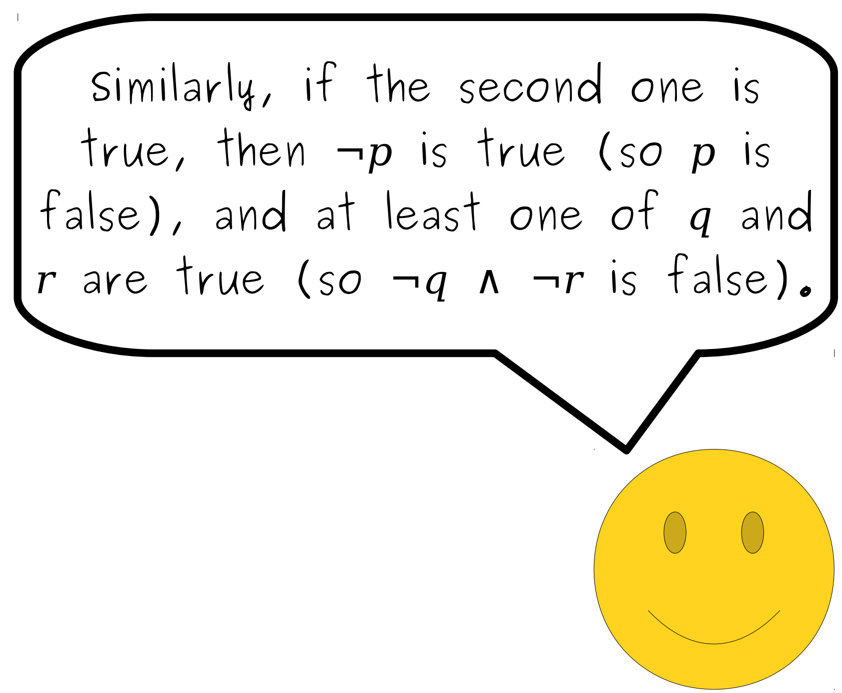

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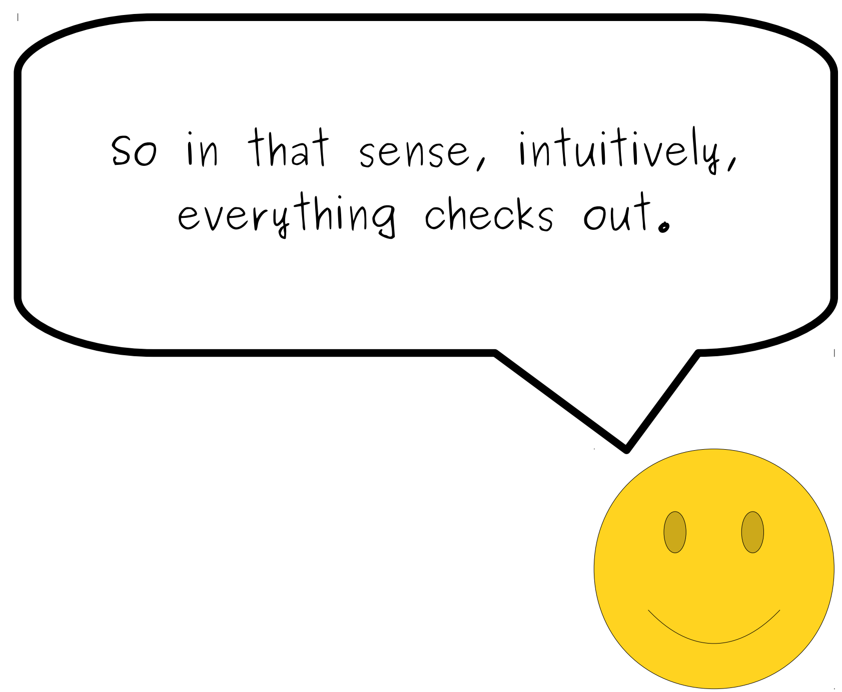

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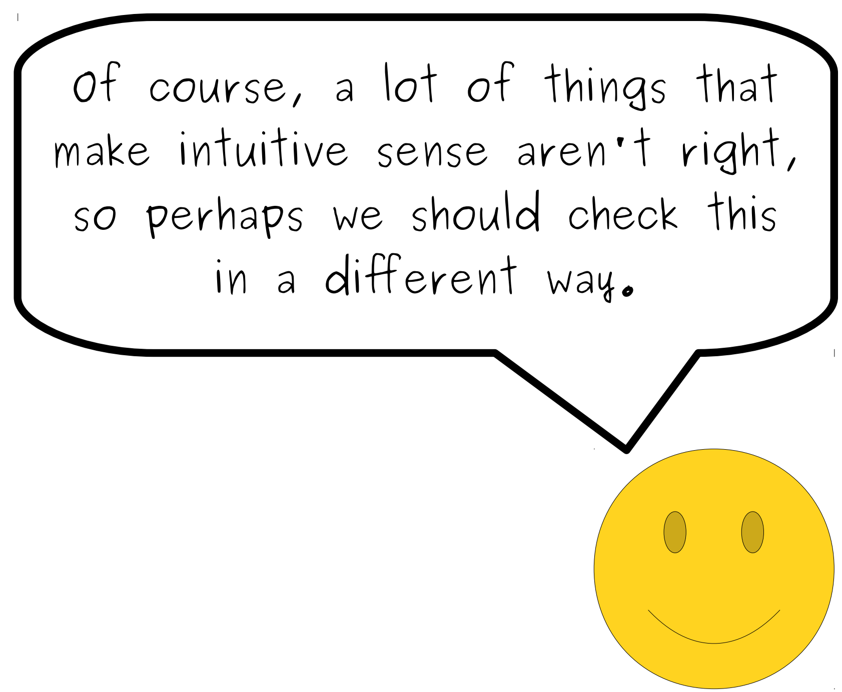

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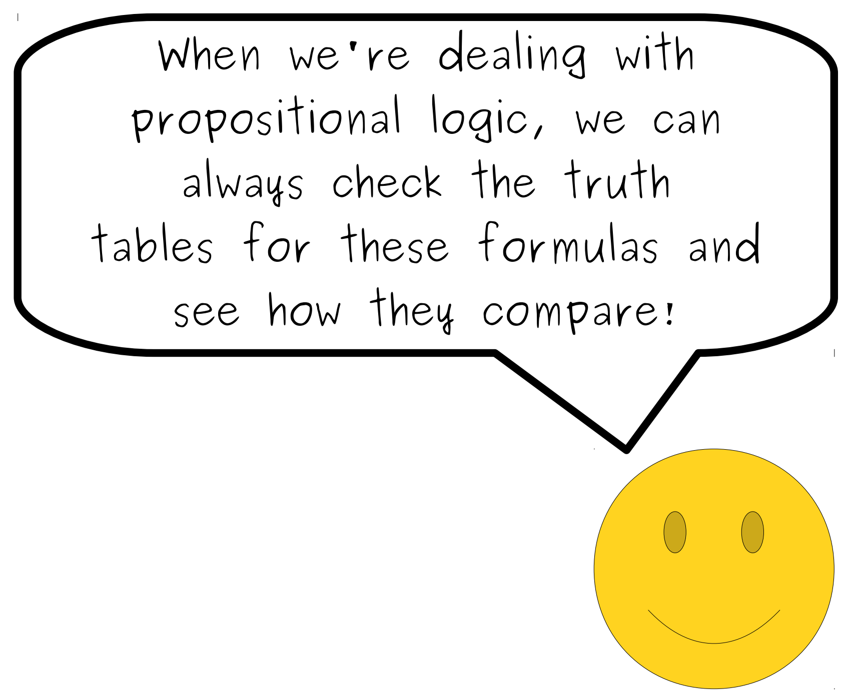


$$\frac{\neg\neg A}{A}$$

p ∨ (¬q ∧ ¬r)

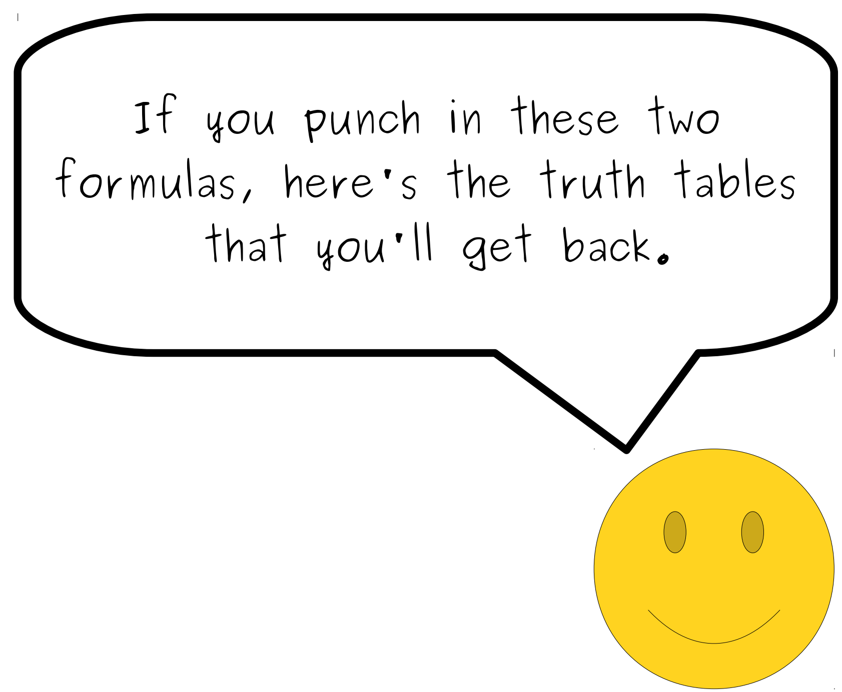
p	q	r	(p ∨ (¬q ∧ ¬r))
F	F	F	**T**
F	F	T	**F**
F	T	F	**F**
F	T	T	**F**
T	F	F	**T**
T	F	T	**T**
T	T	F	**T**
T	T	T	**T**

¬p ∧ (q ∨ r)

p	q	r	(¬p ∧ (q ∨ r))
F	F	F	**F**
F	F	T	**T**
F	T	F	**T**
F	T	T	**T**
T	F	F	**F**
T	F	T	**F**
T	T	F	**F**
T	T	T	**F**

$$p \vee (\neg q \wedge \neg r)$$

$$\neg p \wedge (q \vee r)$$



$$\frac{\neg\neg A}{A}$$

$p \lor (\neg q \land \neg r)$

p	q	r	(p ∨ (¬q ∧ ¬r))
F	F	F	**T**
F	F	T	**F**
F	T	F	**F**
F	T	T	**F**
T	F	F	**T**
T	F	T	**T**
T	T	F	**T**
T	T	T	**T**

$\neg p \land (q \lor r)$

p	q	r	(¬p ∧ (q ∨ r))
F	F	F	**F**
F	F	T	**T**
F	T	F	**T**
F	T	T	**T**
T	F	F	**F**
T	F	T	**F**
T	T	F	**F**
T	T	T	**F**

$$p \lor (\neg q \land \neg r)$$

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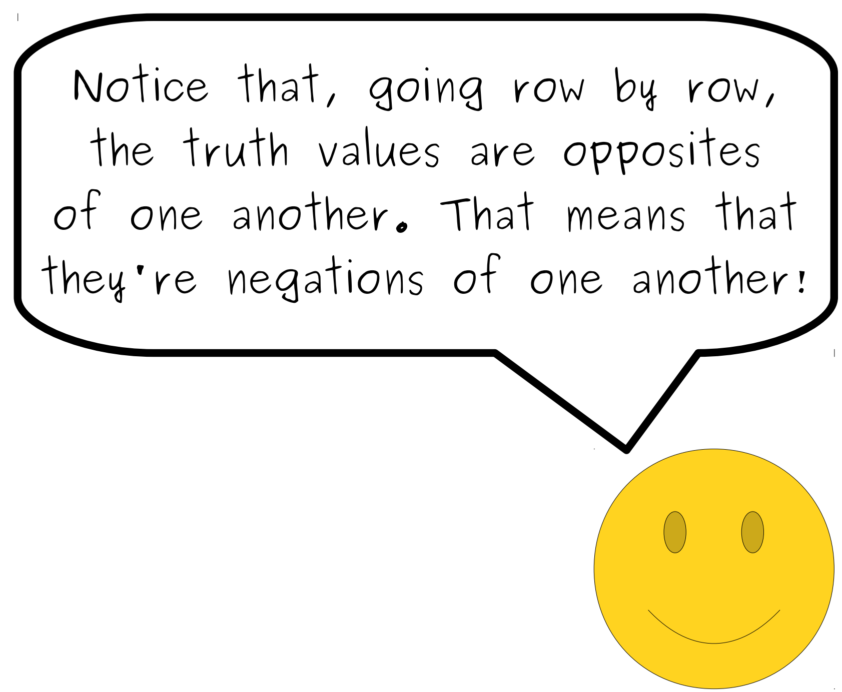

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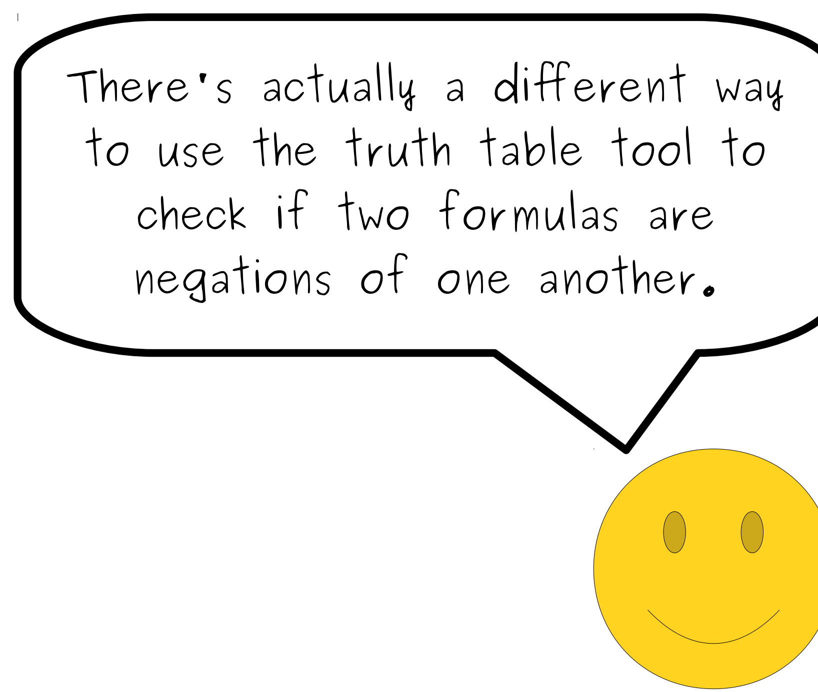
$$p \lor (\neg q \land \neg r)$$

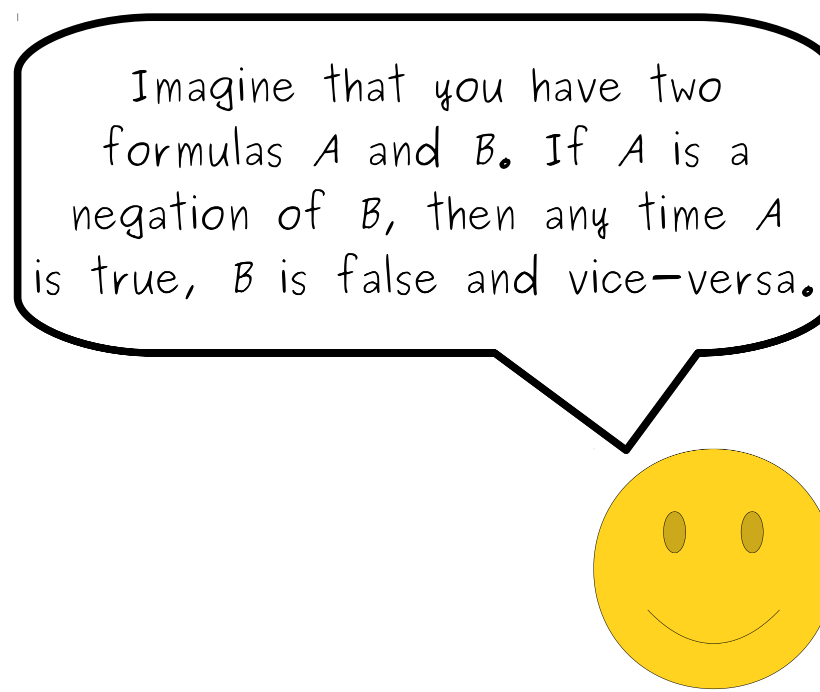
$$\neg p \land (q \lor r)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$p \lor (\neg q \land \neg r)$$

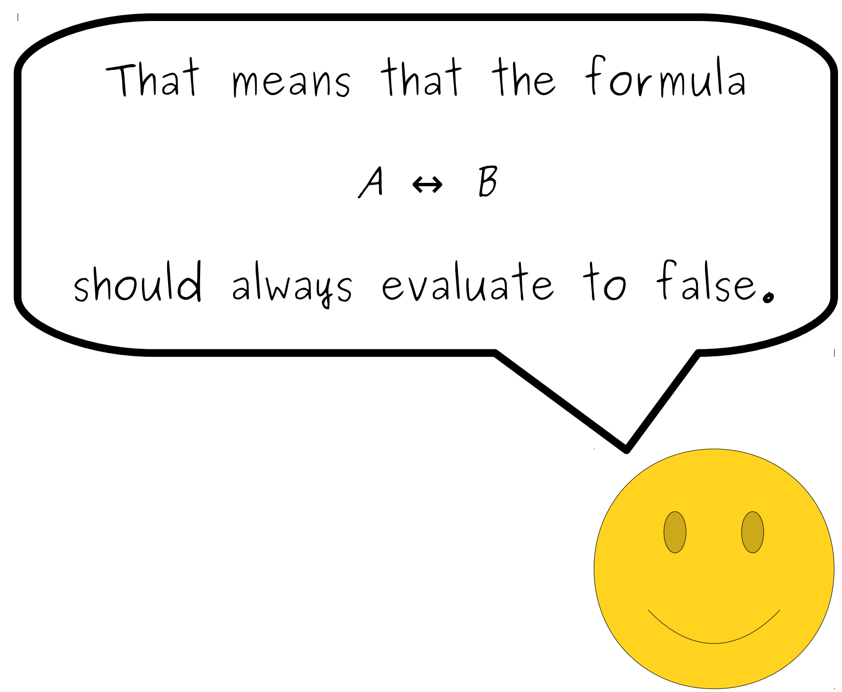
$$\neg p \land (q \lor r)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$p \lor (\neg q \land \neg r)$$

$$\neg p \land (q \lor r)$$

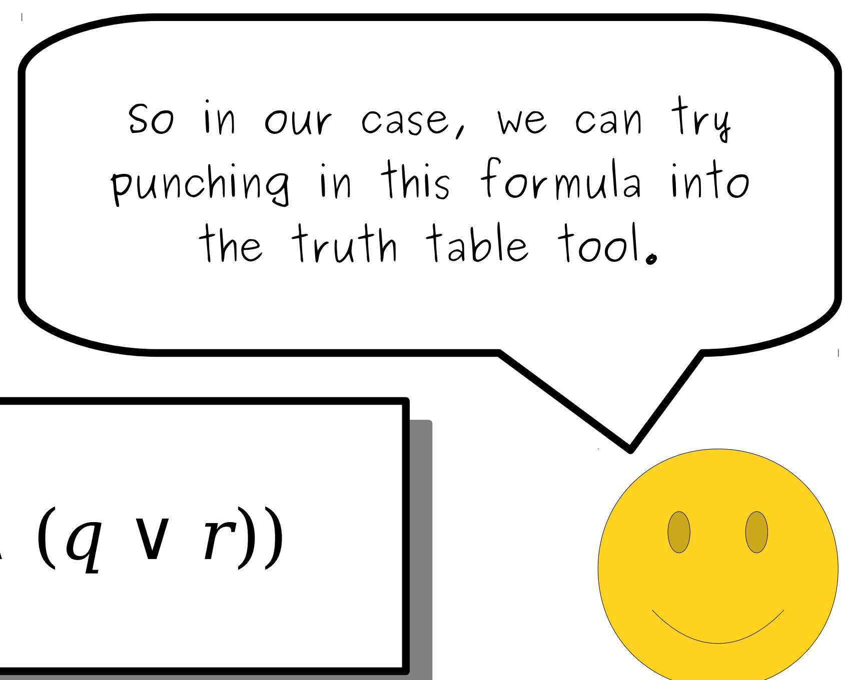


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$p \lor (\neg q \land \neg r)$$

$$\neg p \land (q \lor r)$$

$$(p \lor (\neg q \land \neg r)) \quad \leftrightarrow \quad (\neg p \land (q \lor r))$$



$$\frac{\neg\neg A}{A}$$

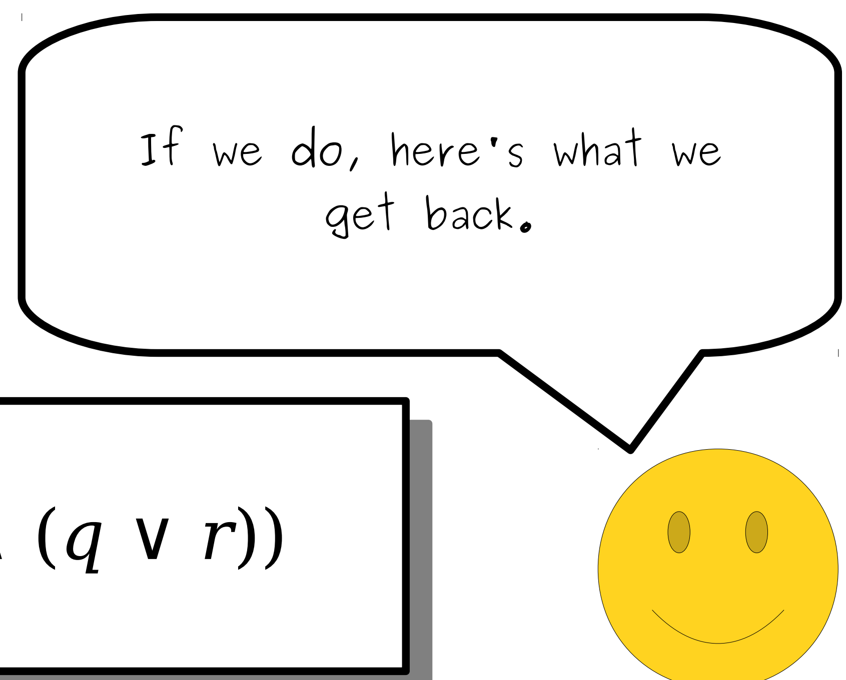
$$\frac{\neg(A \lor B)}{\neg A}$$

$$\neg(A \land B)$$

p	q	r	((p ∨ (¬q ∧ ¬r)) ↔ (¬p ∧ (q ∨ r)))
F	F	F	**F**
F	F	T	**F**
F	T	F	**F**
F	T	T	**F**
T	F	F	**F**
T	F	T	**F**
T	T	F	**F**
T	T	T	**F**

$$p \lor (\neg q \land \neg r)$$

$$\neg p \land (q \lor r)$$



$$(p \lor (\neg q \land \neg r)) \quad \leftrightarrow \quad (\neg p \land (q \lor r))$$

$$\frac{\neg\neg A}{A}$$

$$\frac{\neg(A \lor B)}{\neg A}$$

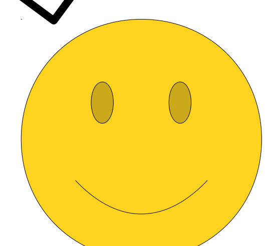
$$\neg(A \land B)$$

p	q	r	((p ∨ (¬q ∧ ¬r)) ↔ (¬p ∧ (q ∨ r)))
F	F	F	**F**
F	F	T	**F**
F	T	F	**F**
F	T	T	**F**
T	F	F	**F**
T	F	T	**F**
T	T	F	**F**
T	T	T	**F**

$$p \lor (\neg q \land \neg r)$$

$$\neg p \land (q \lor r)$$

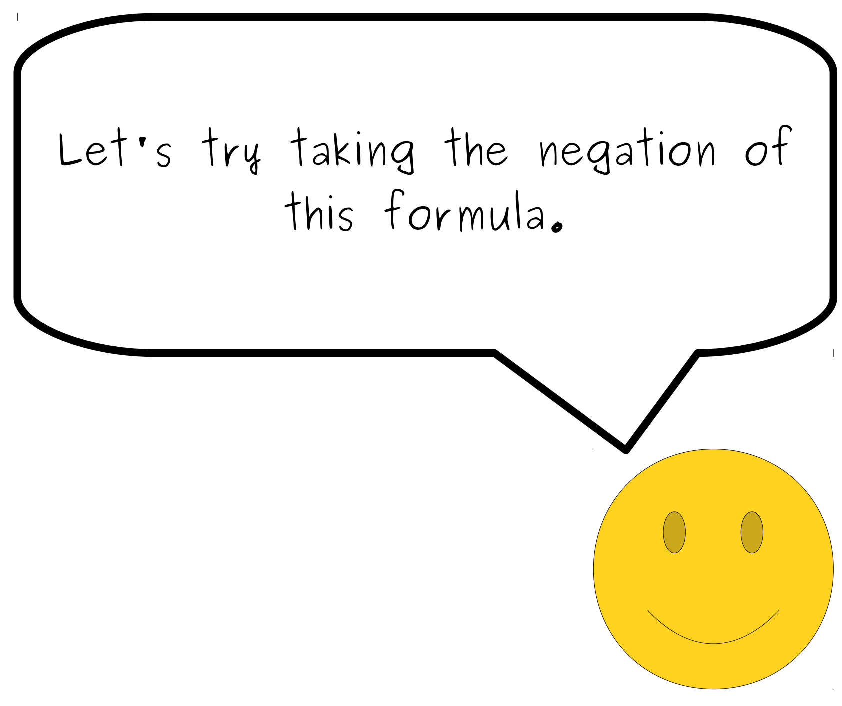
$$(p \lor (\neg q \land \neg r)) \quad \leftrightarrow \quad (\neg p \land (q \lor r))$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$(p \lor q) \land (\neg p \land \neg q)$$

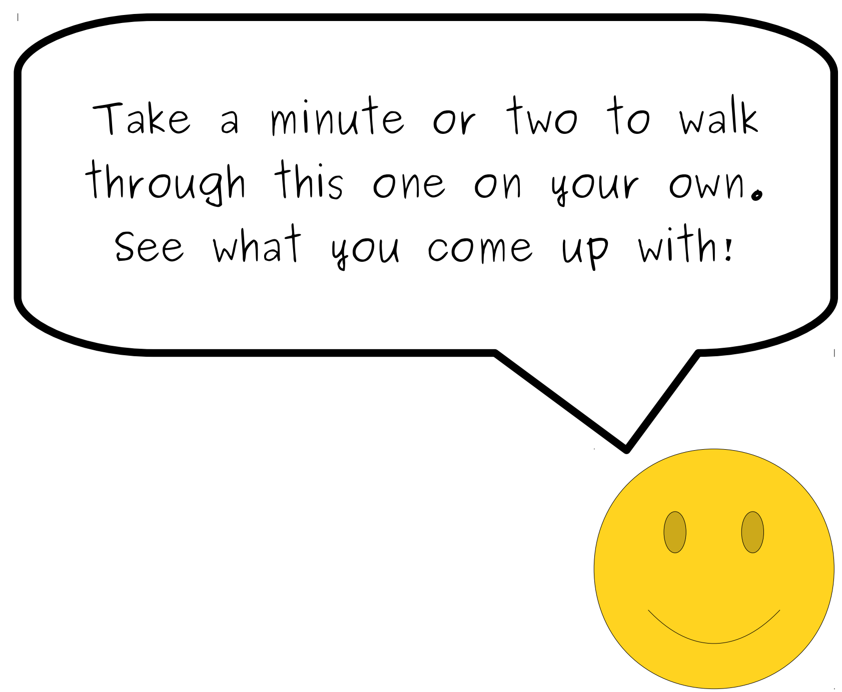


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B}$$

$$(p \vee q) \wedge (\neg p \wedge \neg q)$$

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$(p \lor q) \land (\neg p \land \neg q)$$

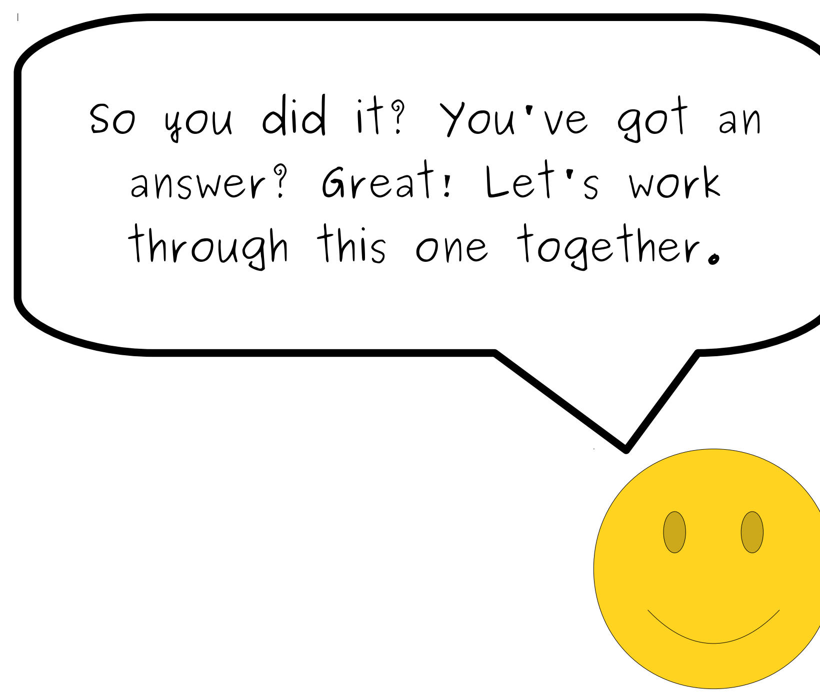


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B}$$

$$(p \vee q) \wedge (\neg p \wedge \neg q)$$

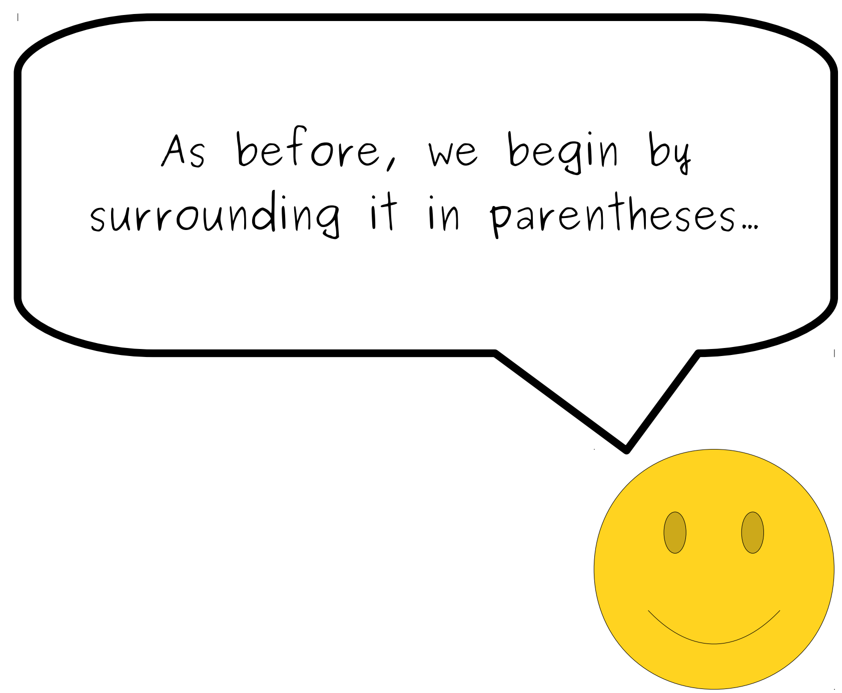
$$\frac{\neg\neg \boldsymbol{A}}{\boldsymbol{A}} \qquad \frac{\neg(\boldsymbol{A} \lor \boldsymbol{B})}{\neg \boldsymbol{A} \land \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\neg \boldsymbol{A} \lor \neg \boldsymbol{B}}$$

$$(p \lor q) \land (\neg p \land \neg q)$$



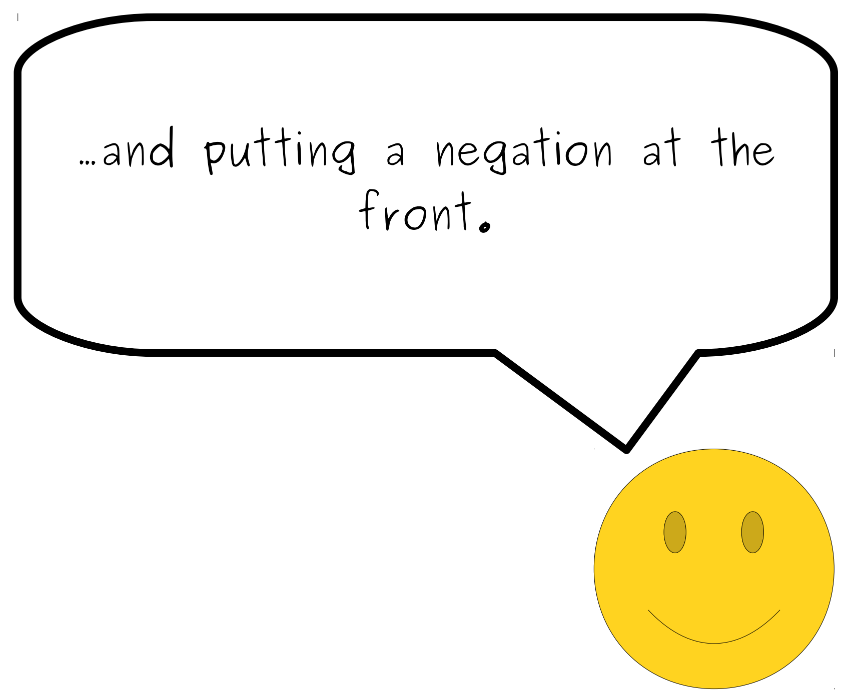
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$((p \lor q) \land (\neg p \land \neg q))$$



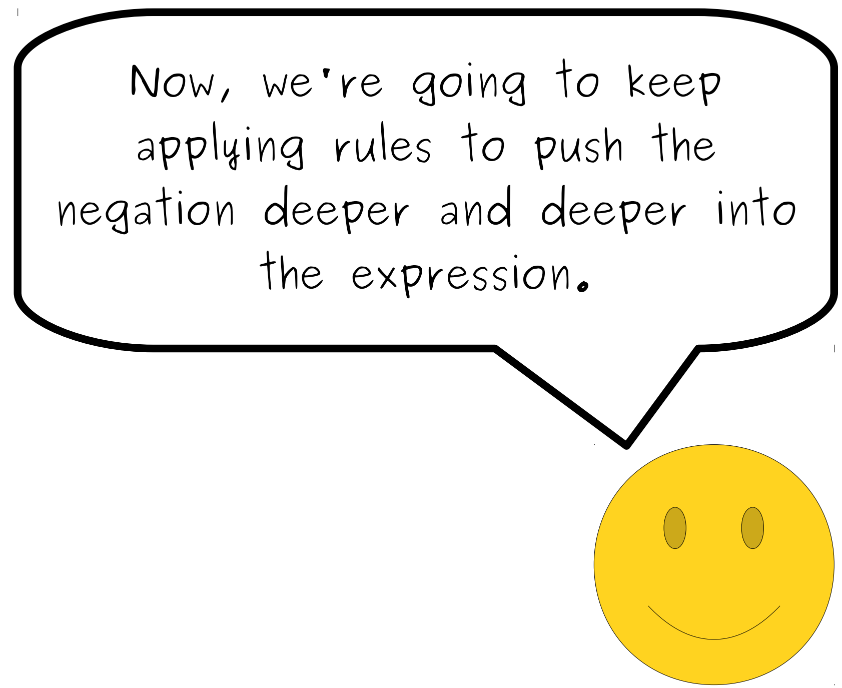
$$\frac{\neg\neg \boldsymbol{A}}{\boldsymbol{A}} \qquad \frac{\neg(\boldsymbol{A} \lor \boldsymbol{B})}{\neg \boldsymbol{A} \land \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\neg \boldsymbol{A} \lor \neg \boldsymbol{B}}$$

$$\neg((p \lor q) \land (\neg p \land \neg q))$$



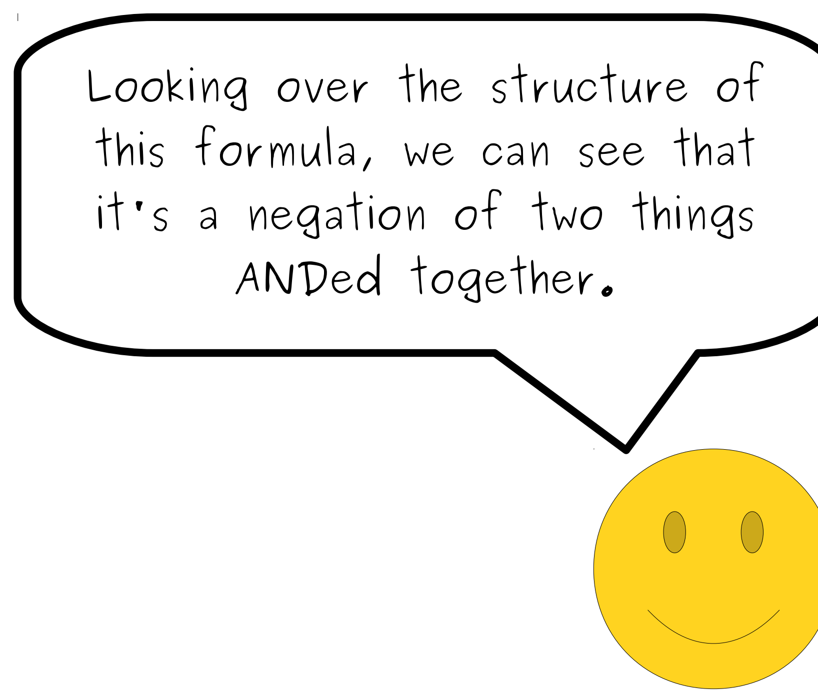
$$\frac{\neg\neg \mathbf{A}}{\mathbf{A}} \qquad \frac{\neg(\mathbf{A} \lor \mathbf{B})}{\neg \mathbf{A} \land \neg \mathbf{B}} \qquad \frac{\neg(\mathbf{A} \land \mathbf{B})}{\neg \mathbf{A} \lor \neg \mathbf{B}}$$

$$\neg((p \lor q) \land (\neg p \land \neg q))$$



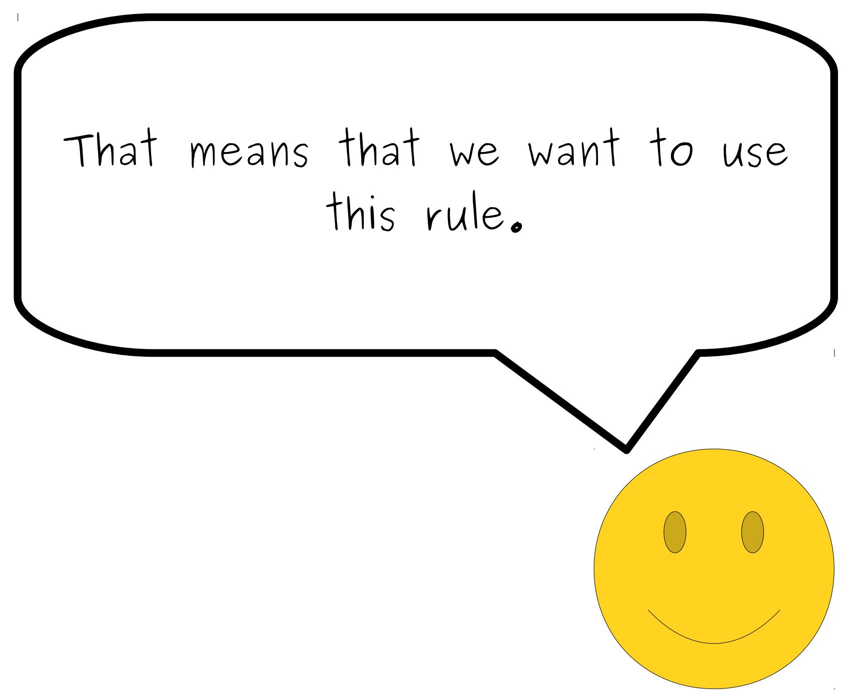
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\neg((p \lor q) \land (\neg p \land \neg q))$$



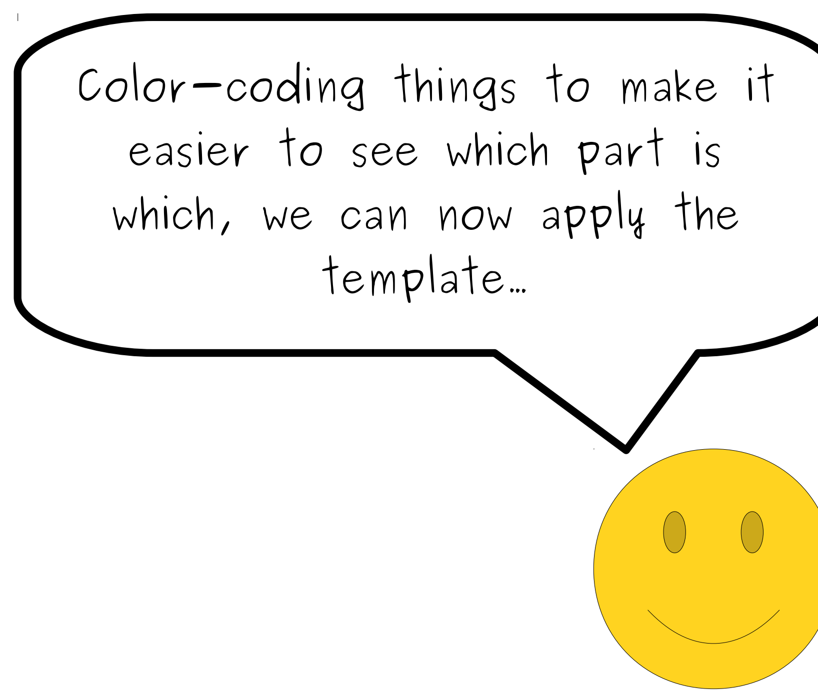
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\neg((p \lor q) \land (\neg p \land \neg q))$$



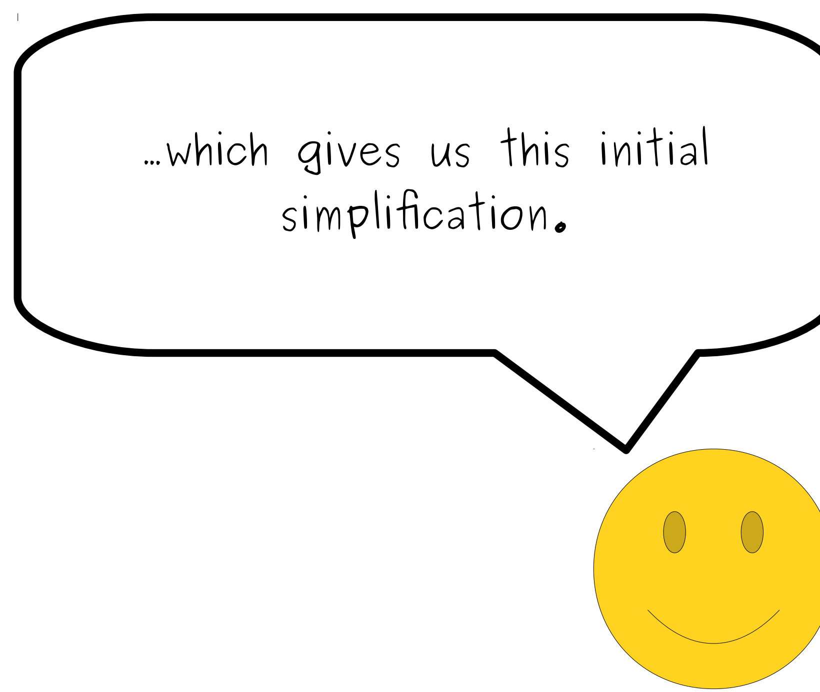
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\neg((p \lor q) \land (\neg p \land \neg q))$$



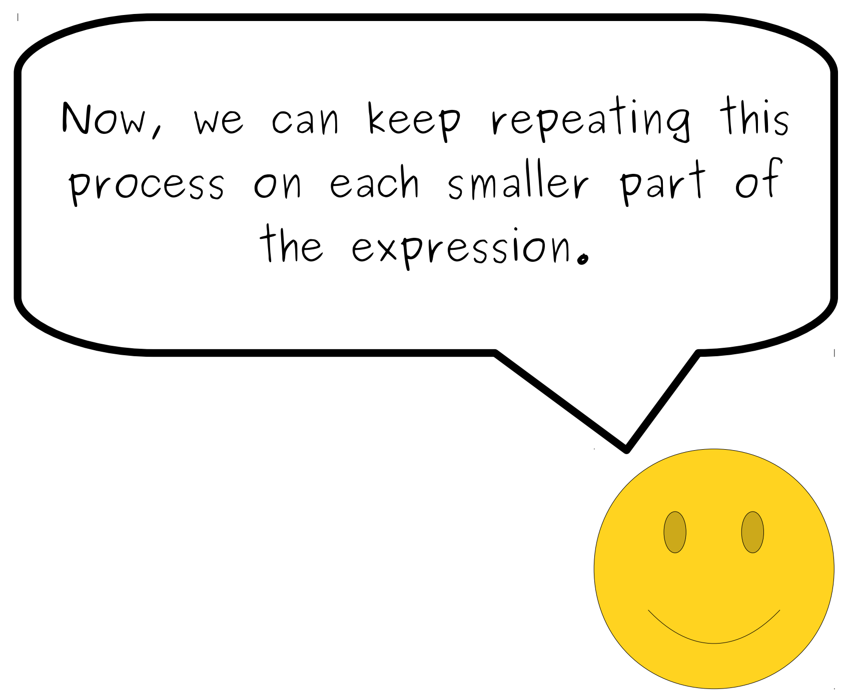
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B}$$

$$\neg(p \vee q) \vee \neg(\neg p \wedge \neg q)$$



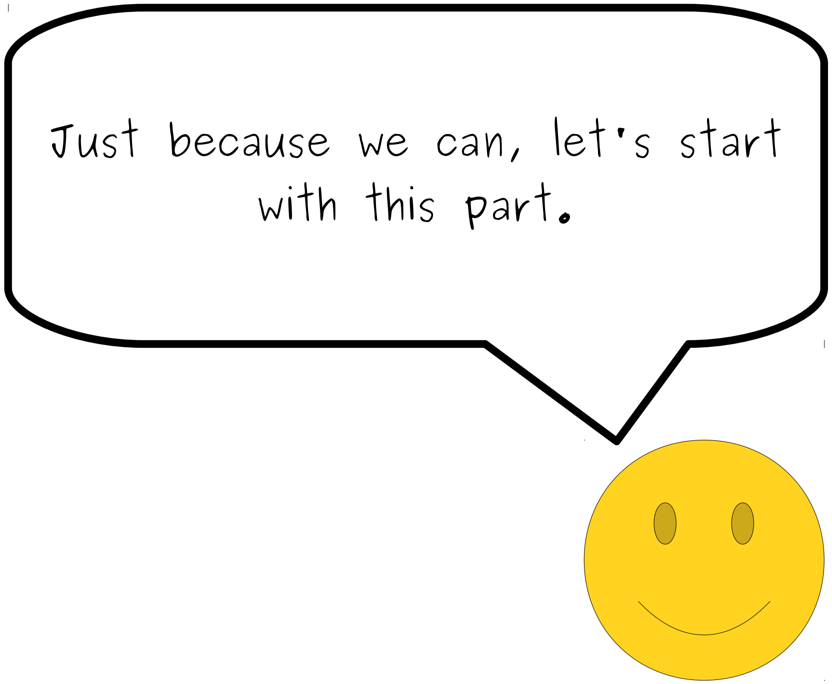
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\neg(p \lor q) \lor \neg(\neg p \land \neg q)$$



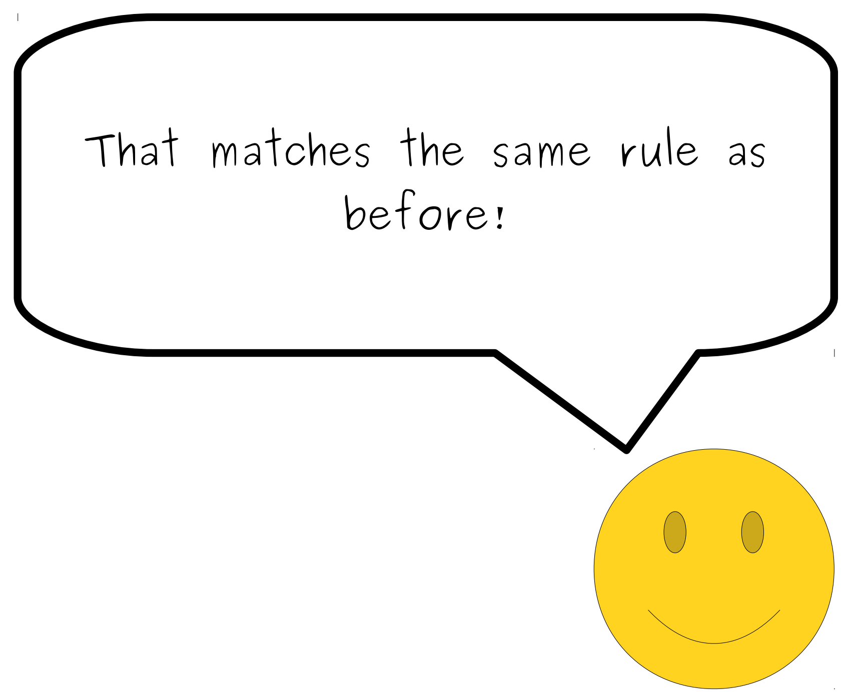
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B}$$

$$\neg(p \vee q) \vee \neg(\neg p \wedge \neg q)$$



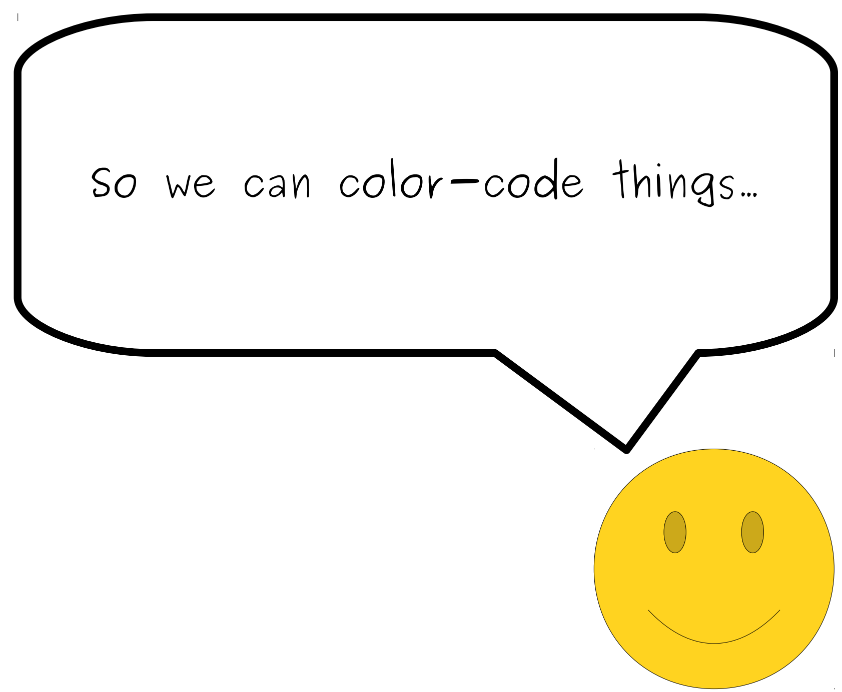
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\neg(p \lor q) \lor \neg(\neg p \land \neg q)$$



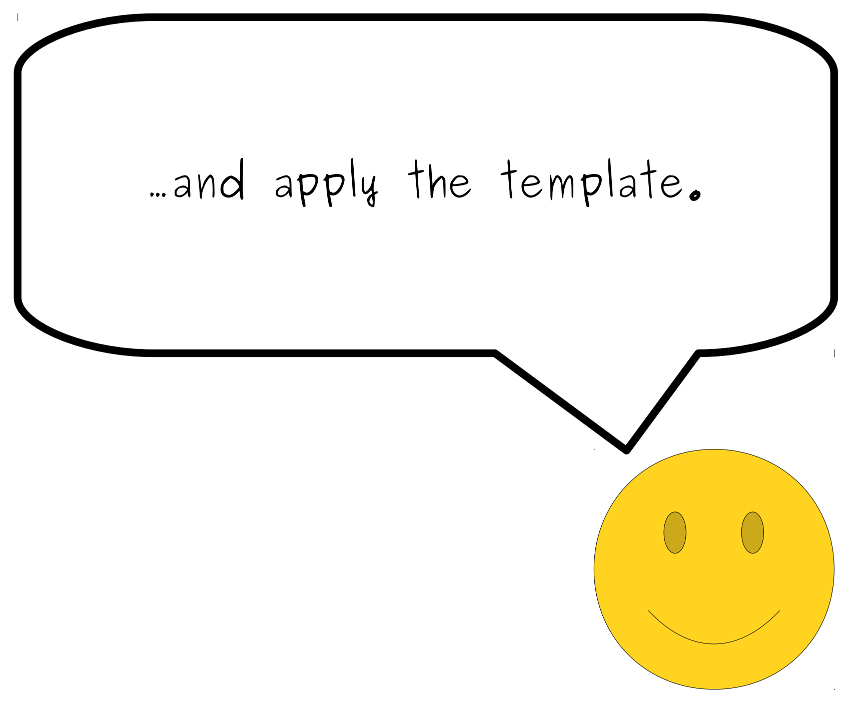
$$\frac{\neg\neg \boldsymbol{A}}{\boldsymbol{A}} \qquad \frac{\neg(\boldsymbol{A} \lor \boldsymbol{B})}{\neg \boldsymbol{A} \land \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\neg \boldsymbol{A} \lor \neg \boldsymbol{B}}$$

$$\neg(p \lor q) \lor \neg(\neg p \land \neg q)$$



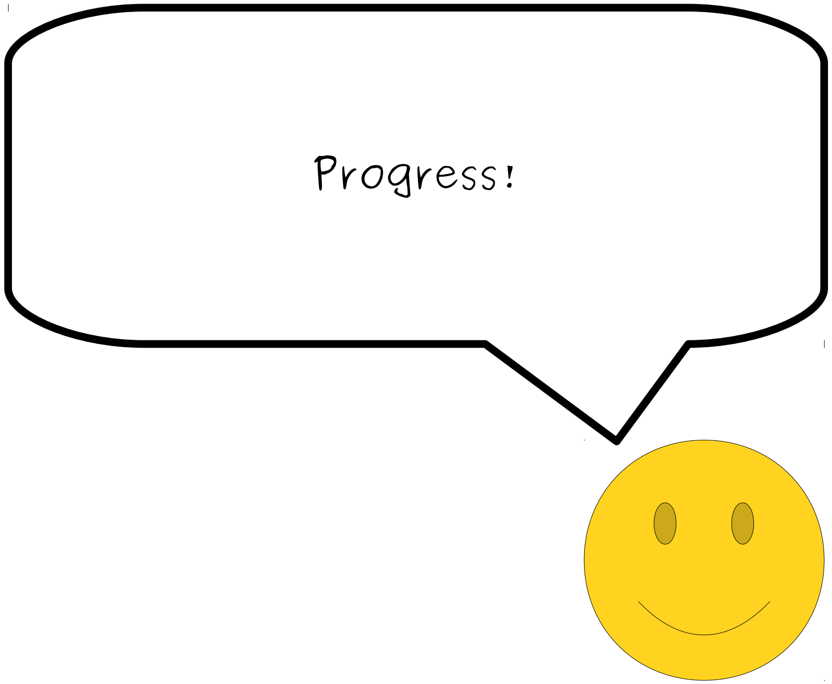
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\neg(p \lor q) \lor (\neg\neg p \lor \neg\neg q)$$



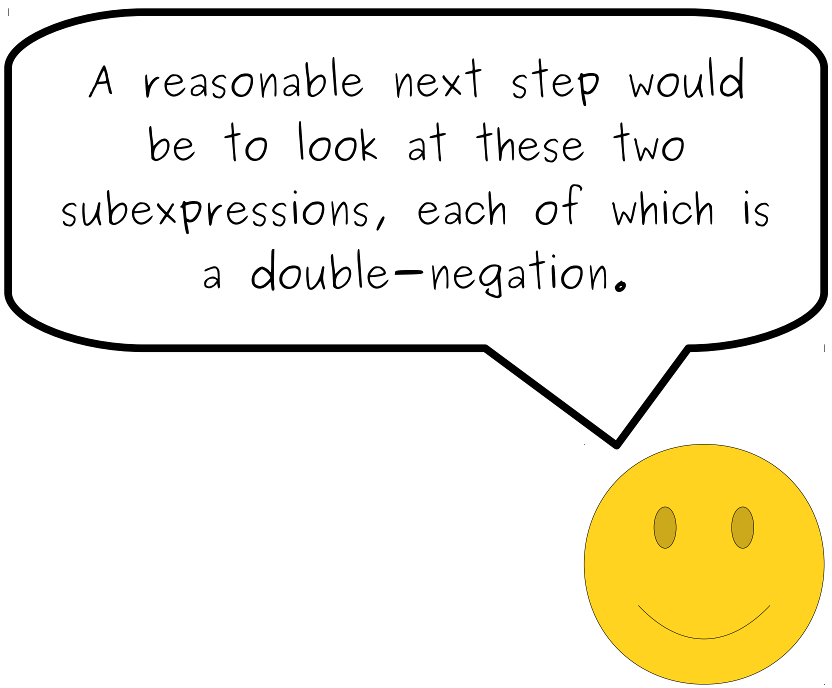
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\neg(p \lor q) \lor (\neg\neg p \lor \neg\neg q)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\neg(p \lor q) \lor (\neg\neg p \lor \neg\neg q)$$

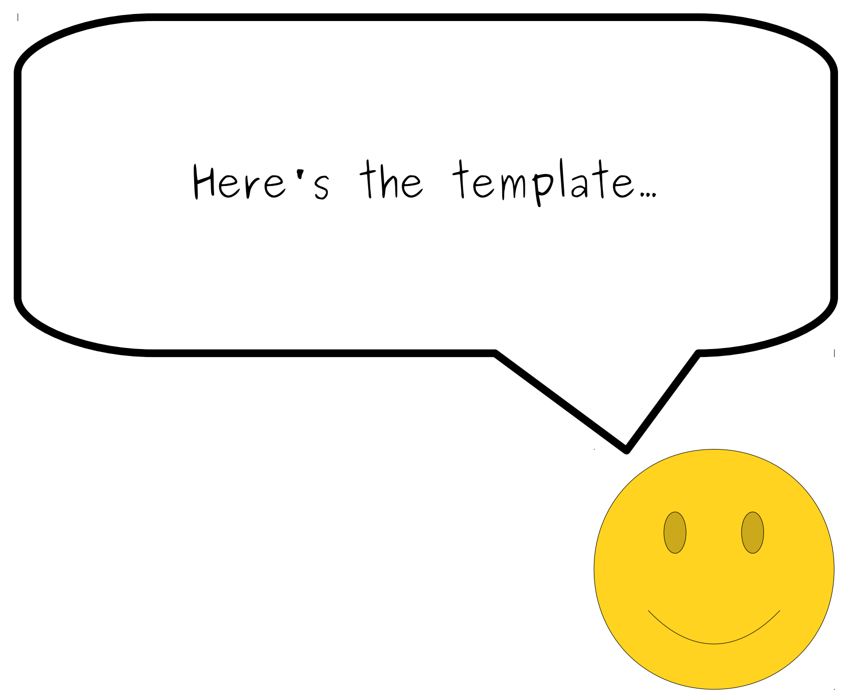


$$\frac{\neg\neg A}{A}$$

$$\frac{\neg(A \lor B)}{\neg A \land \neg B}$$

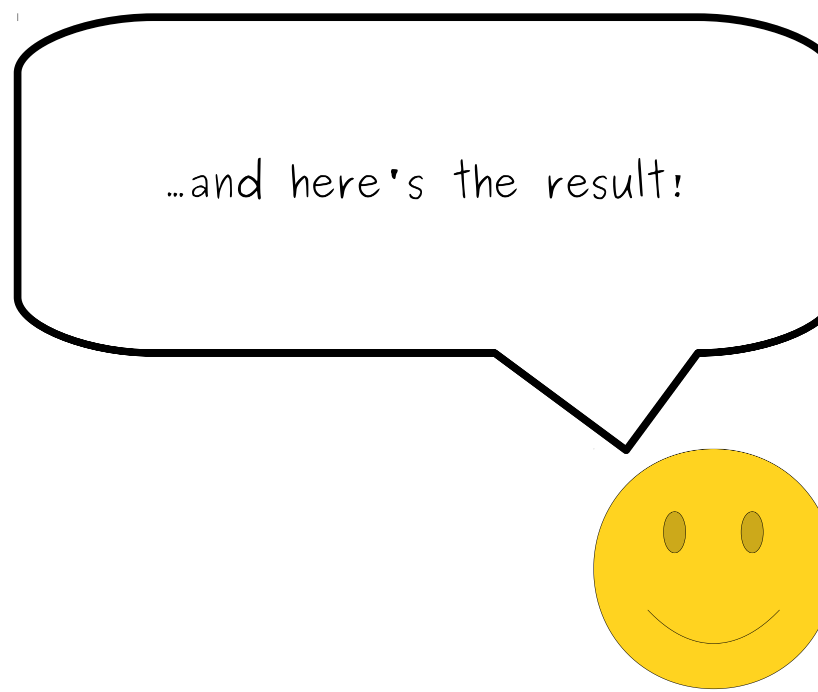
$$\frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\neg(p \lor q) \lor (\neg\neg p \lor \neg\neg q)$$



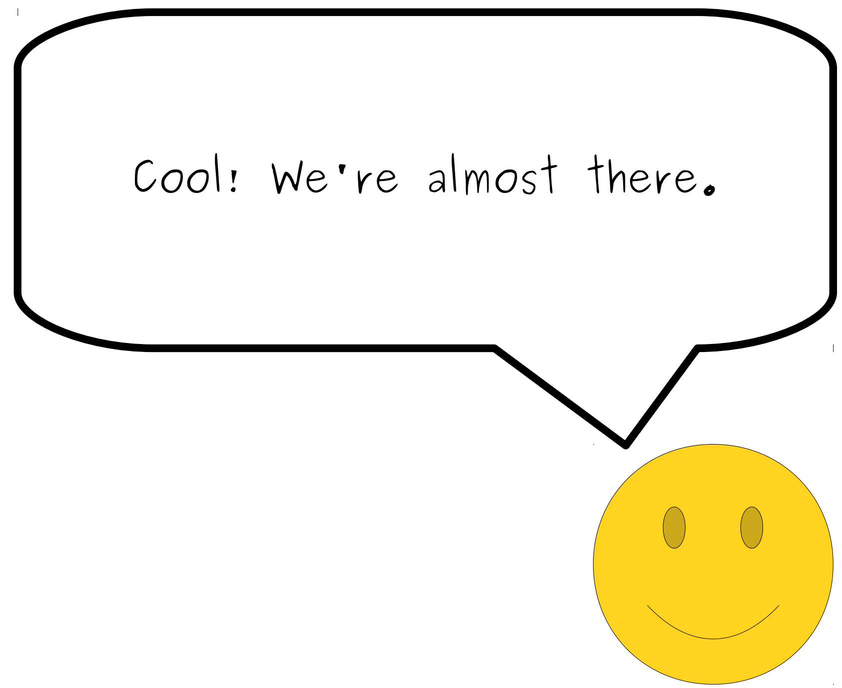
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\neg(p \lor q) \lor (p \lor q)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\neg(p \lor q) \lor (p \lor q)$$

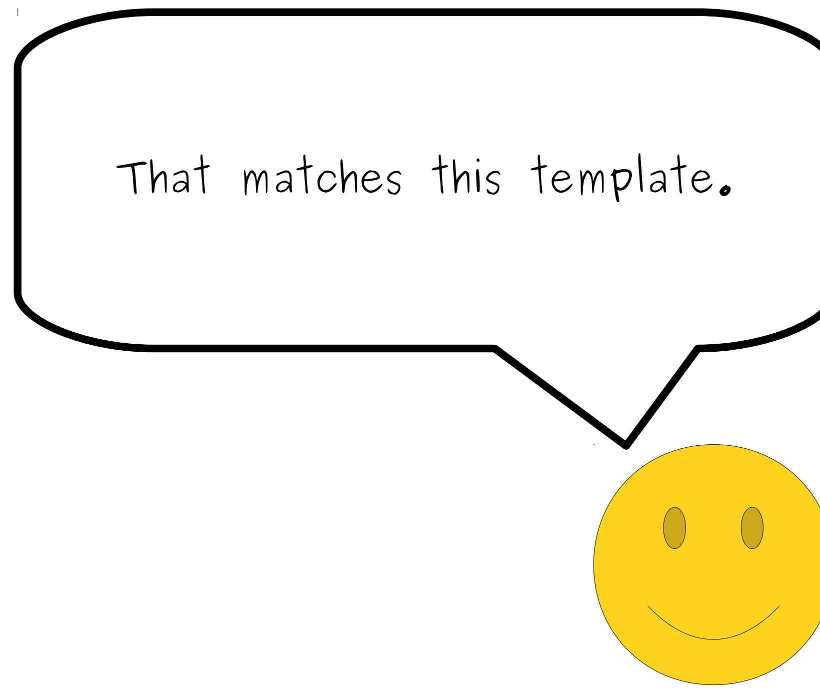


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\neg(p \lor q) \lor (p \lor q)$$

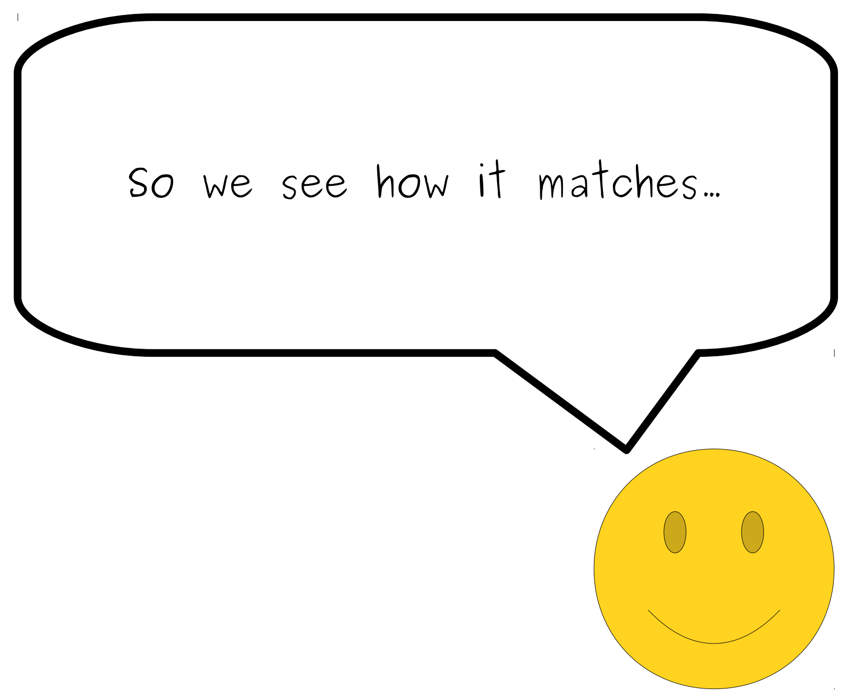
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\neg(p \lor q) \lor (p \lor q)$$



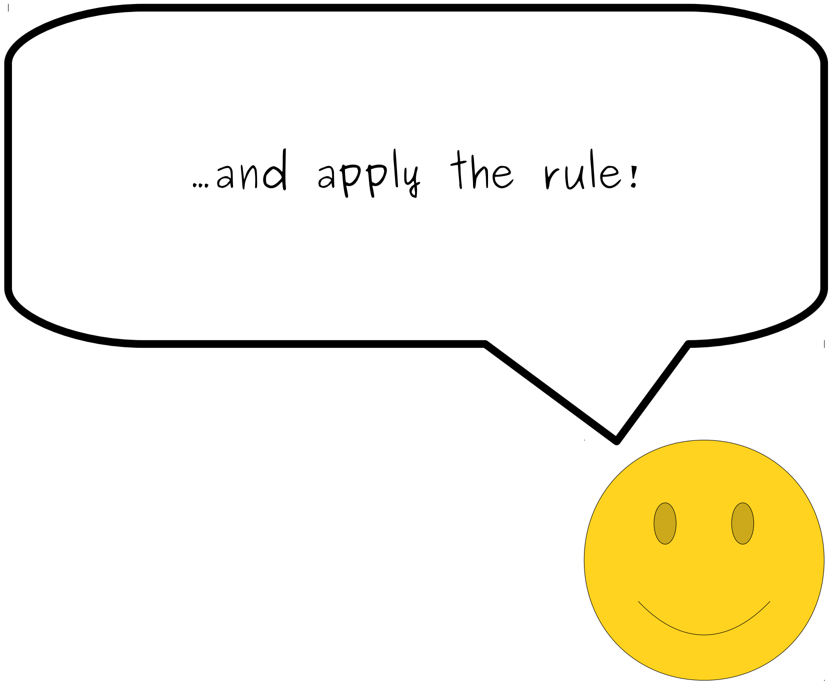
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\neg(p \lor q) \lor (p \lor q)$$



$$\frac{\neg\neg \boldsymbol{A}}{\boldsymbol{A}} \qquad \frac{\neg(\boldsymbol{A} \lor \boldsymbol{B})}{\neg\boldsymbol{A} \land \neg\boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\neg\boldsymbol{A} \lor \neg\boldsymbol{B}}$$

$$(\neg p \land \neg q) \lor (p \lor q)$$

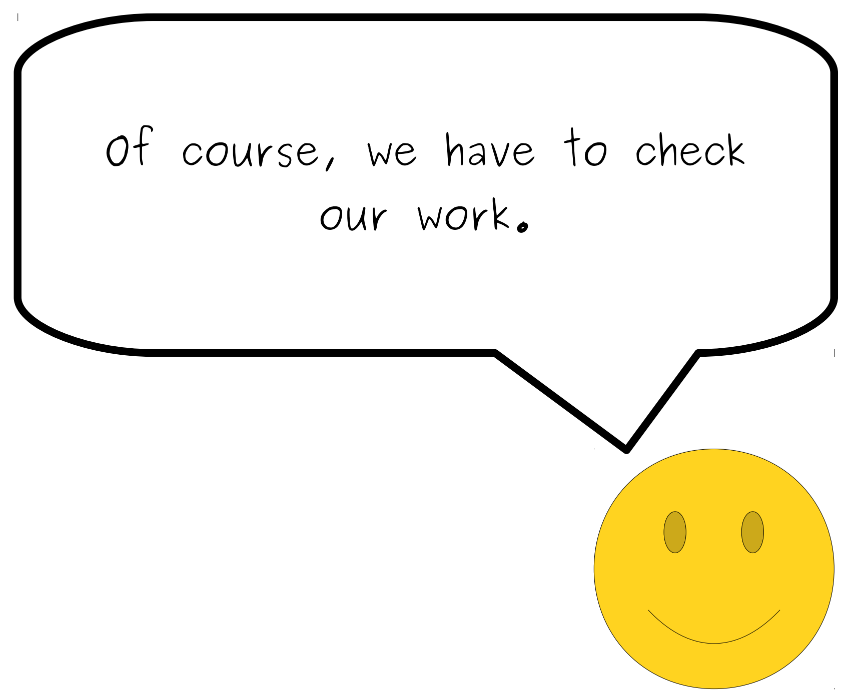


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B}$$

$$(\neg p \wedge \neg q) \vee (p \vee q)$$

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

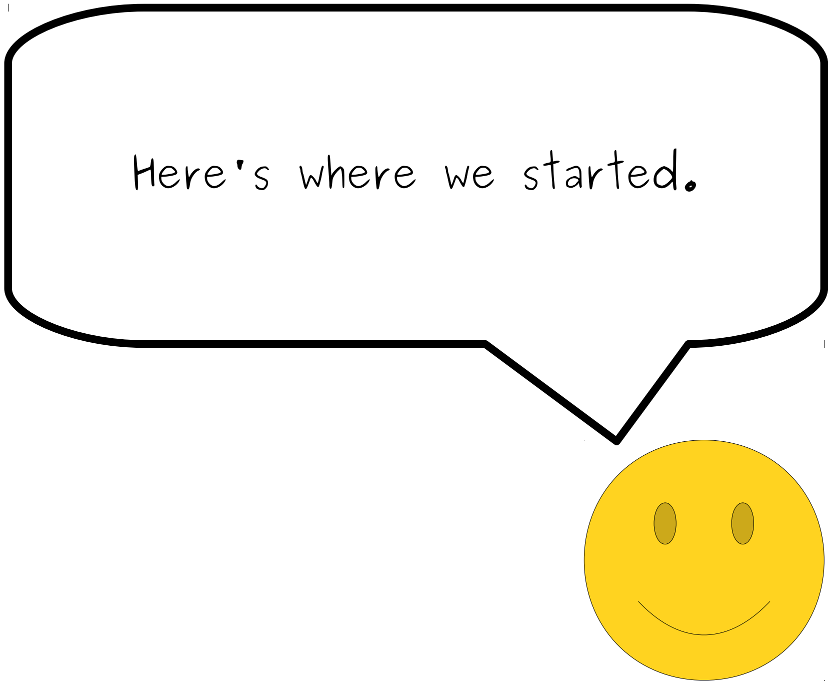
$$(\neg p \land \neg q) \lor (p \lor q)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$(\neg p \land \neg q) \lor (p \lor q)$$

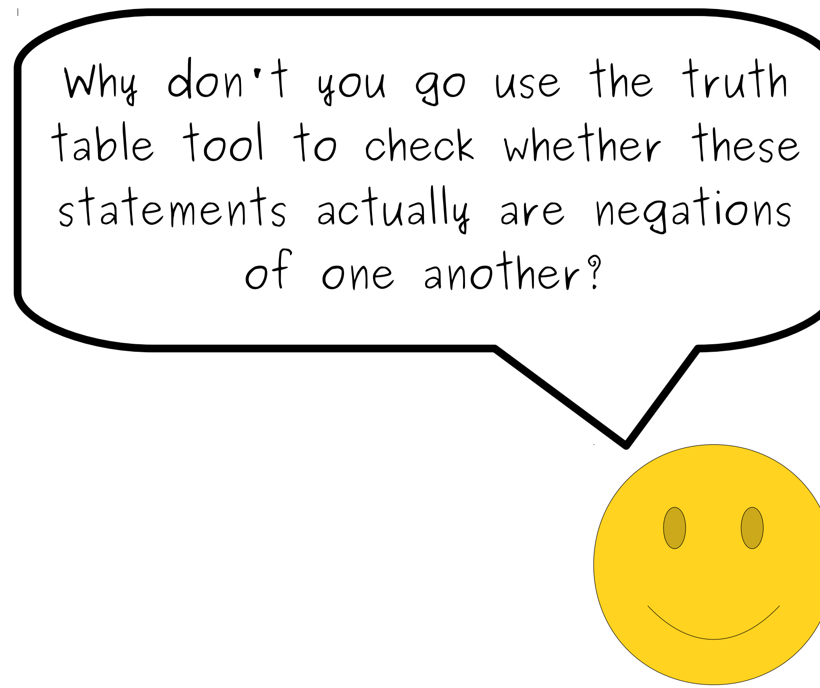
$$(p \lor q) \land (\neg p \land \neg q)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$(\neg p \land \neg q) \lor (p \lor q)$$

$$(p \lor q) \land (\neg p \land \neg q)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

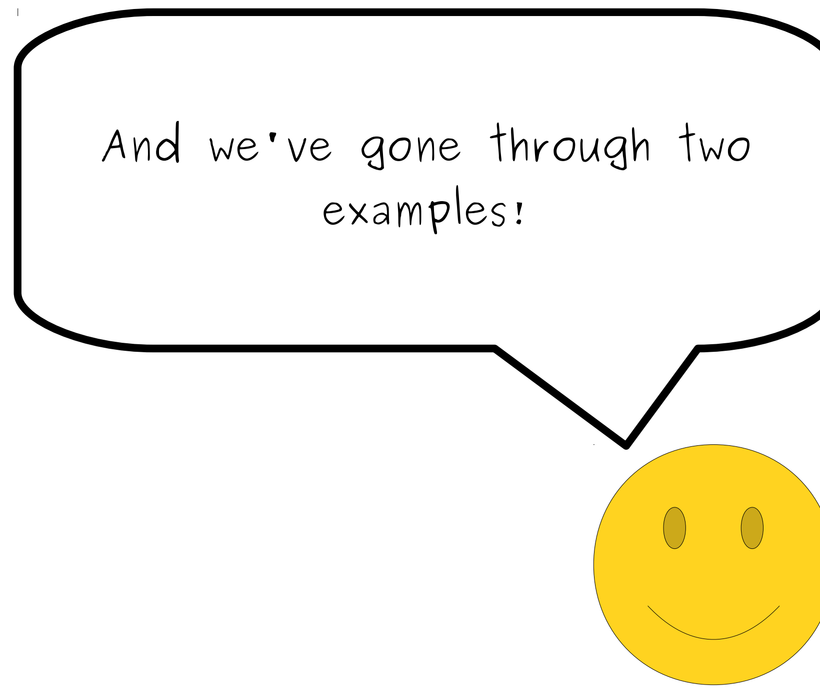
$$\frac{\neg\neg A}{A}$$

$$\frac{\neg(A \lor B)}{\neg A \land \neg B}$$

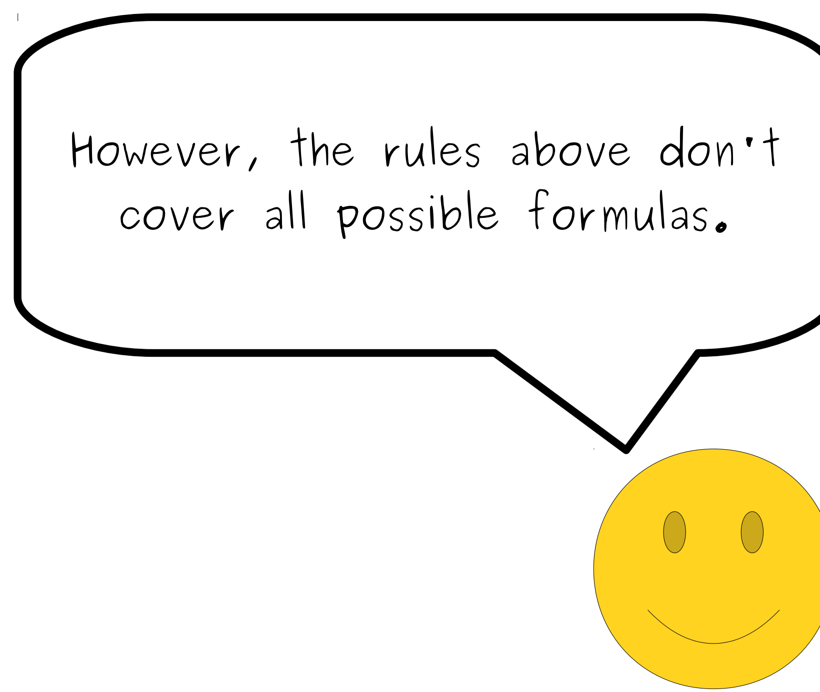
$$\frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

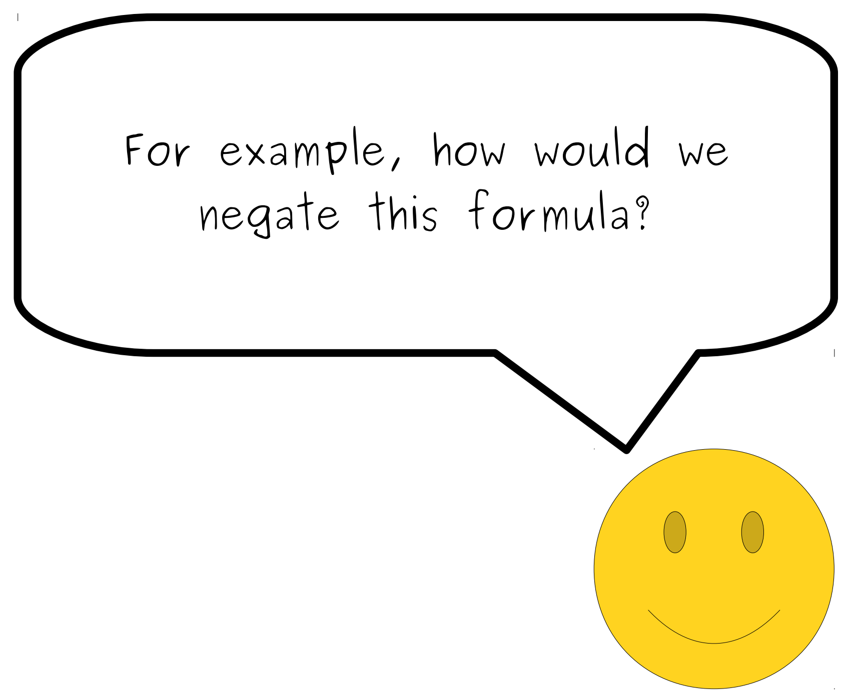


$$\frac{\neg\neg \boldsymbol{A}}{\boldsymbol{A}} \qquad \frac{\neg(\boldsymbol{A} \lor \boldsymbol{B})}{\neg \boldsymbol{A} \land \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\neg \boldsymbol{A} \lor \neg \boldsymbol{B}}$$



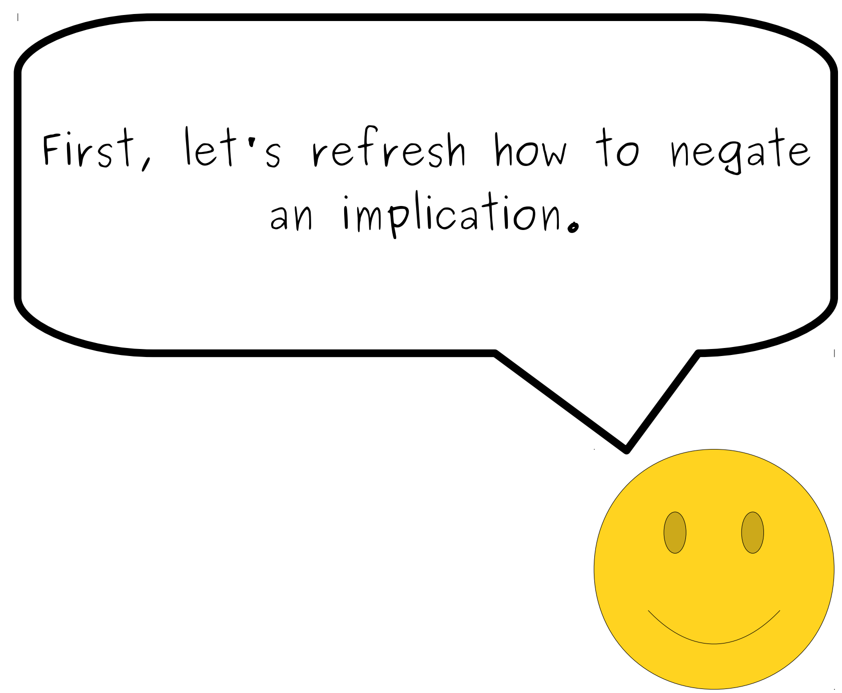
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$p \land q \to r$$



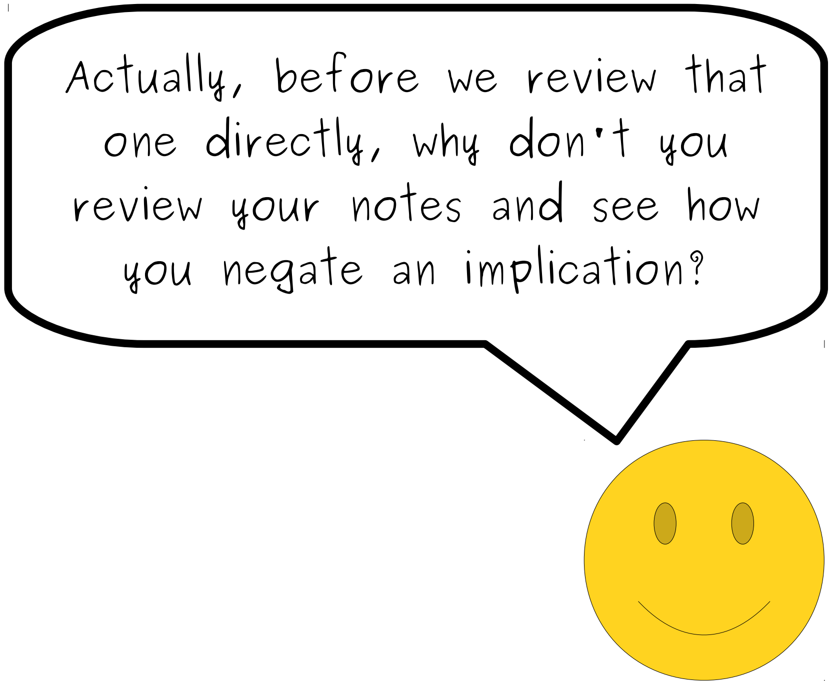
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$p \land q \to r$$



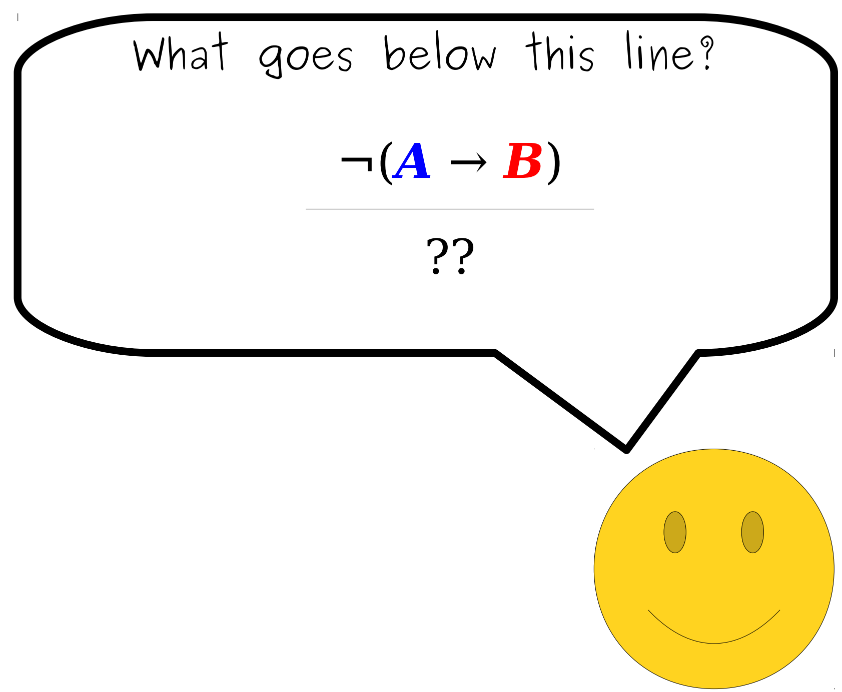
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$p \land q \to r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B}$$

$$p \wedge q \to r$$

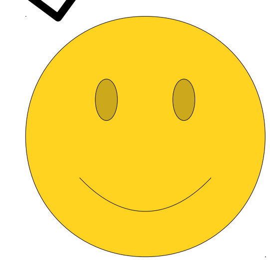


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$p \land q \to r$$

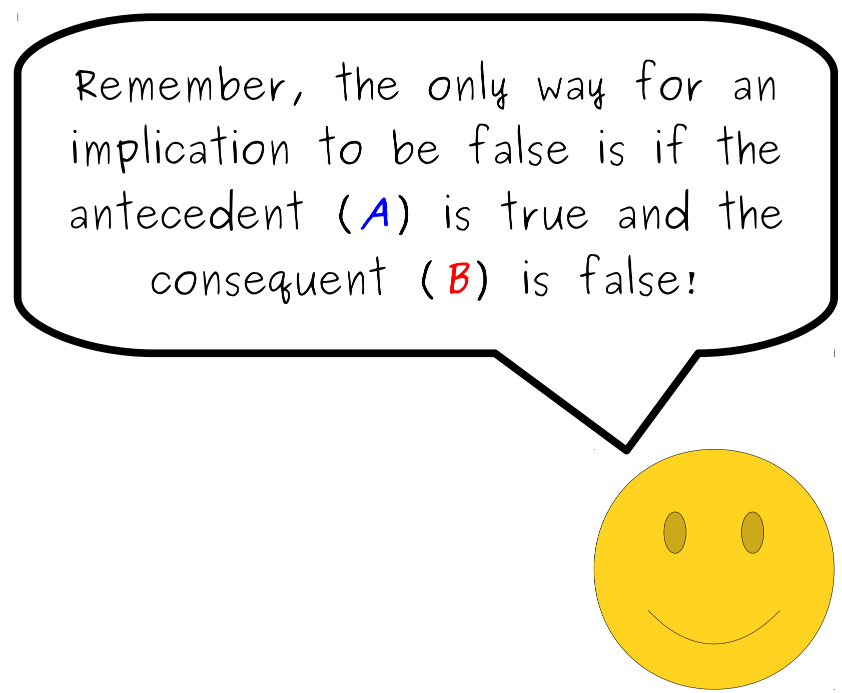
Here's the answer:

$$\frac{\neg(A \to B)}{A \land \neg B}$$



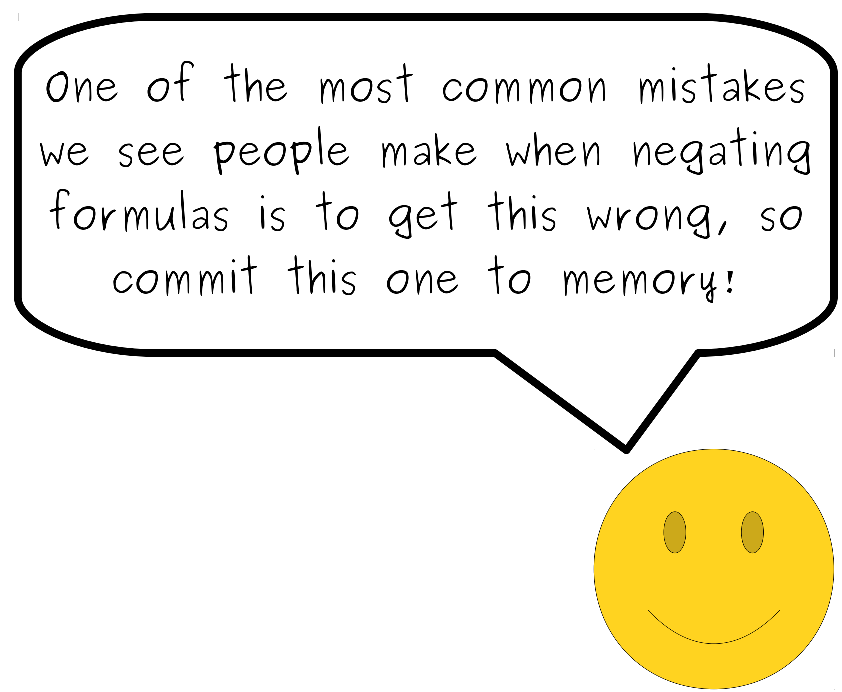
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \to B)}{A \wedge \neg B}$$

$$p \wedge q \to r$$



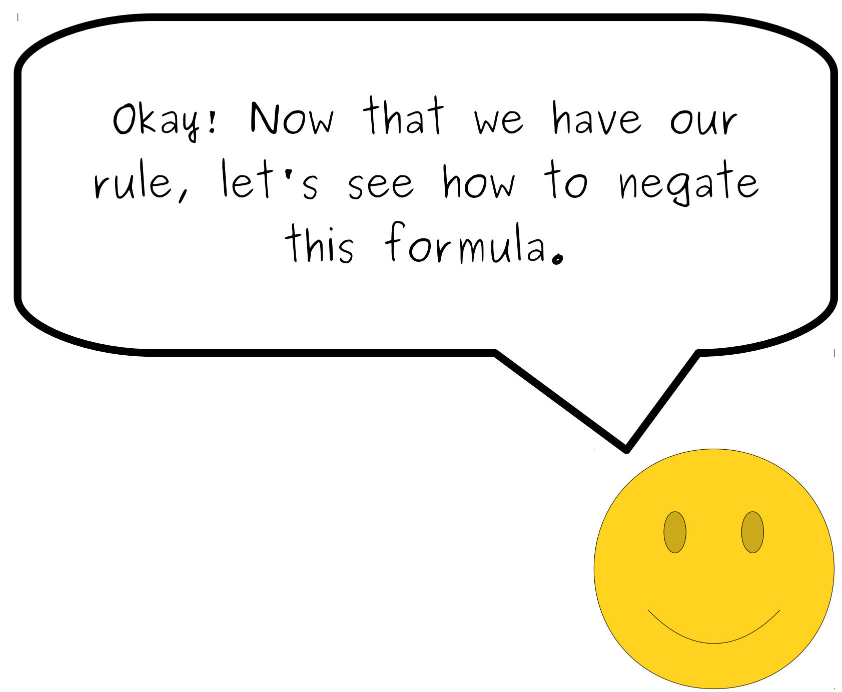
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land q \to r$$



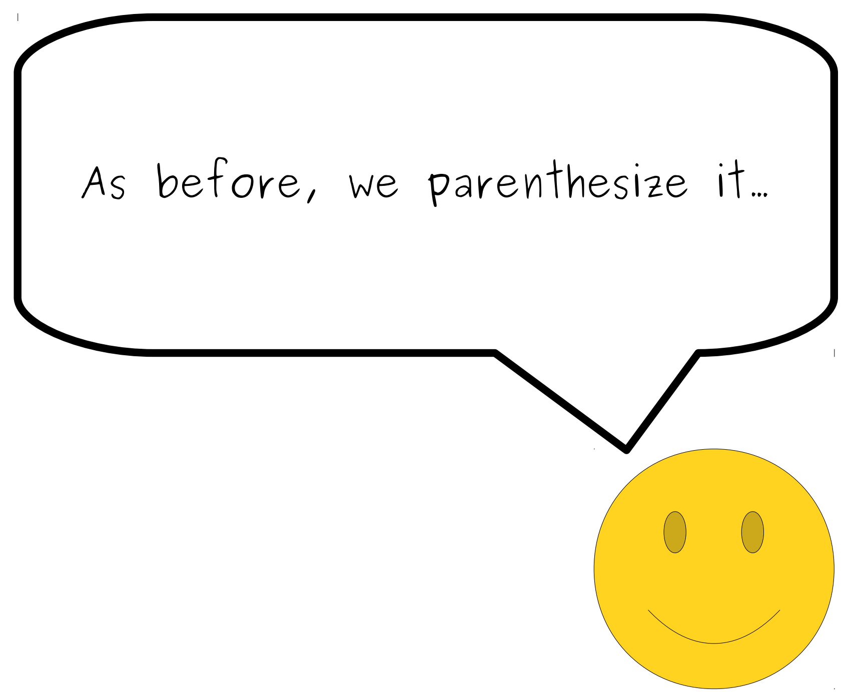
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land q \to r$$



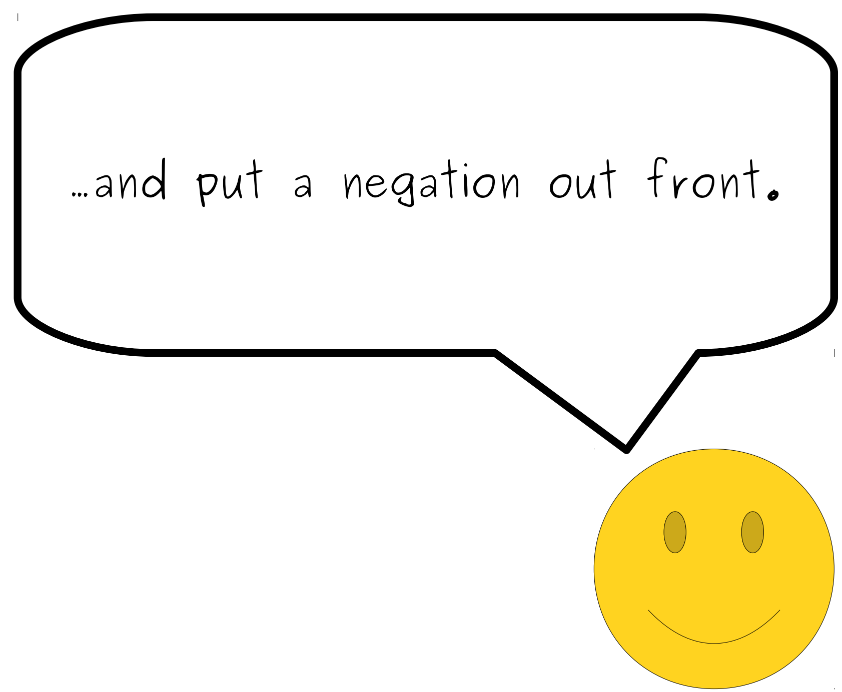
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$(p \land q \to r)$$



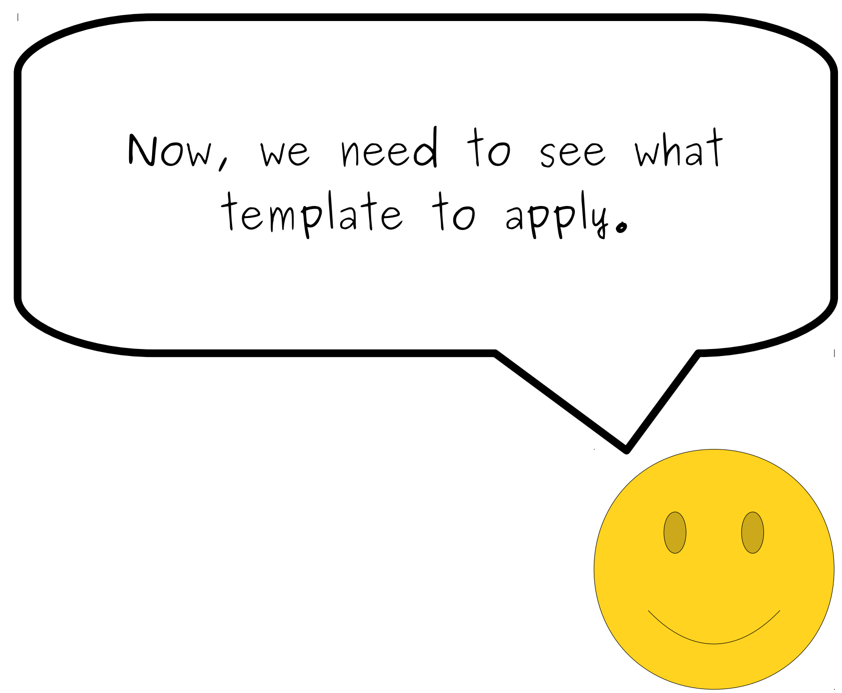
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\neg(p \land q \to r)$$



$$\frac{\neg\neg \boldsymbol{A}}{\boldsymbol{A}} \qquad \frac{\neg(\boldsymbol{A} \lor \boldsymbol{B})}{\neg \boldsymbol{A} \land \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\neg \boldsymbol{A} \lor \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \to \boldsymbol{B})}{\boldsymbol{A} \land \neg \boldsymbol{B}}$$

$$\neg(p \land q \to r)$$



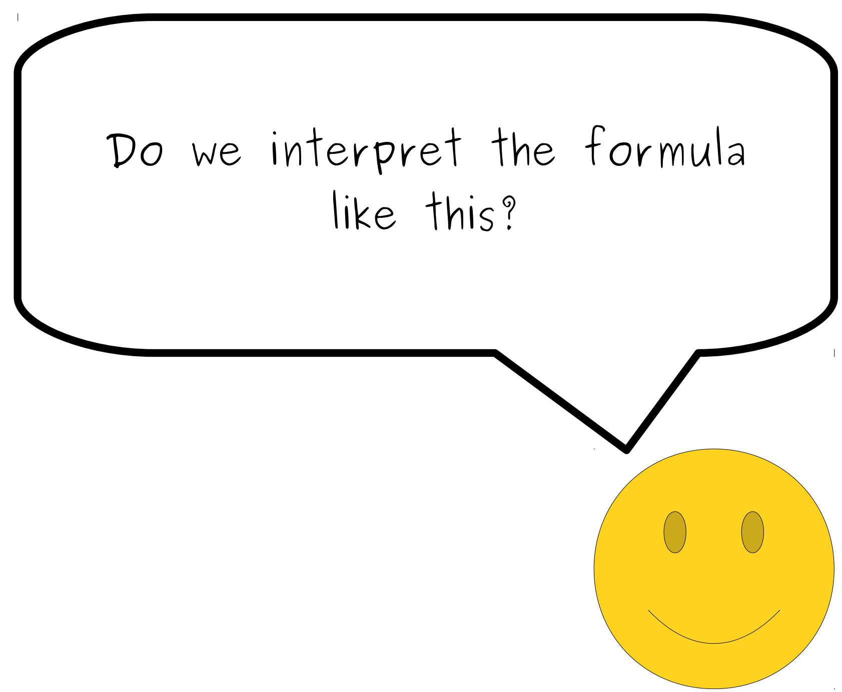
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\neg(p \land q \to r)$$

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

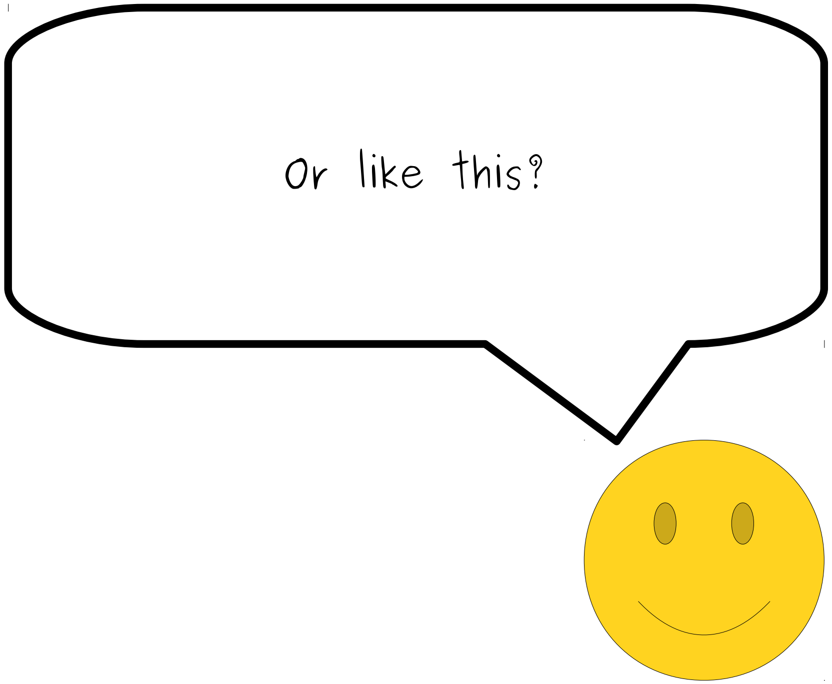
$$\neg(p \land q \to r)$$

$$\neg((p \land q) \to r) \quad (?)$$



$$\frac{\neg\neg \boldsymbol{A}}{\boldsymbol{A}} \qquad \frac{\neg(\boldsymbol{A} \lor \boldsymbol{B})}{\neg \boldsymbol{A} \land \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\neg \boldsymbol{A} \lor \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \to \boldsymbol{B})}{\boldsymbol{A} \land \neg \boldsymbol{B}}$$

$$\neg(p \land q \to r)$$

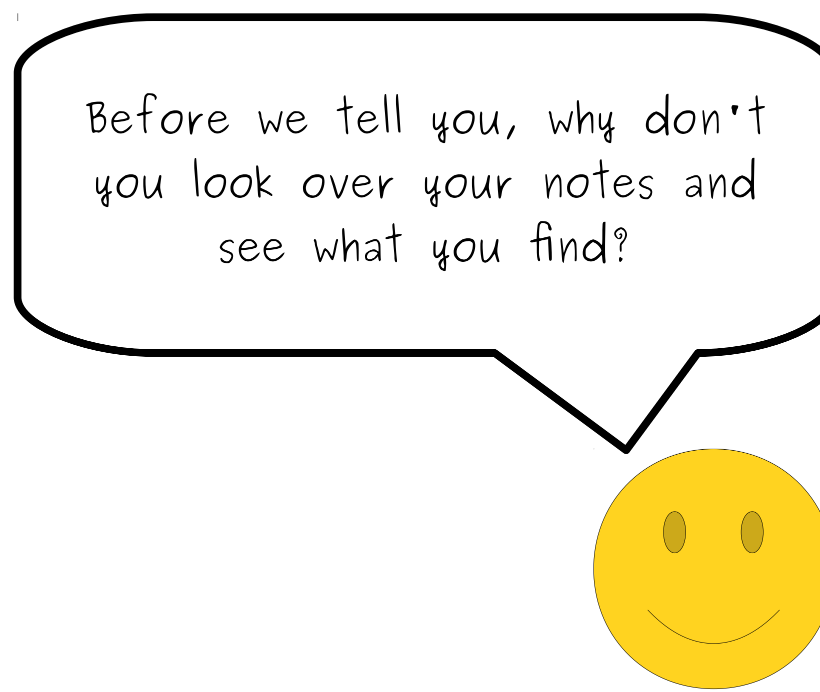


$$\neg((p \land q) \to r) \quad (?)$$
$$\neg(p \land (q \to r)) \quad (?)$$

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \to B)}{A \wedge \neg B}$$

$$\neg(p \wedge q \to r)$$

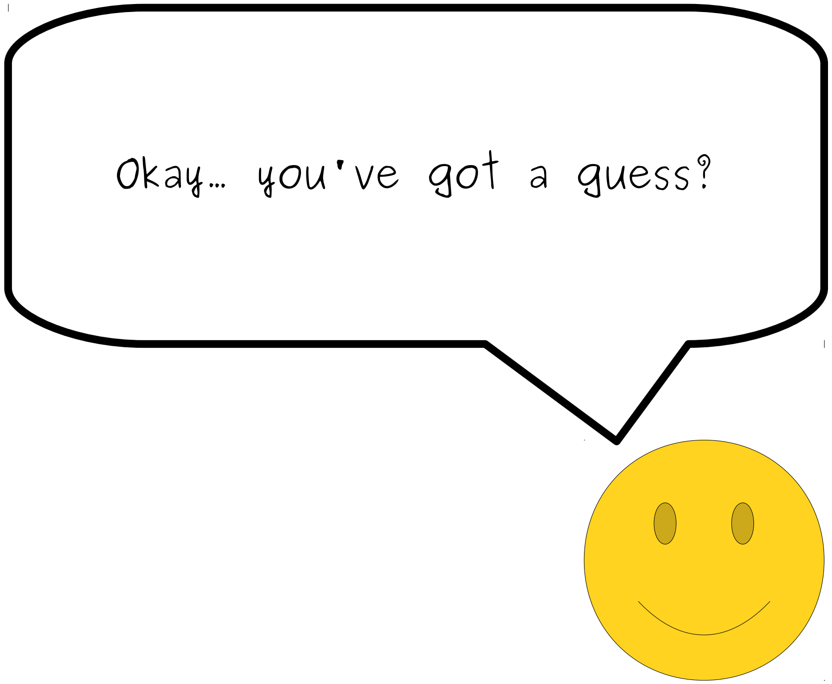
$$\neg((p \wedge q) \to r) \qquad (?)$$
$$\neg(p \wedge (q \to r)) \qquad (?)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\neg(p \land q \to r)$$

$$\neg((p \land q) \to r) \qquad (?)$$
$$\neg(p \land (q \to r)) \qquad (?)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\neg(p \land q \to r)$$

$$\neg((p \land q) \to r)$$

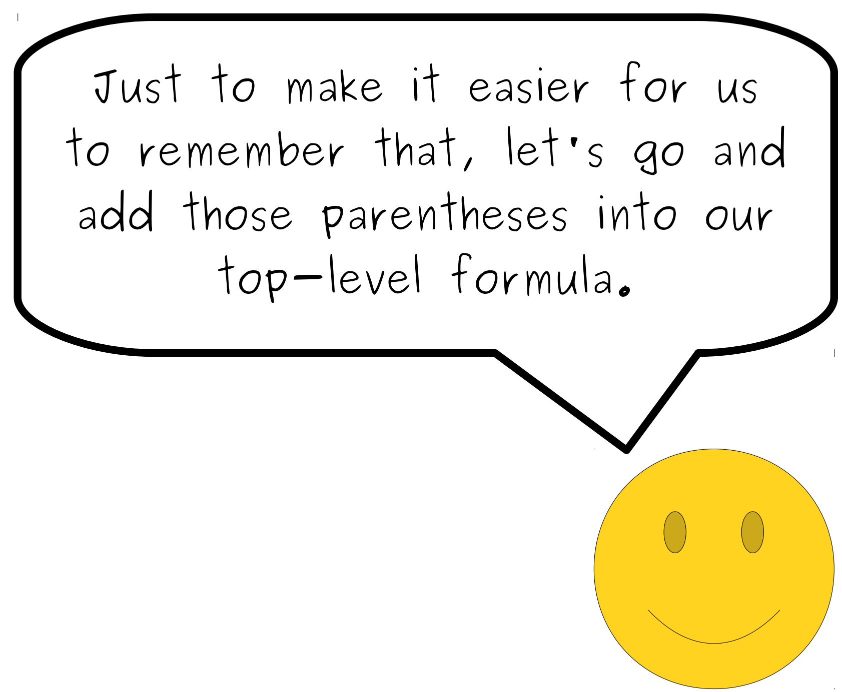
~~$\neg(p \land (q \to r))$~~ ~~(?)~~

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\neg((p \land q) \to r)$$

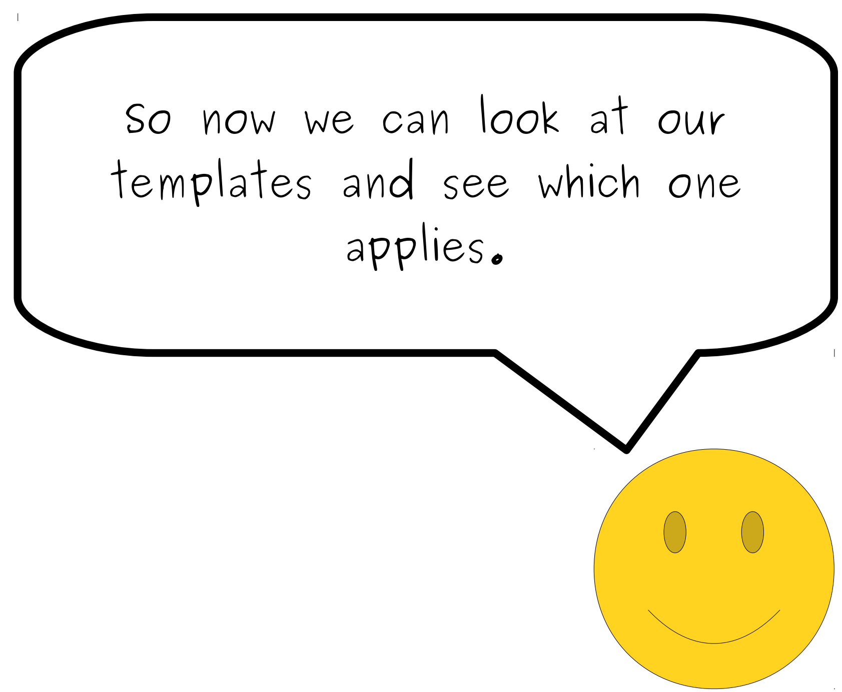
$$\neg((p \land q) \to r)$$

~~$\neg(p \land (q \to r))$~~ ~~(?)~~



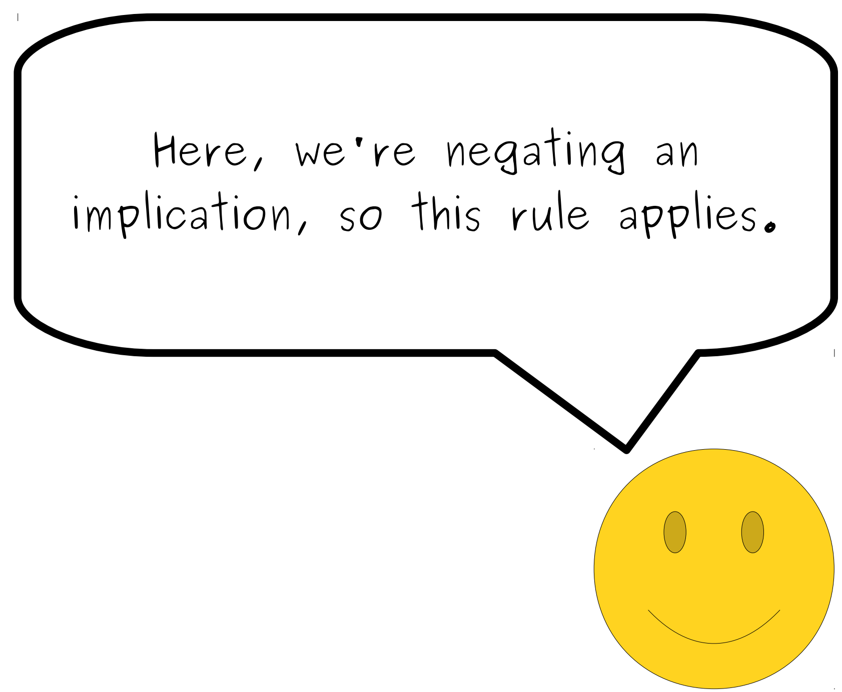
$$\frac{\neg\neg \boldsymbol{A}}{\boldsymbol{A}} \qquad \frac{\neg(\boldsymbol{A} \lor \boldsymbol{B})}{\neg \boldsymbol{A} \land \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\neg \boldsymbol{A} \lor \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \to \boldsymbol{B})}{\boldsymbol{A} \land \neg \boldsymbol{B}}$$

$$\neg((p \land q) \to r)$$



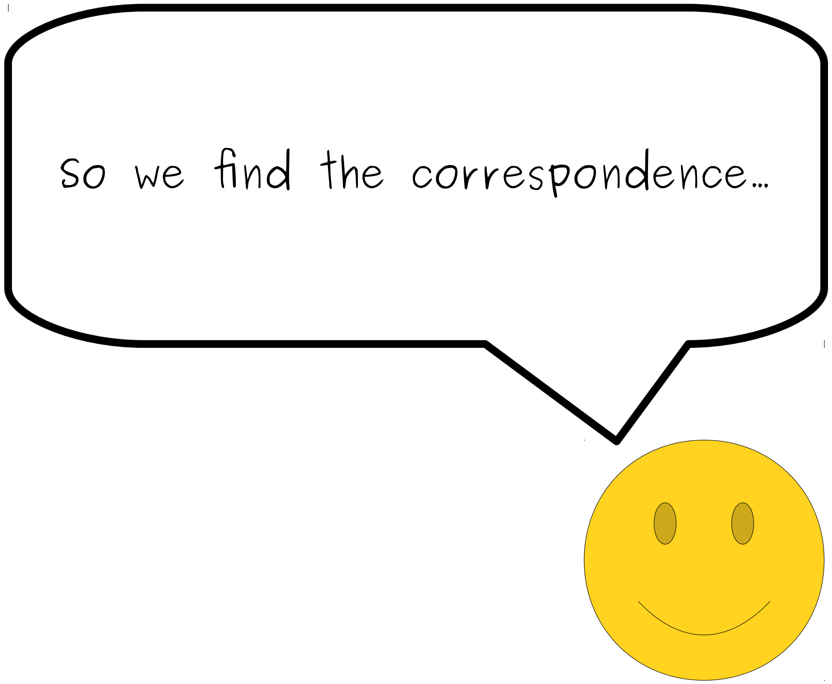
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\neg((p \land q) \to r)$$



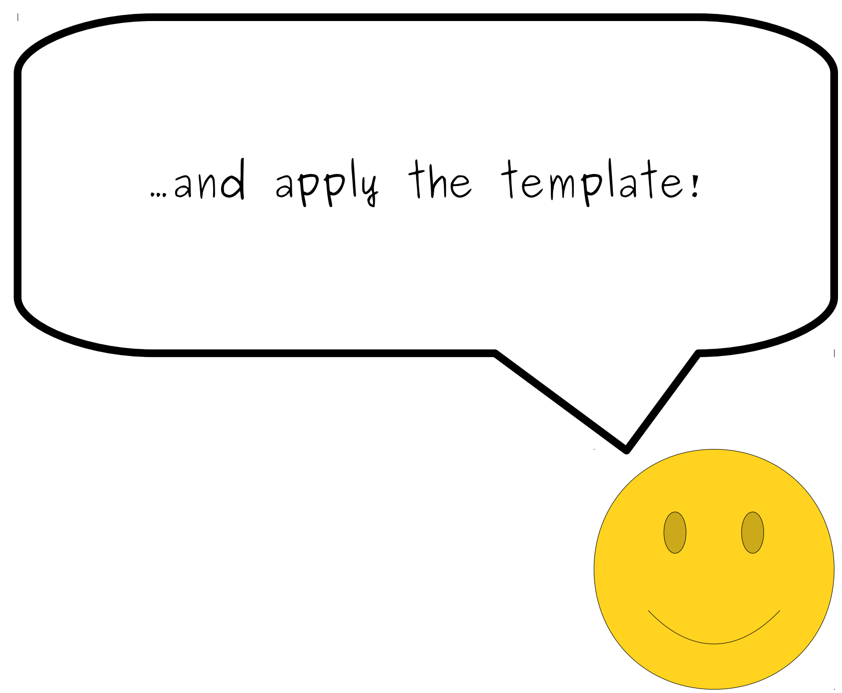
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \to B)}{A \wedge \neg B}$$

$$\neg((p \wedge q) \to r)$$



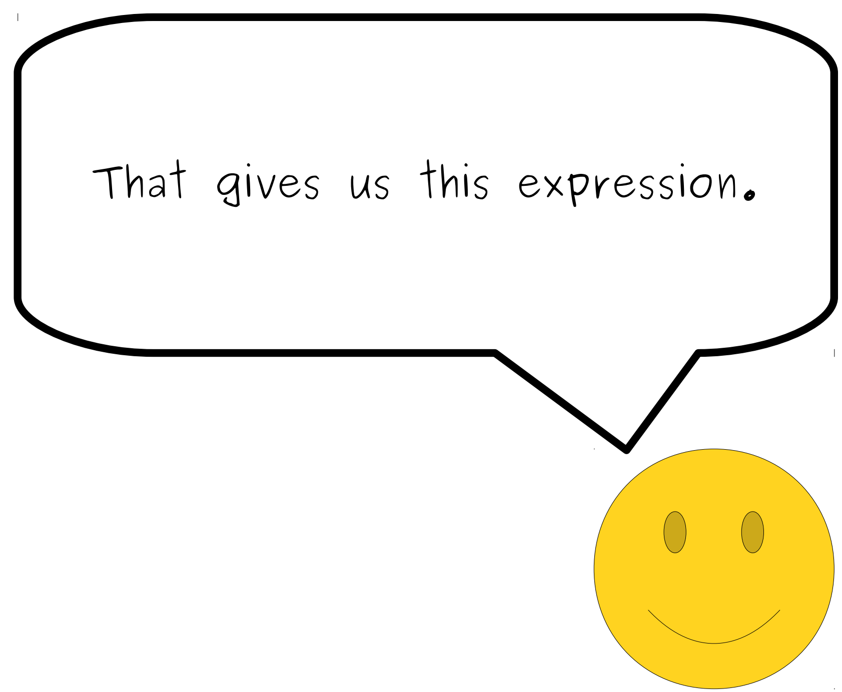
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$(p \land q) \land \neg r$$



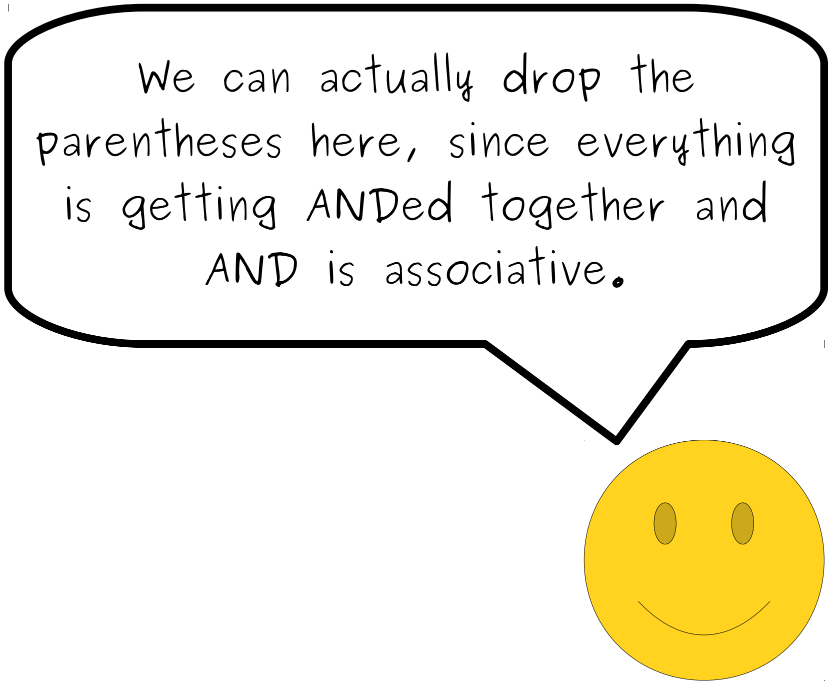
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$(p \land q) \land \neg r$$



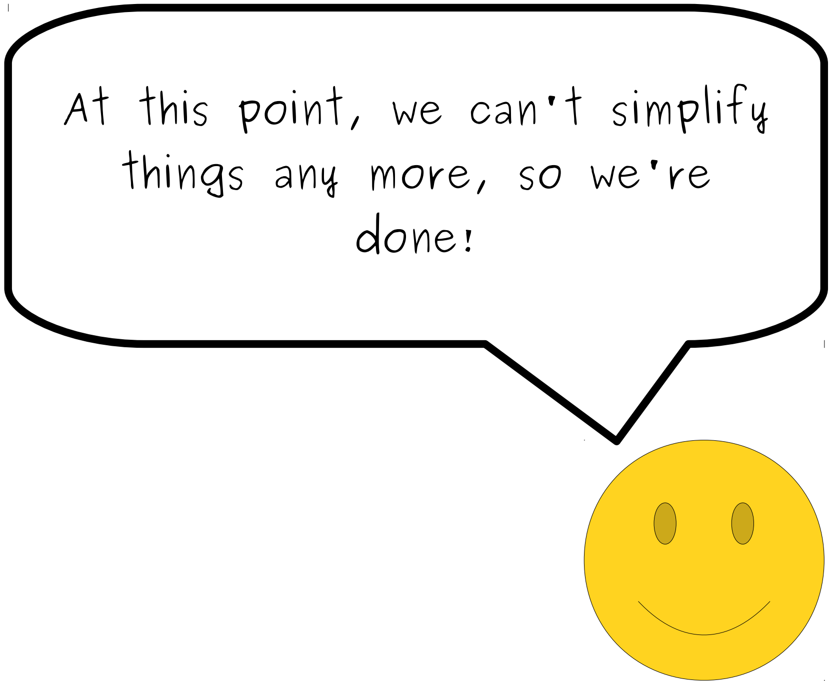
$$\frac{\lnot\lnot A}{A} \qquad \frac{\lnot(A \lor B)}{\lnot A \land \lnot B} \qquad \frac{\lnot(A \land B)}{\lnot A \lor \lnot B} \qquad \frac{\lnot(A \to B)}{A \land \lnot B}$$

$$p \land q \land \lnot r$$



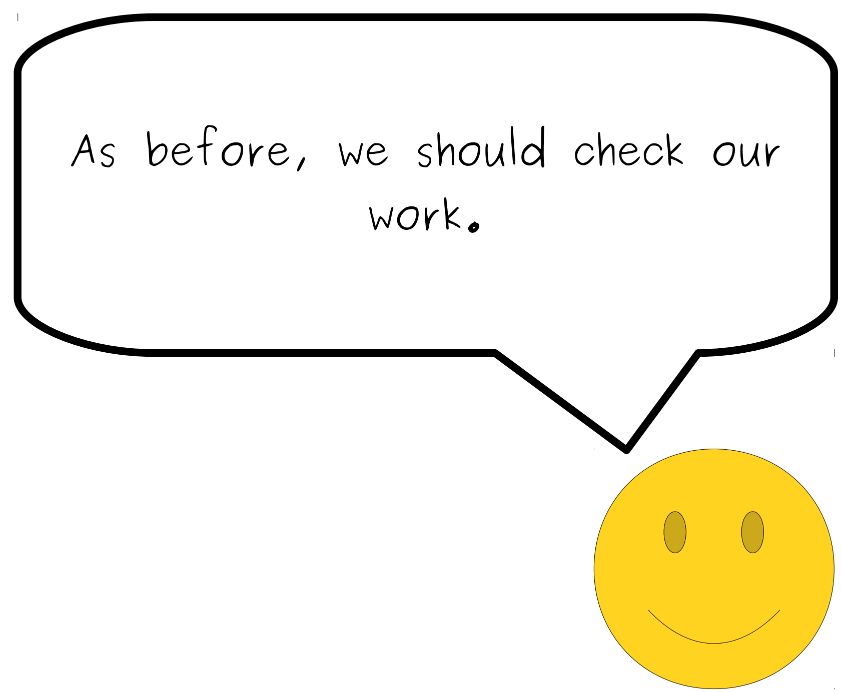
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land q \land \neg r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

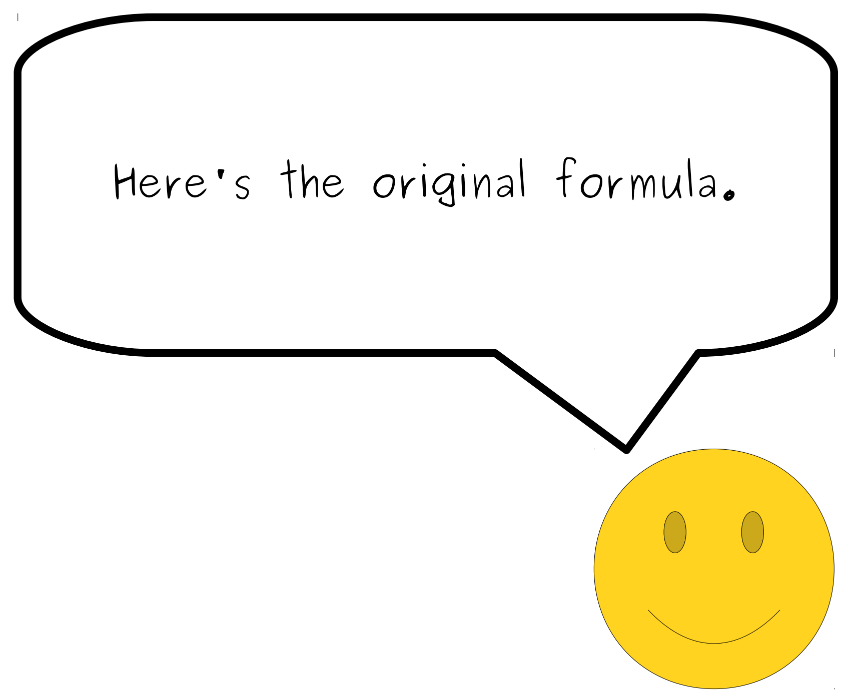
$$p \land q \land \neg r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

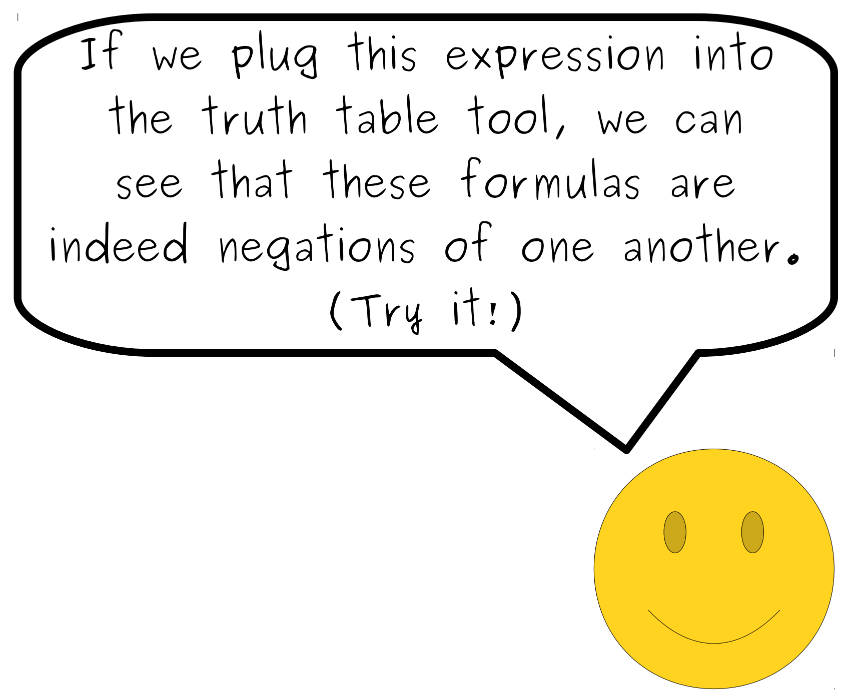
$$p \land q \land \neg r$$

$$p \land q \to r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

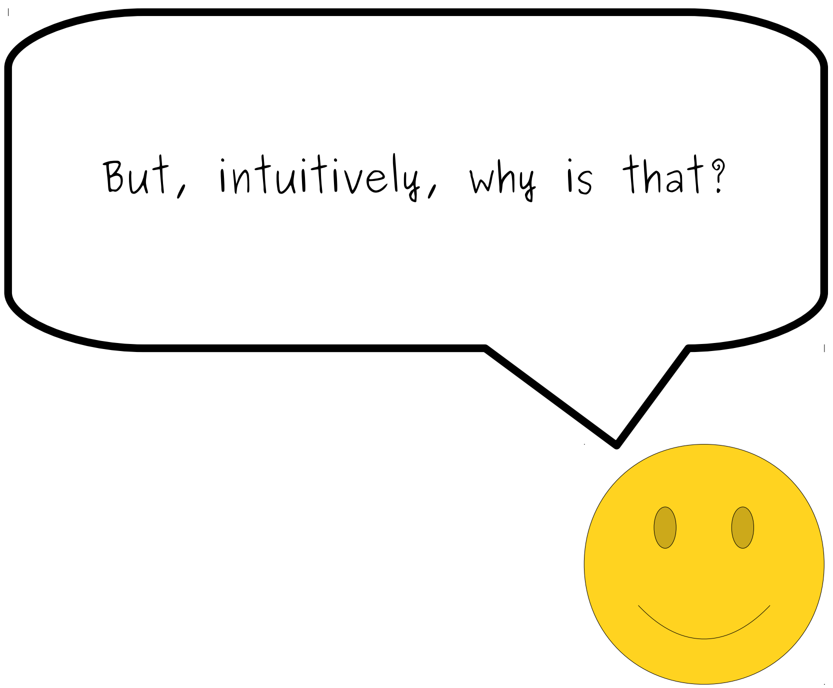
$$(p \land q \land \neg r)$$
$$\leftrightarrow$$
$$(p \land q \to r)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land q \land \neg r$$

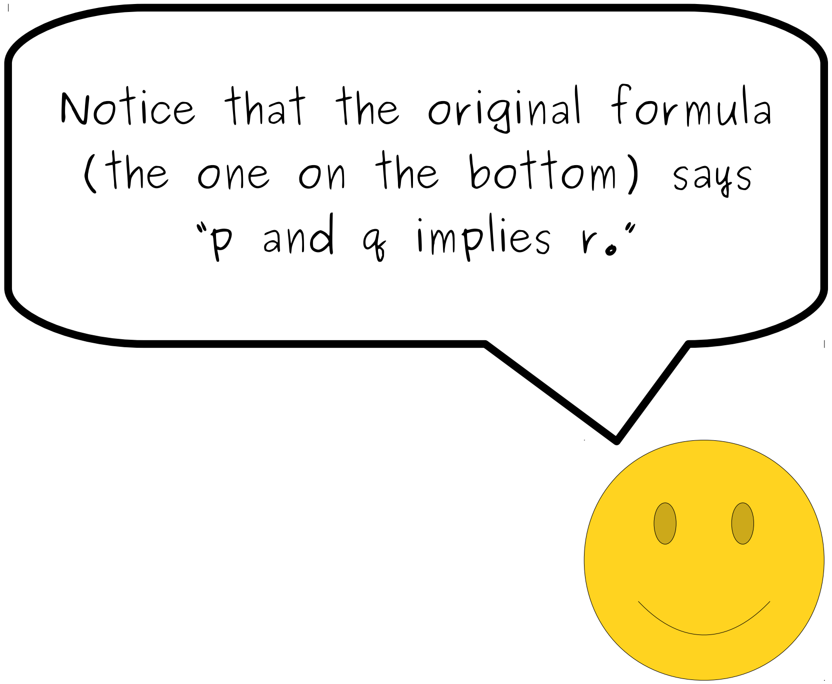
$$p \land q \to r$$



$$\frac{\neg\neg \boldsymbol{A}}{\boldsymbol{A}} \qquad \frac{\neg(\boldsymbol{A} \lor \boldsymbol{B})}{\neg \boldsymbol{A} \land \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\neg \boldsymbol{A} \lor \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \to \boldsymbol{B})}{\boldsymbol{A} \land \neg \boldsymbol{B}}$$

$$p \land q \land \neg r$$

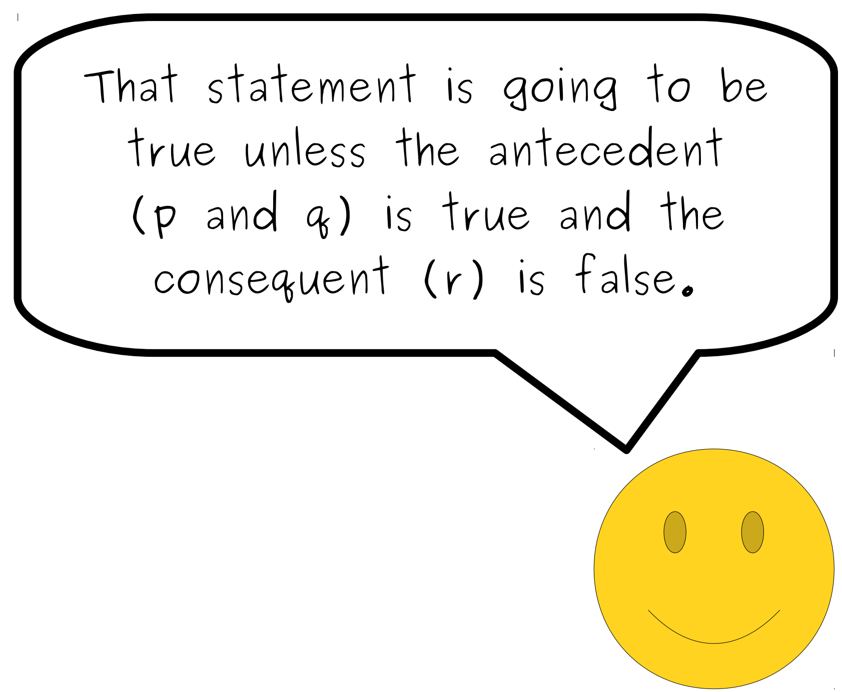
$$p \land q \to r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \to B)}{A \wedge \neg B}$$

$$p \wedge q \wedge \neg r$$

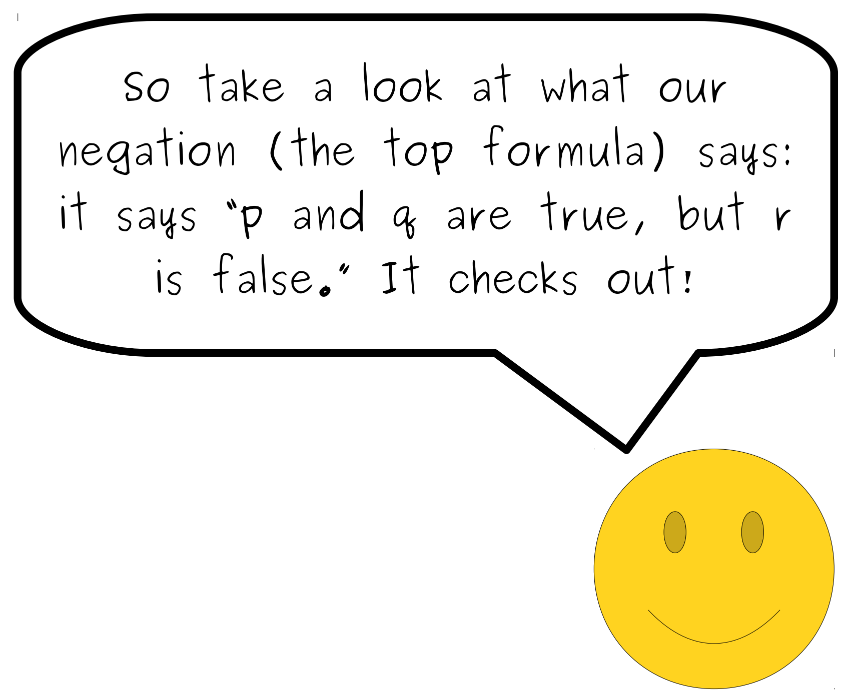
$$p \wedge q \to r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \rightarrow B)}{A \wedge \neg B}$$

$$p \wedge q \wedge \neg r$$

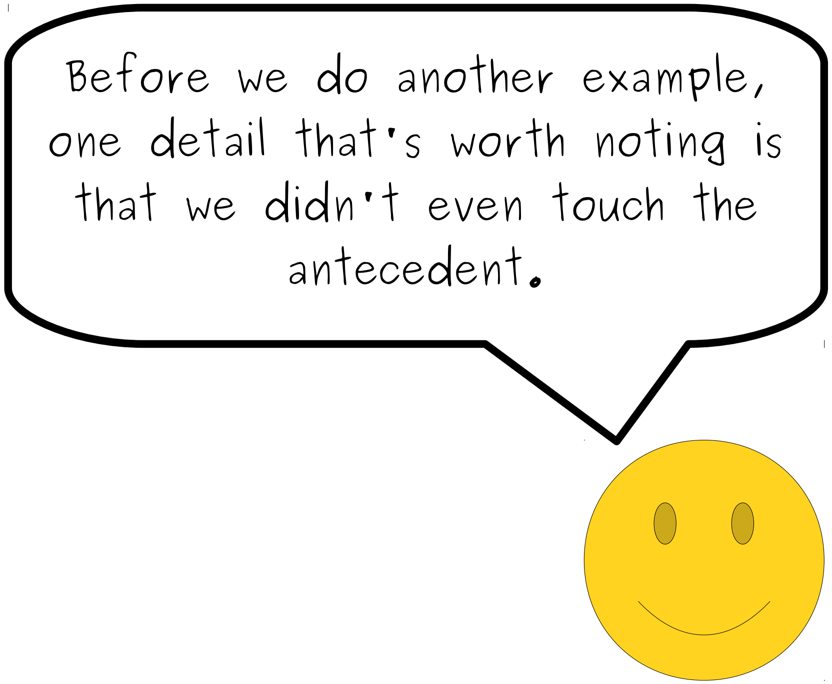
$$p \wedge q \rightarrow r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land q \land \neg r$$

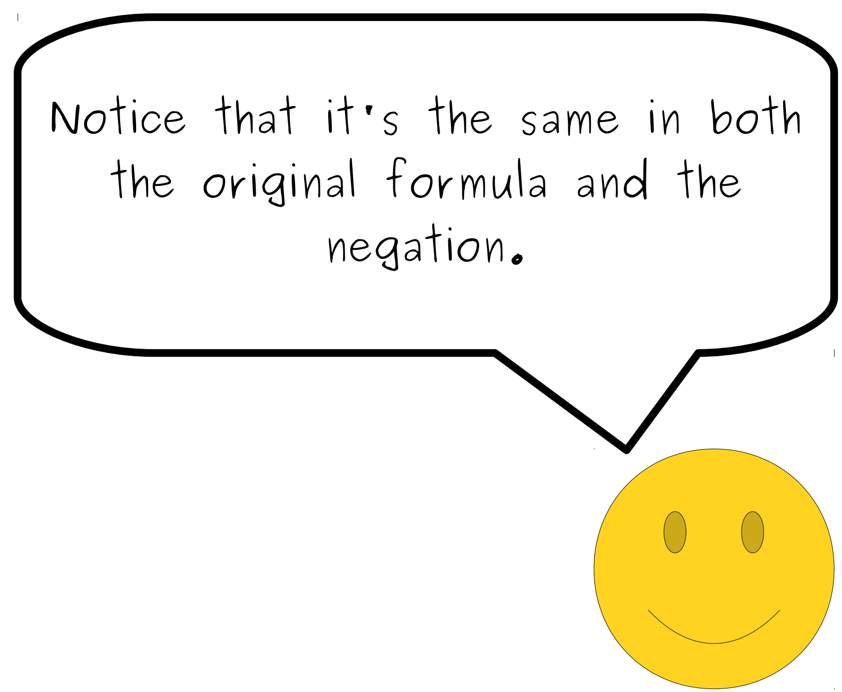
$$p \land q \to r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land q \land \neg r$$

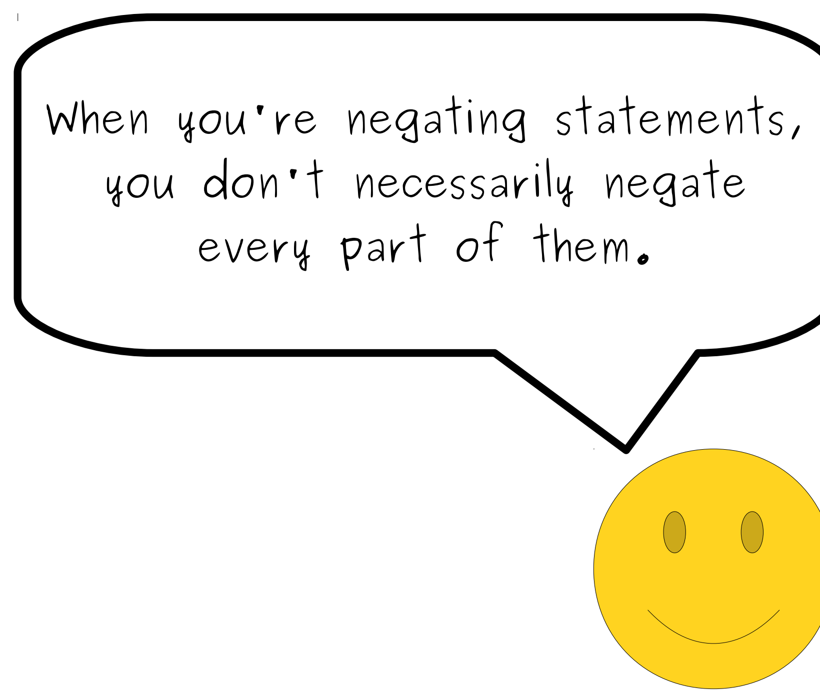
$$p \land q \to r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land q \land \neg r$$

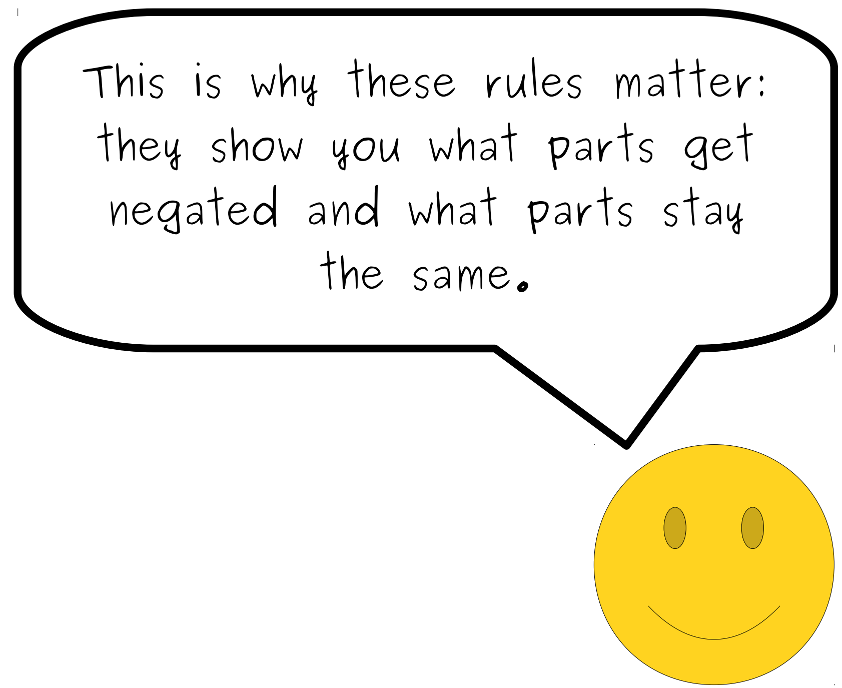
$$p \land q \to r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \to B)}{A \wedge \neg B}$$

$$p \wedge q \wedge \neg r$$

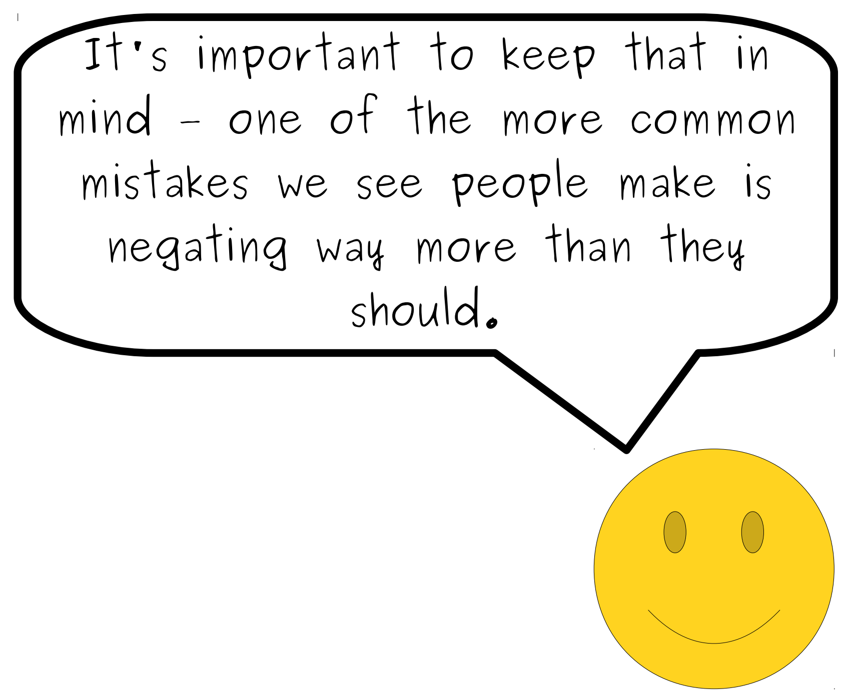
$$p \wedge q \to r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land q \land \neg r$$

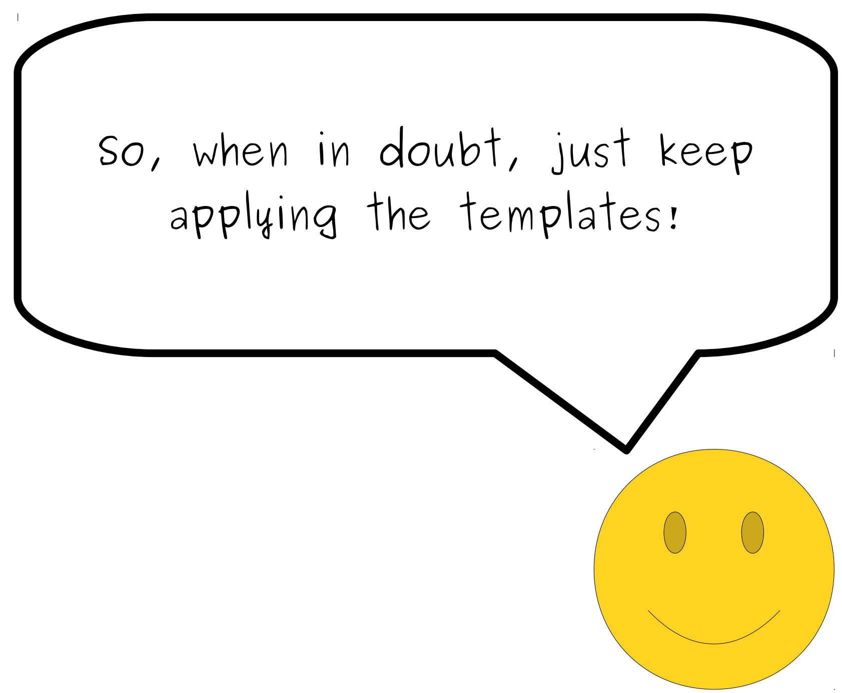
$$p \land q \to r$$



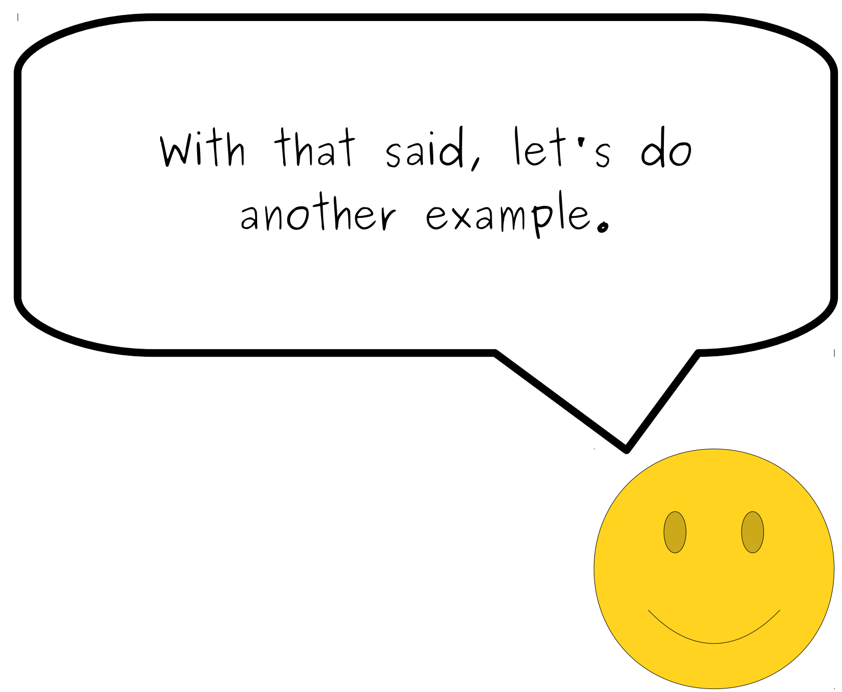
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \to B)}{A \wedge \neg B}$$

$$p \wedge q \wedge \neg r$$

$$p \wedge q \to r$$

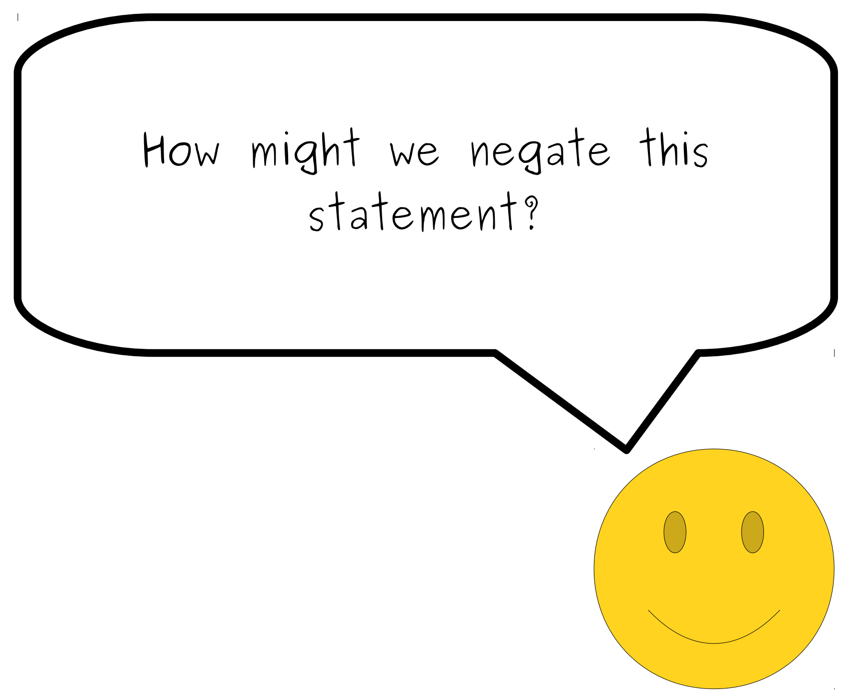


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$



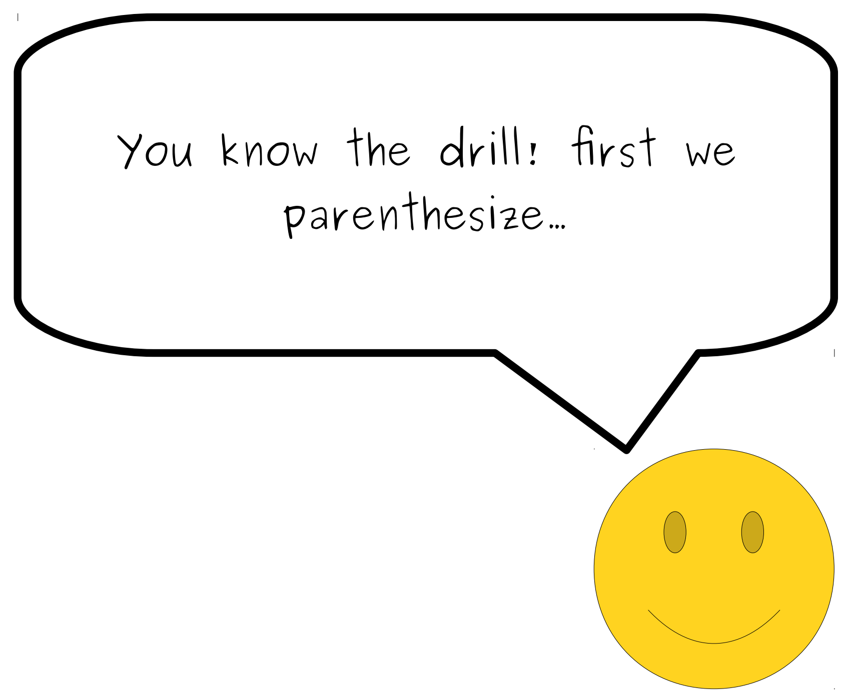
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \to q \to r$$



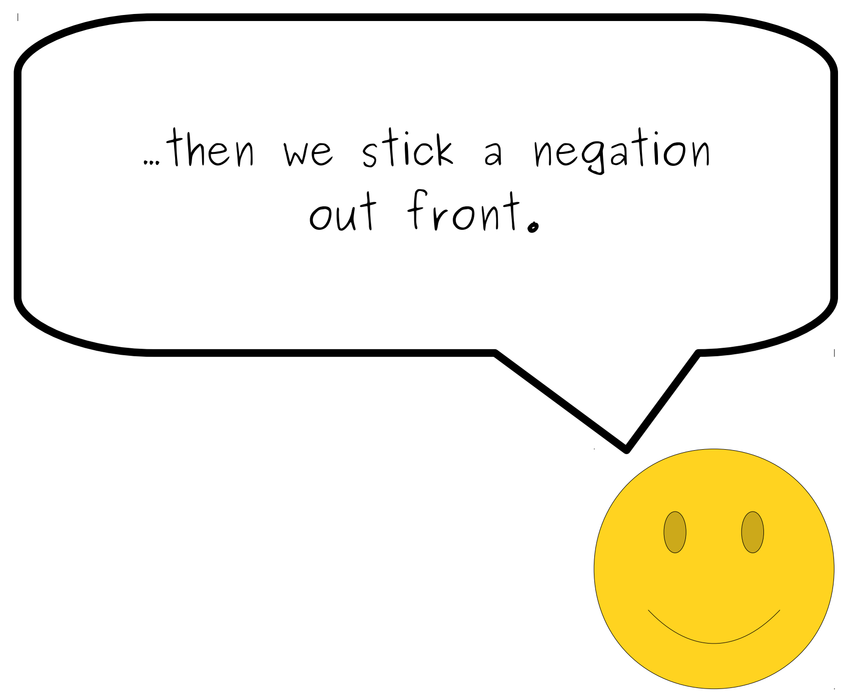
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$(p \to q \to r)$$



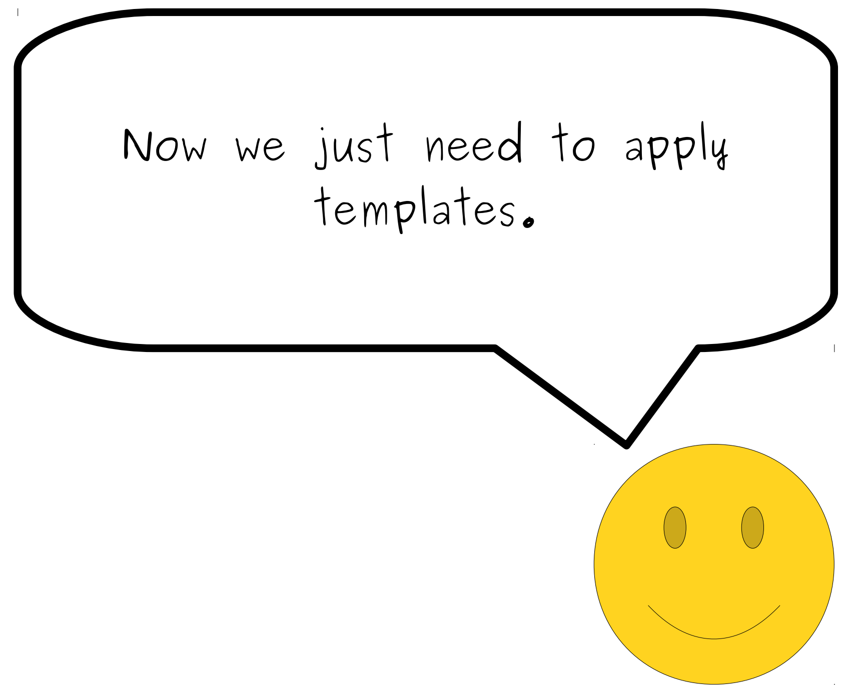
$$\frac{\neg\neg \boldsymbol{A}}{\boldsymbol{A}} \qquad \frac{\neg(\boldsymbol{A} \lor \boldsymbol{B})}{\neg \boldsymbol{A} \land \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\neg \boldsymbol{A} \lor \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \to \boldsymbol{B})}{\boldsymbol{A} \land \neg \boldsymbol{B}}$$

$$\neg(p \to q \to r)$$



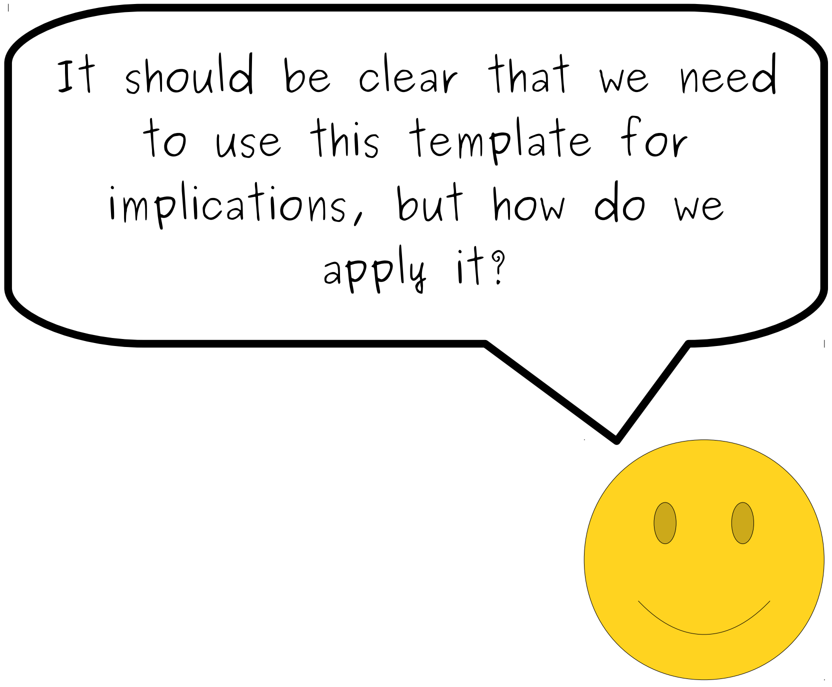
$$\frac{\neg\neg \boldsymbol{A}}{\boldsymbol{A}} \qquad \frac{\neg(\boldsymbol{A} \lor \boldsymbol{B})}{\neg\boldsymbol{A} \land \neg\boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\neg\boldsymbol{A} \lor \neg\boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \to \boldsymbol{B})}{\boldsymbol{A} \land \neg\boldsymbol{B}}$$

$$\neg(p \to q \to r)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \boxed{\frac{\neg(A \to B)}{A \land \neg B}}$$

$$\neg(p \to q \to r)$$

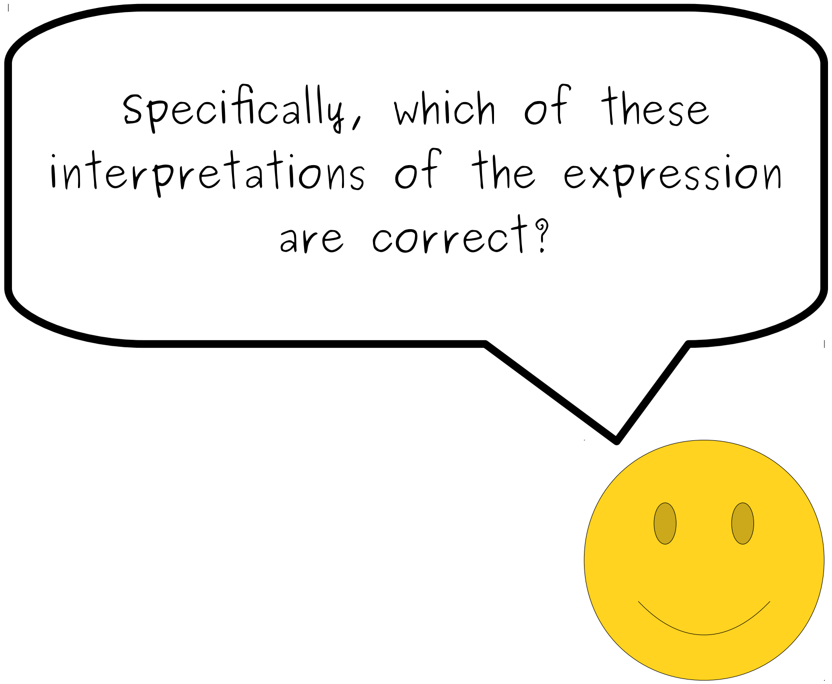


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\neg(p \to q \to r)$$

$$\neg((p \to q) \to r) \quad (?)$$

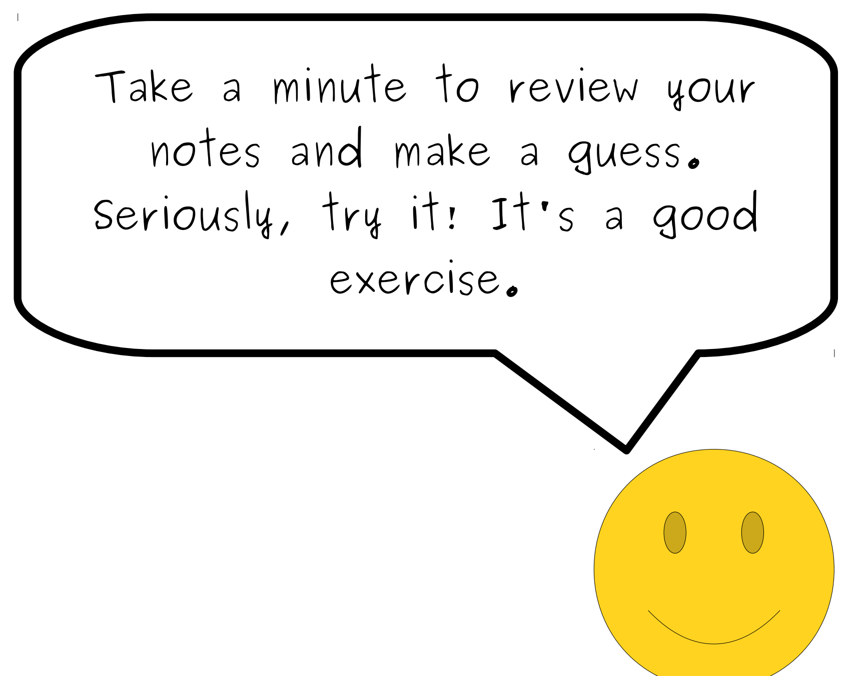
$$\neg(p \to (q \to r)) \quad (?)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \boxed{\frac{\neg(A \to B)}{A \land \neg B}}$$

$$\neg(p \to q \to r)$$

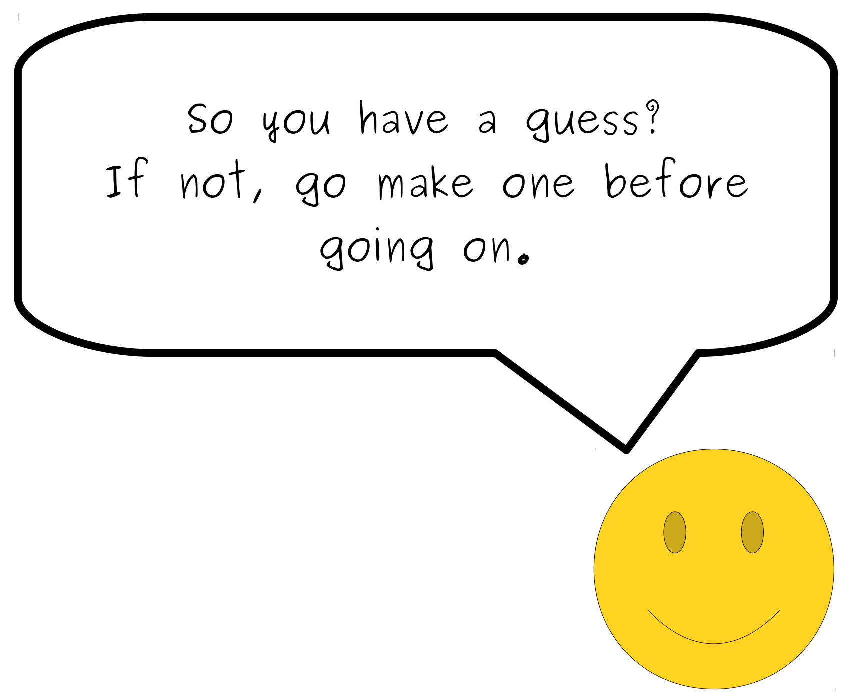
$$\neg((p \to q) \to r) \qquad (?)$$
$$\neg(p \to (q \to r)) \qquad (?)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \boxed{\frac{\neg(A \to B)}{A \land \neg B}}$$

$$\neg(p \to q \to r)$$

$$\neg((p \to q) \to r) \quad (?)$$
$$\neg(p \to (q \to r)) \quad (?)$$

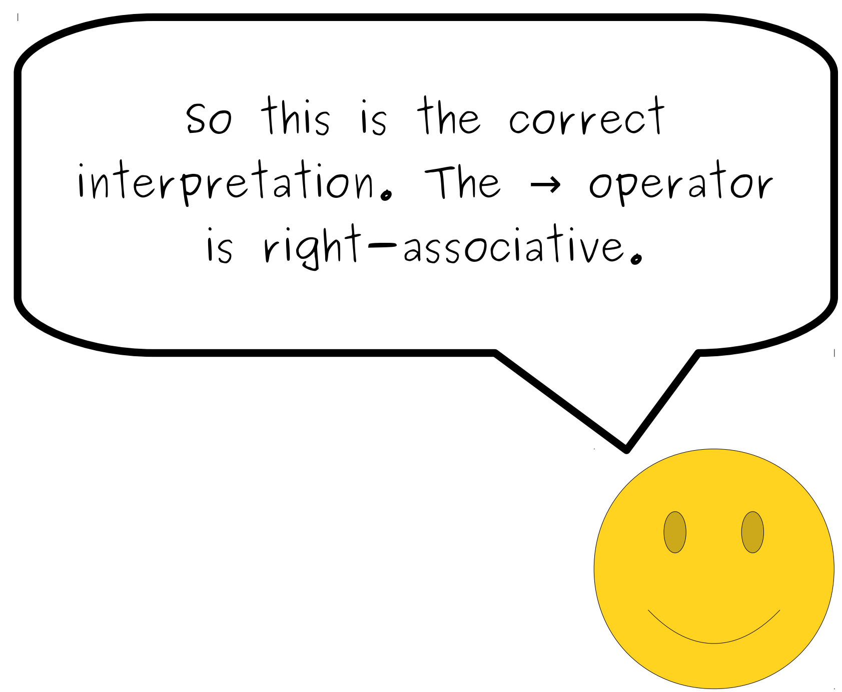


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\neg(p \to q \to r)$$

~~$\neg((p \to q) \to r)$ (?)~~

$\neg(p \to (q \to r))$

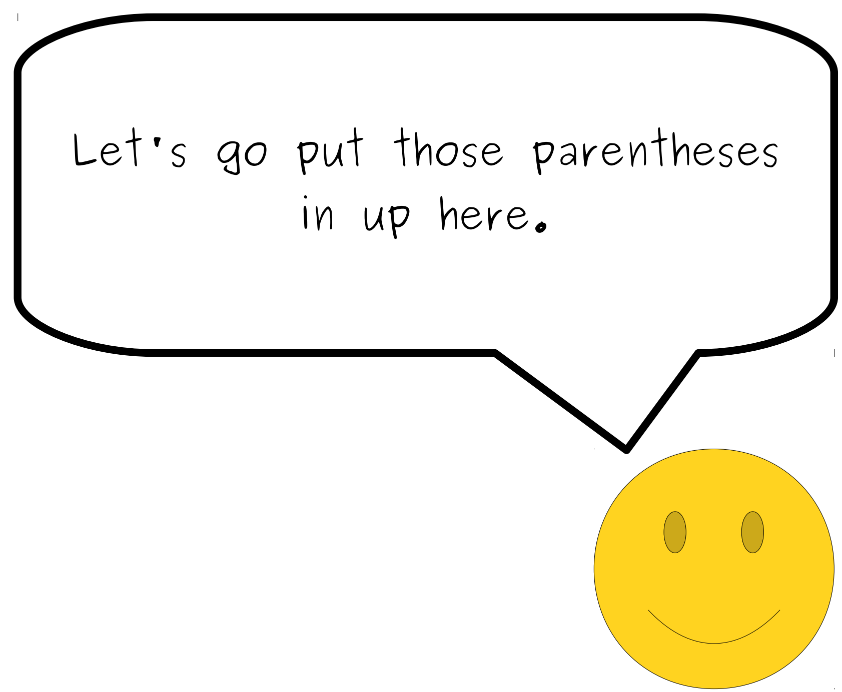


$$\frac{\neg\neg \boldsymbol{A}}{\boldsymbol{A}} \qquad \frac{\neg(\boldsymbol{A} \lor \boldsymbol{B})}{\neg \boldsymbol{A} \land \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\neg \boldsymbol{A} \lor \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \to \boldsymbol{B})}{\boldsymbol{A} \land \neg \boldsymbol{B}}$$

$$\neg(p \to (q \to r))$$

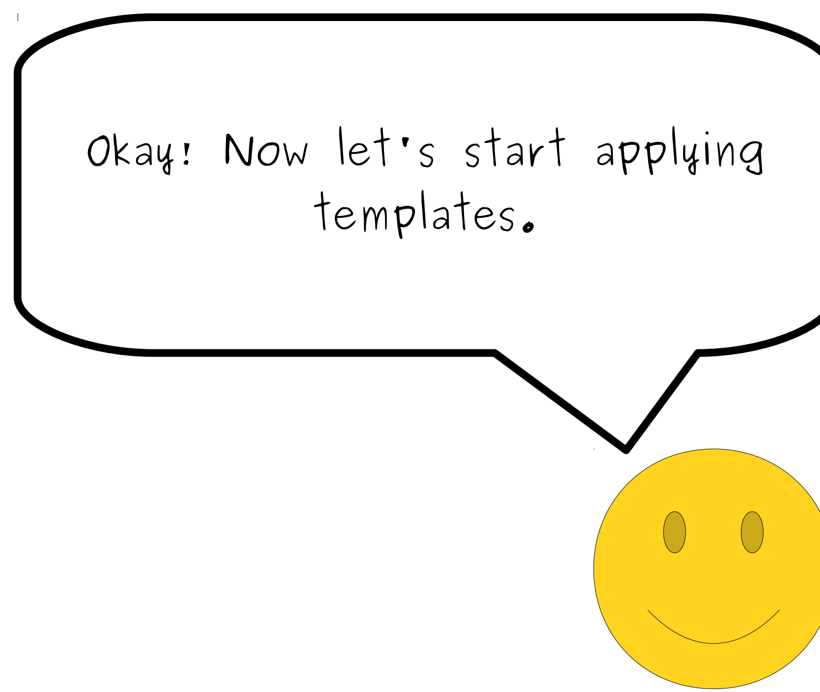
~~$\neg((p \to q) \to r)$~~ ~~(?)~~

$\neg(p \to (q \to r))$



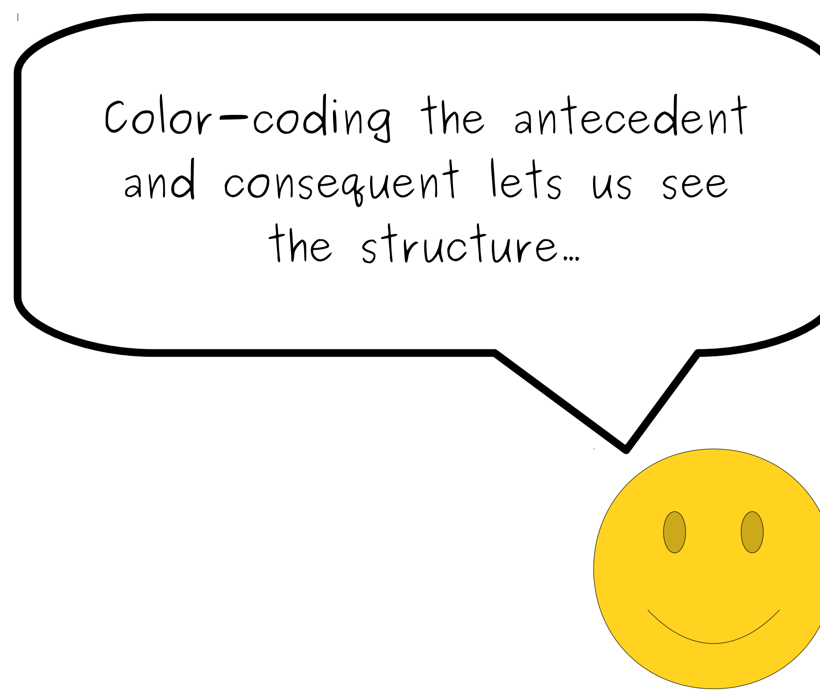
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\neg(p \to (q \to r))$$



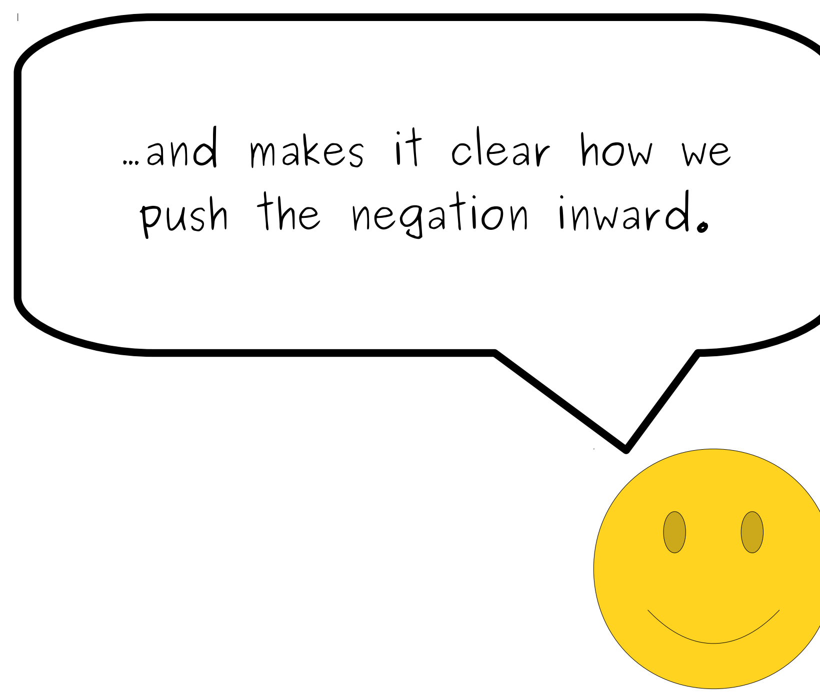
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \to B)}{A \wedge \neg B}$$

$$\neg(p \to (q \to r))$$



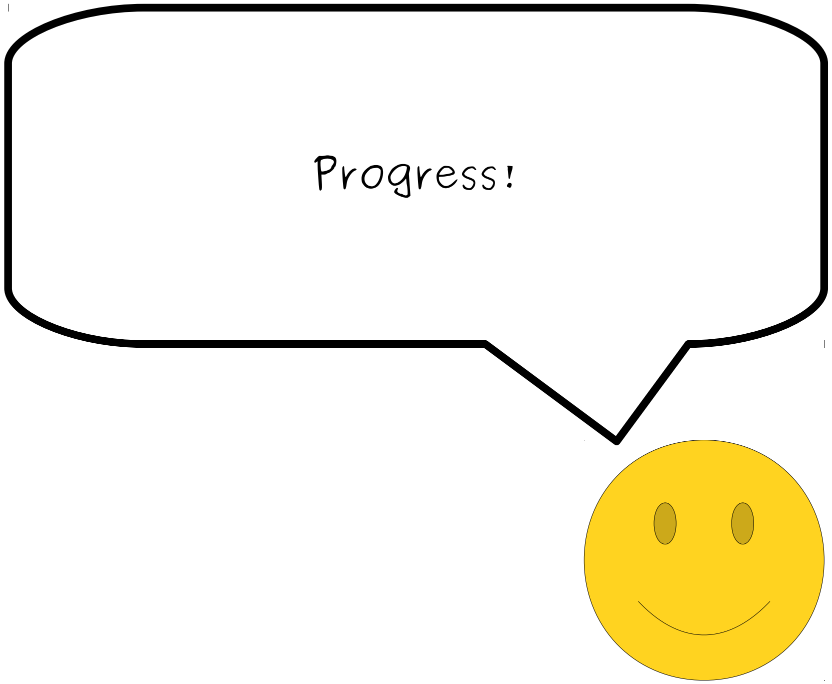
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land \neg(q \to r)$$



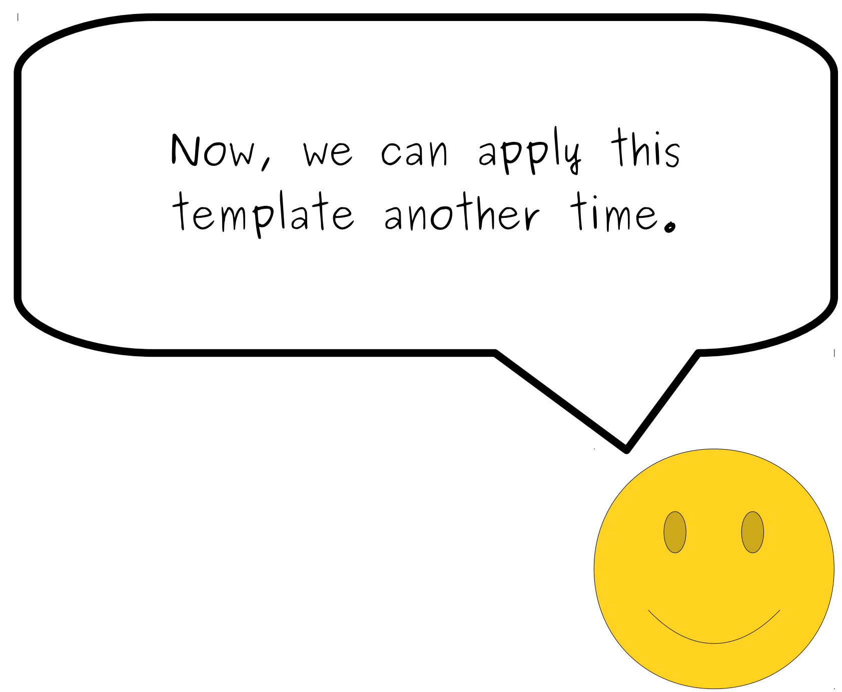
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land \neg(q \to r)$$



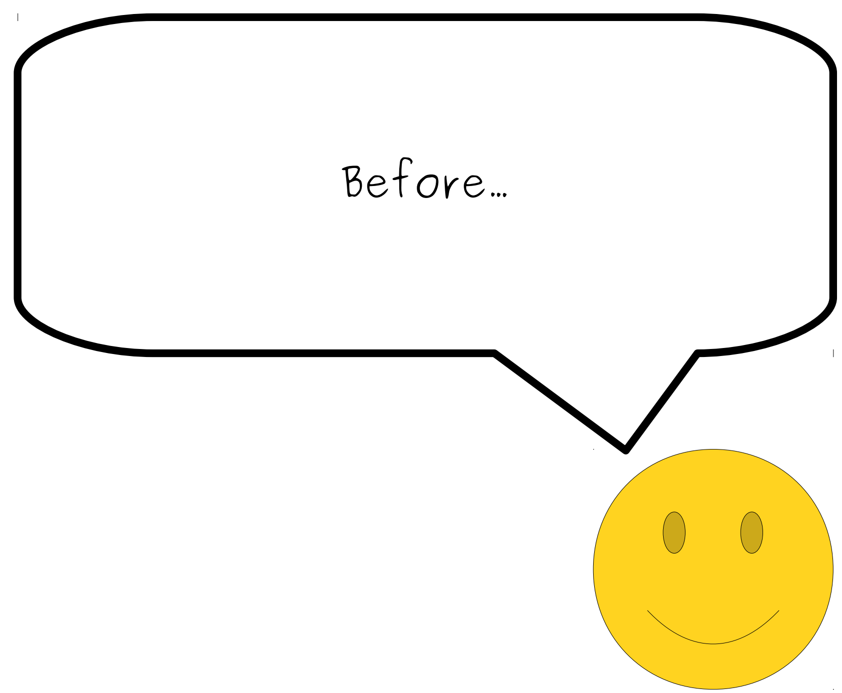
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land \neg(q \to r)$$



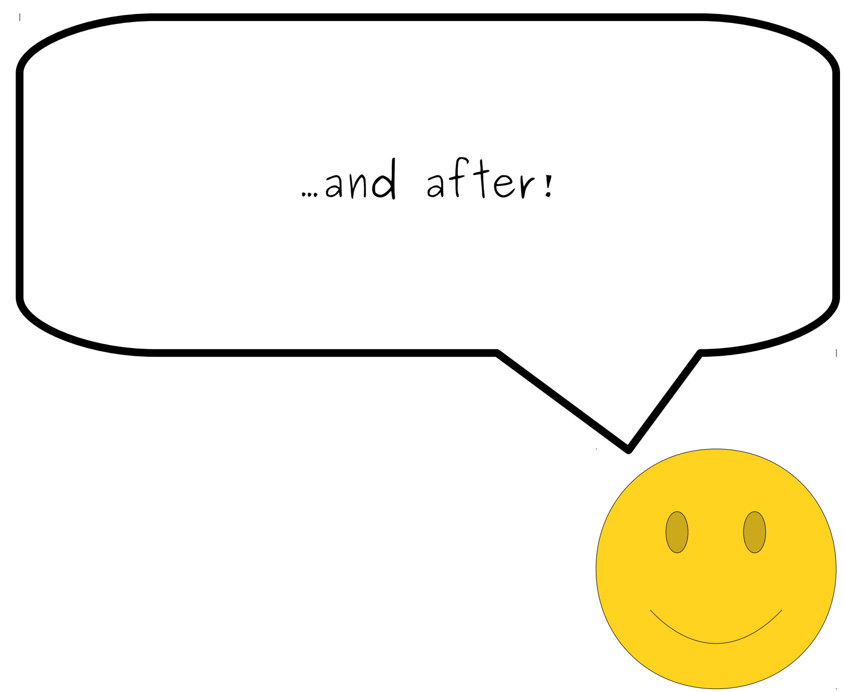
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land \neg(q \to r)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land q \land \neg r$$



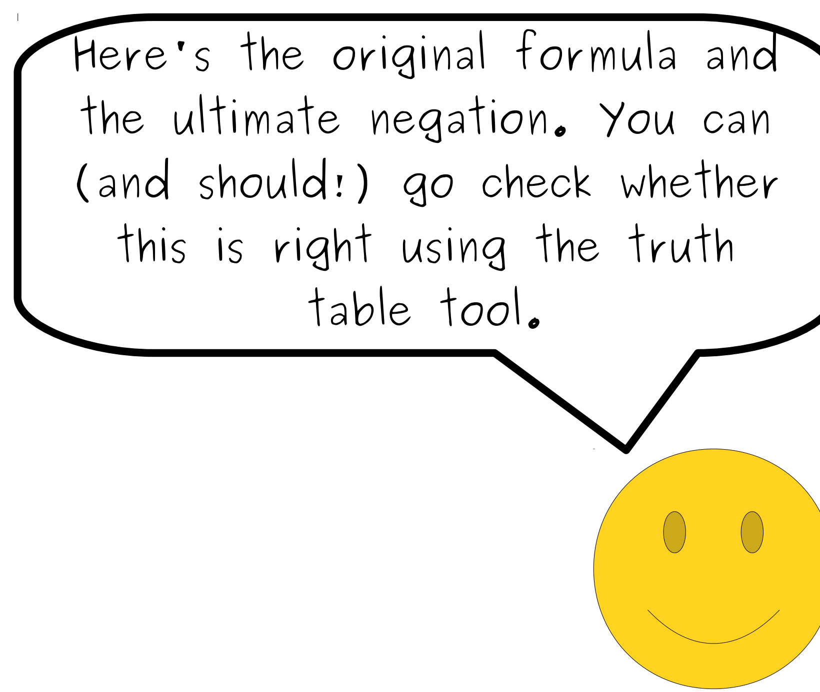
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land q \land \neg r$$

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land q \land \neg r$$

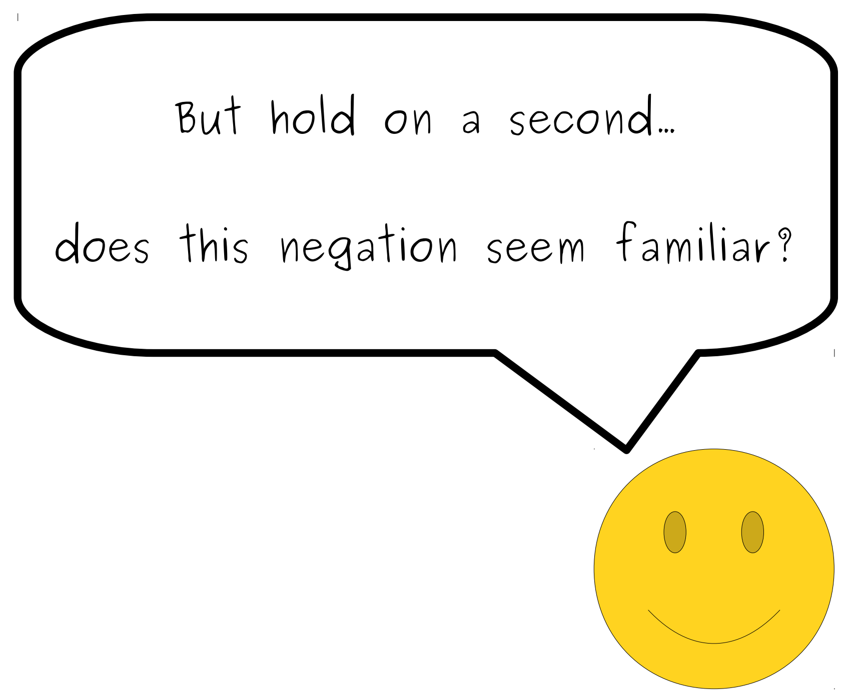
$$p \to q \to r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land q \land \neg r$$

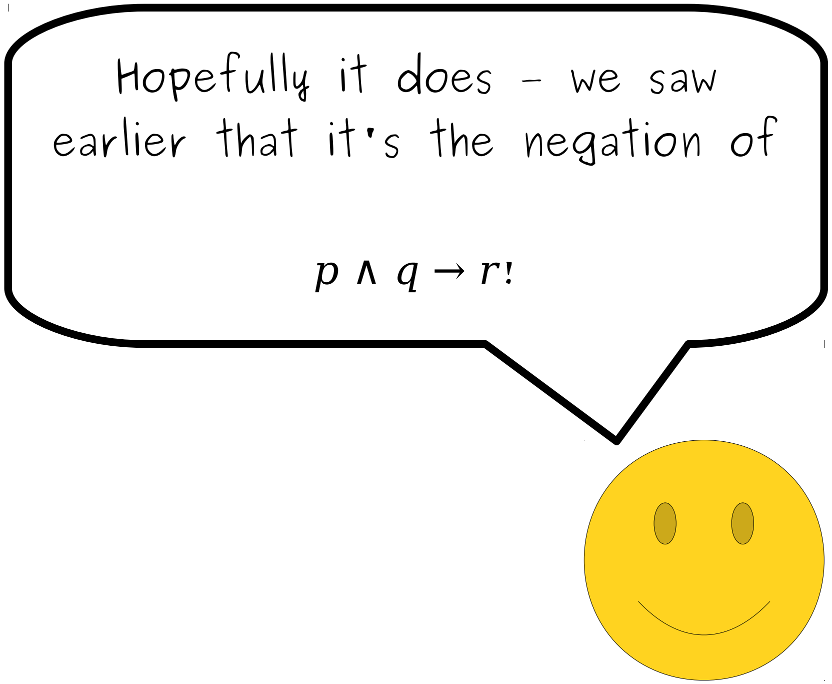
$$p \to q \to r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land q \land \neg r$$

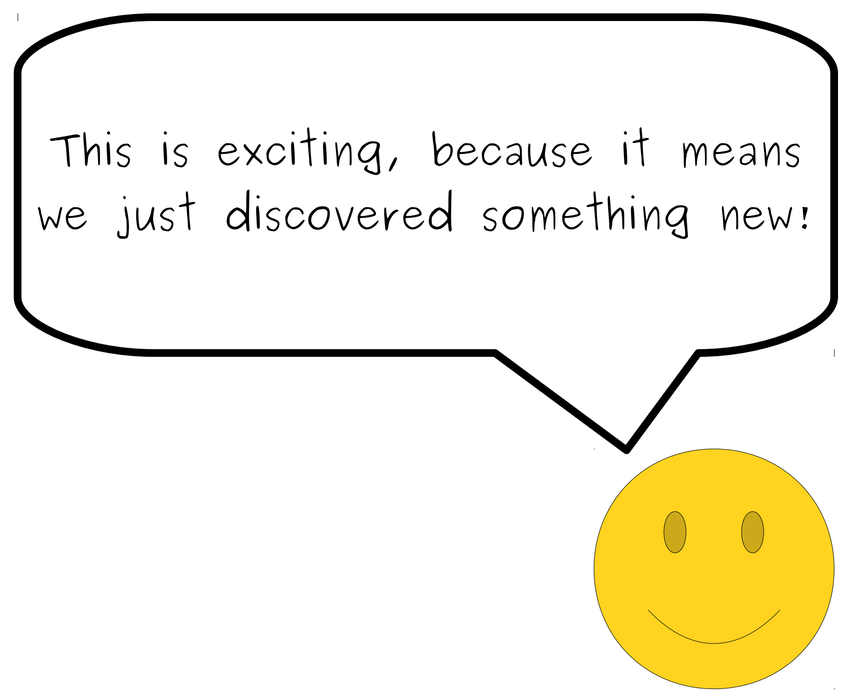
$$p \to q \to r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land q \land \neg r$$

$$p \to q \to r$$

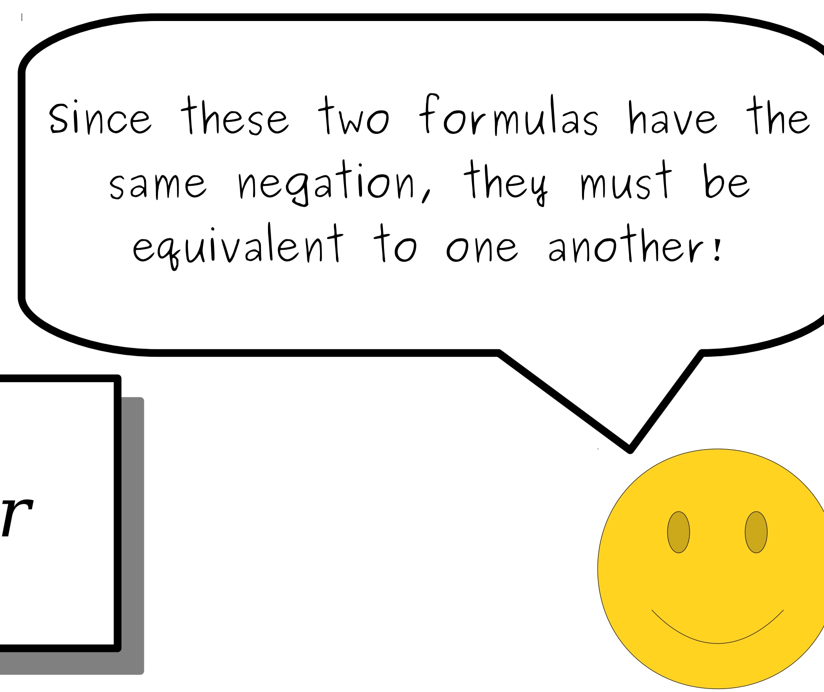


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$p \land q \land \neg r$$

$$p \to q \to r$$

$$p \to q \to r \quad \equiv \quad p \land q \to r$$

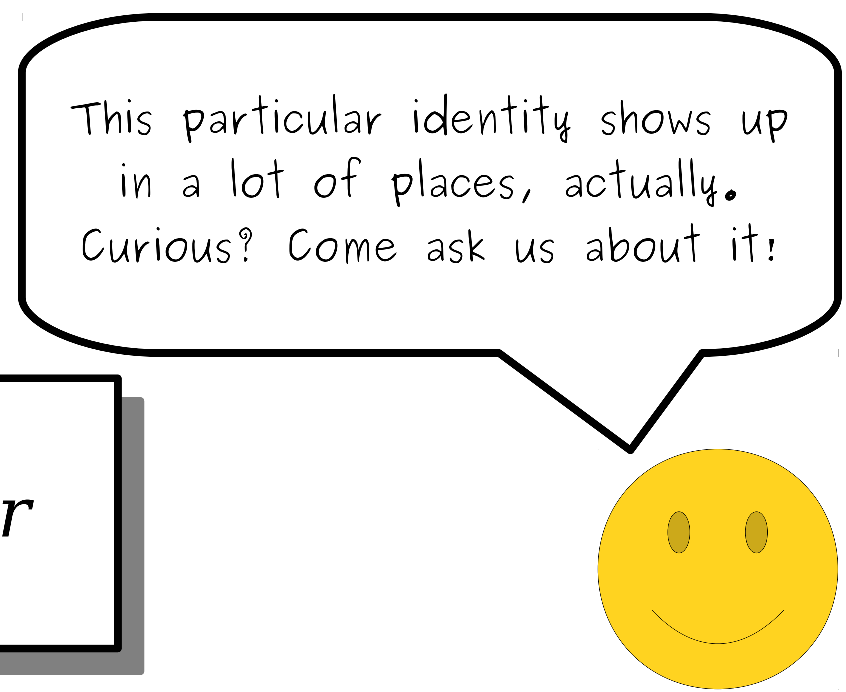


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

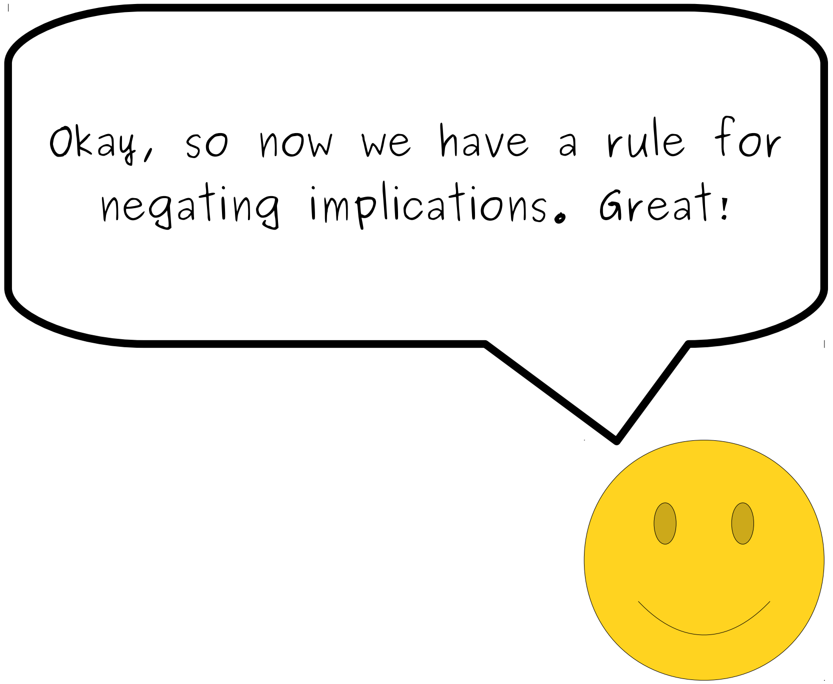
$$p \land q \land \neg r$$

$$p \to q \to r$$

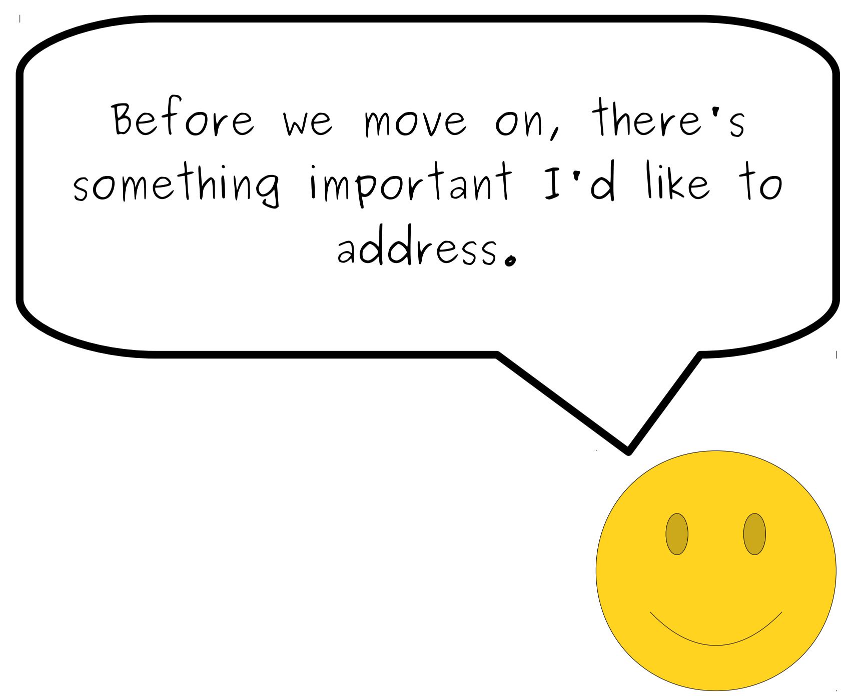
$$p \to q \to r \quad \equiv \quad p \land q \to r$$



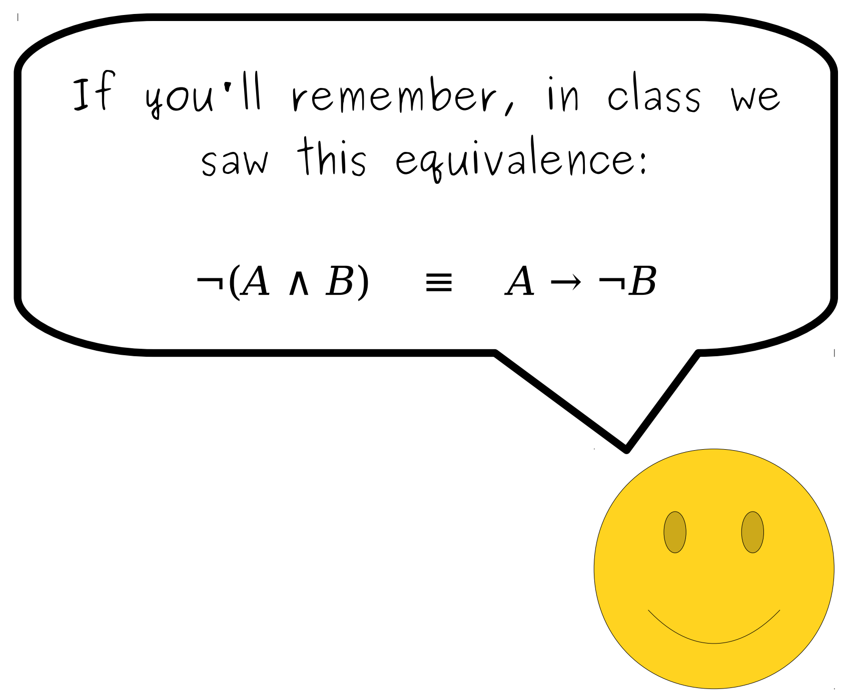
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

We can treat this as a new template for negating AND:

$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\frac{\neg\neg A}{A}$$

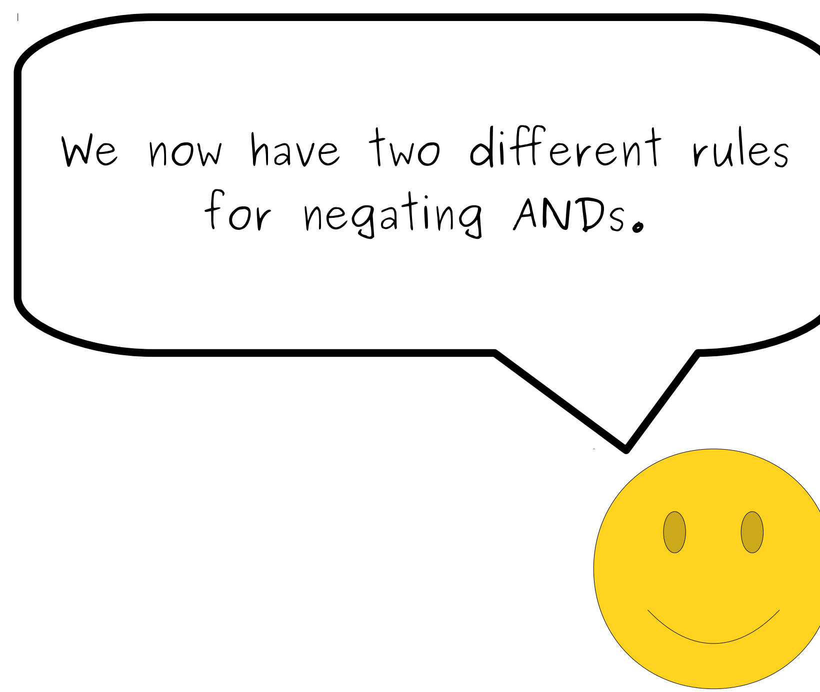
$$\frac{\neg(A \lor B)}{\neg A \land \neg B}$$

$$\frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\frac{\neg(A \land B)}{A \rightarrow \neg B}$$

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

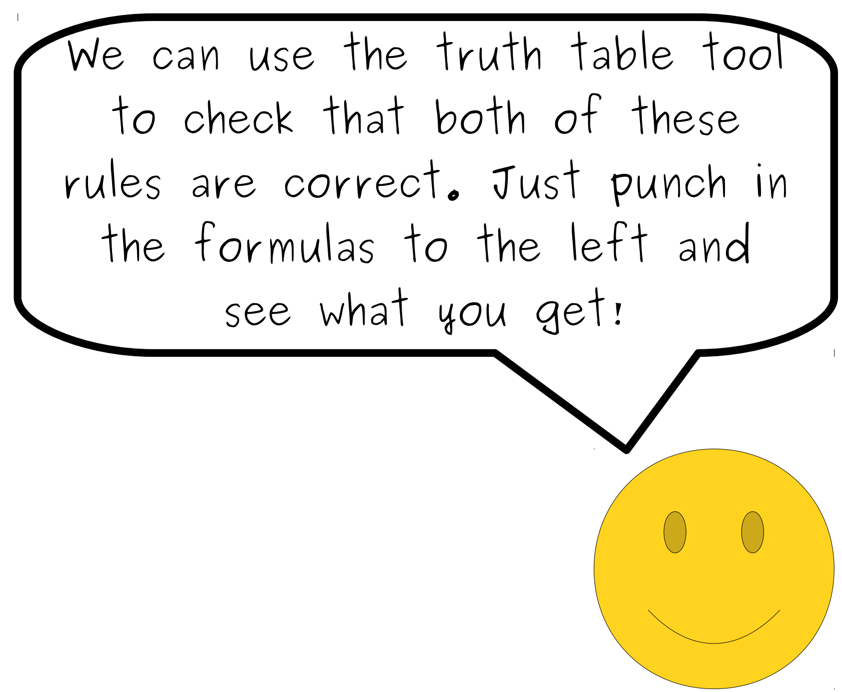


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \to B)}{A \wedge \neg B}$$

$$\frac{\neg(A \wedge B)}{A \to \neg B}$$

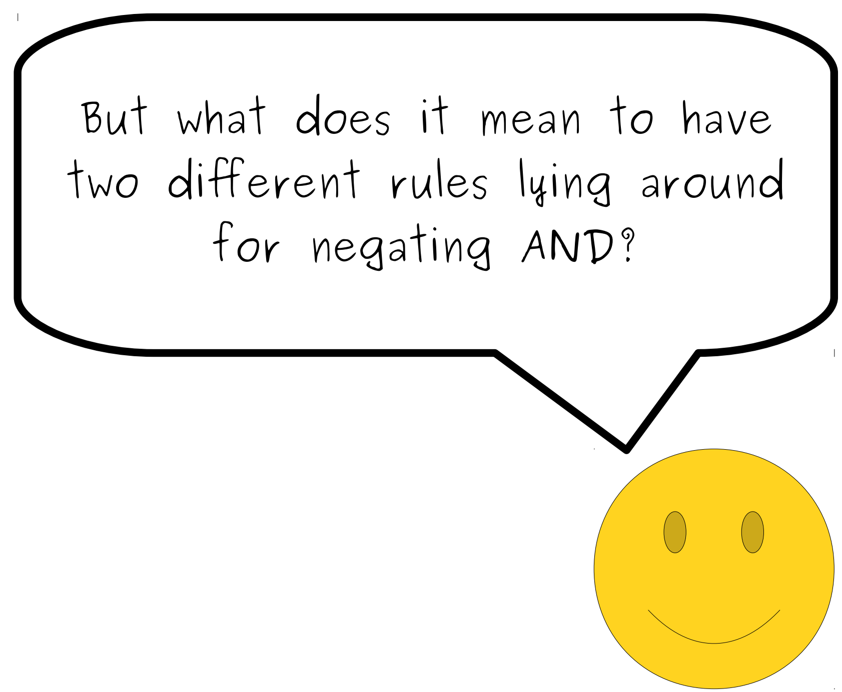
$$\neg(A \wedge B) \quad \leftrightarrow \quad (\neg A \vee \neg B)$$

$$\neg(A \wedge B) \quad \leftrightarrow \quad (A \to \neg B)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \to B)}{A \wedge \neg B}$$

$$\frac{\neg(A \wedge B)}{A \to \neg B}$$



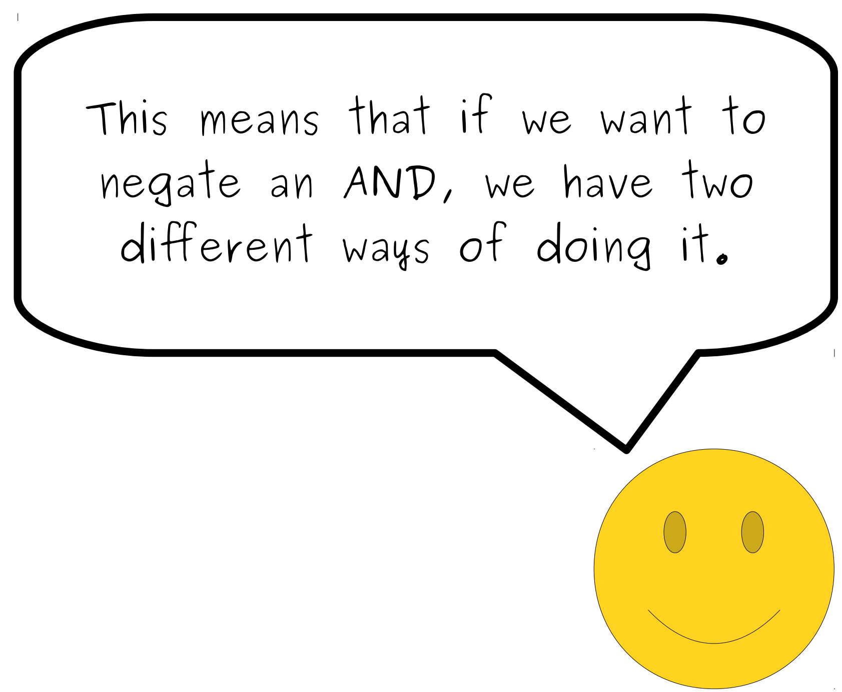
$$\frac{\neg\neg A}{A}$$

$$\frac{\neg(A \lor B)}{\neg A \land \neg B}$$

$$\frac{\neg(A \land B)}{\neg A \lor \neg B}$$

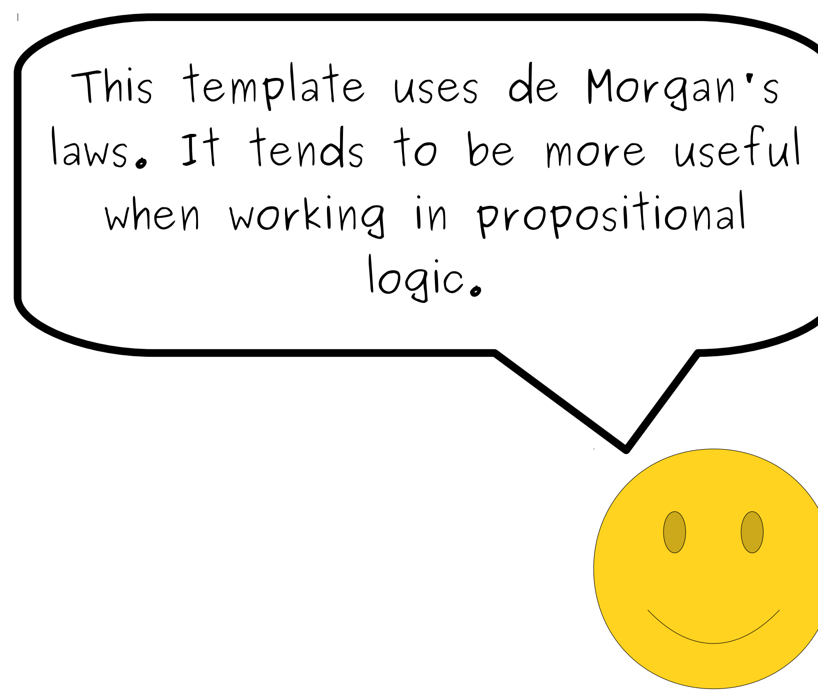
$$\frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$



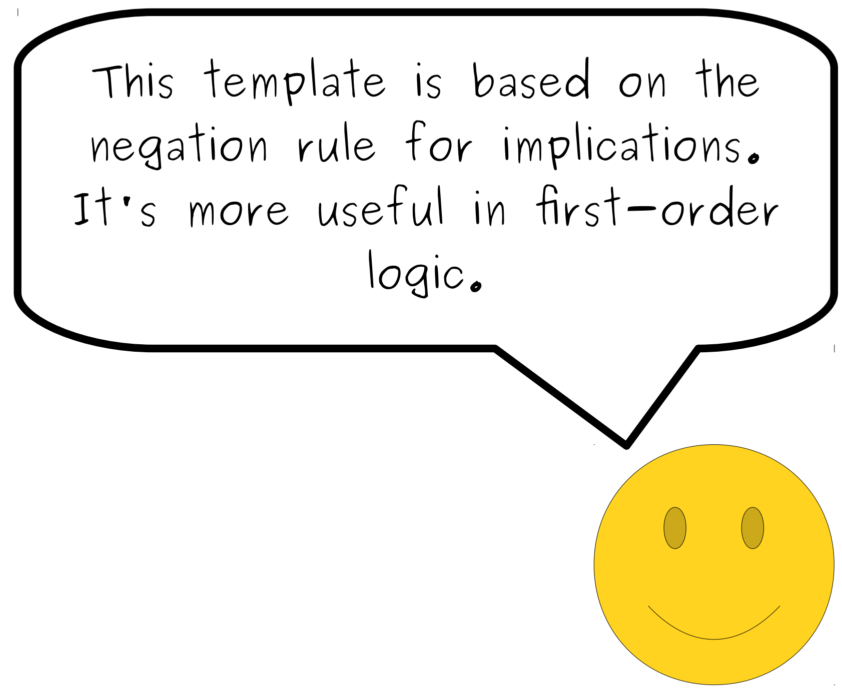
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$



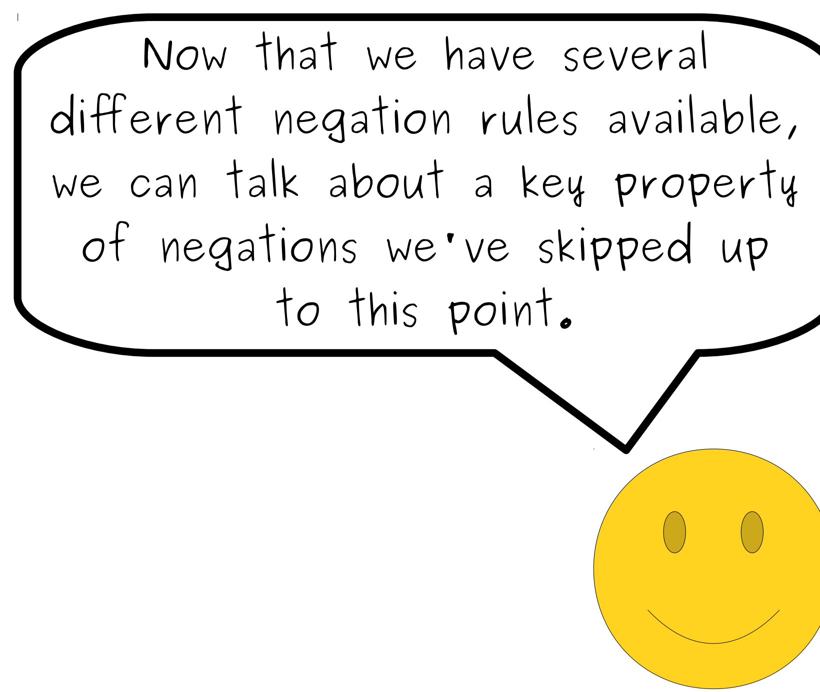
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \to B)}{A \wedge \neg B}$$

$$\frac{\neg(A \wedge B)}{A \to \neg B}$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

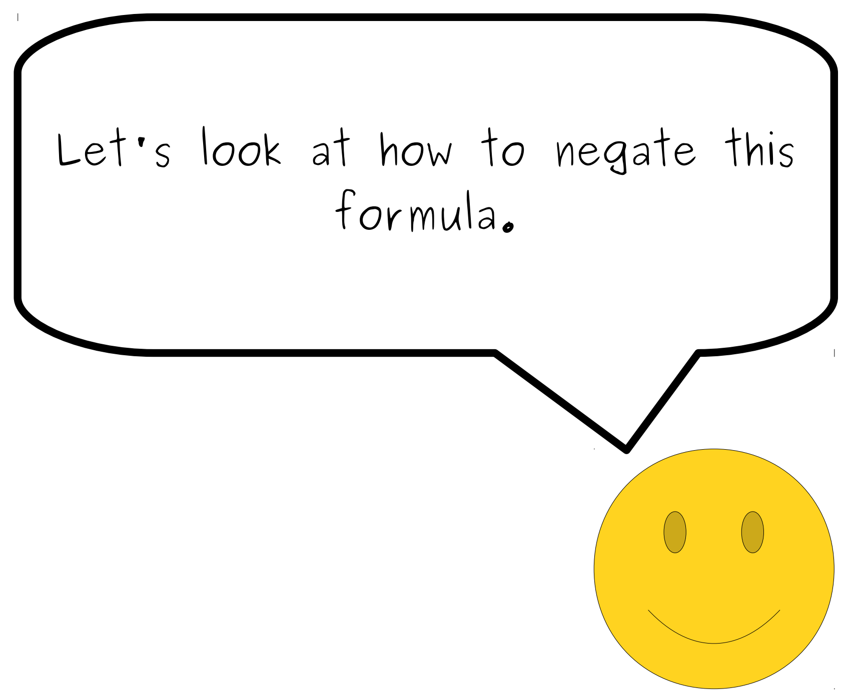
$$\frac{\neg(A \land B)}{A \to \neg B}$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

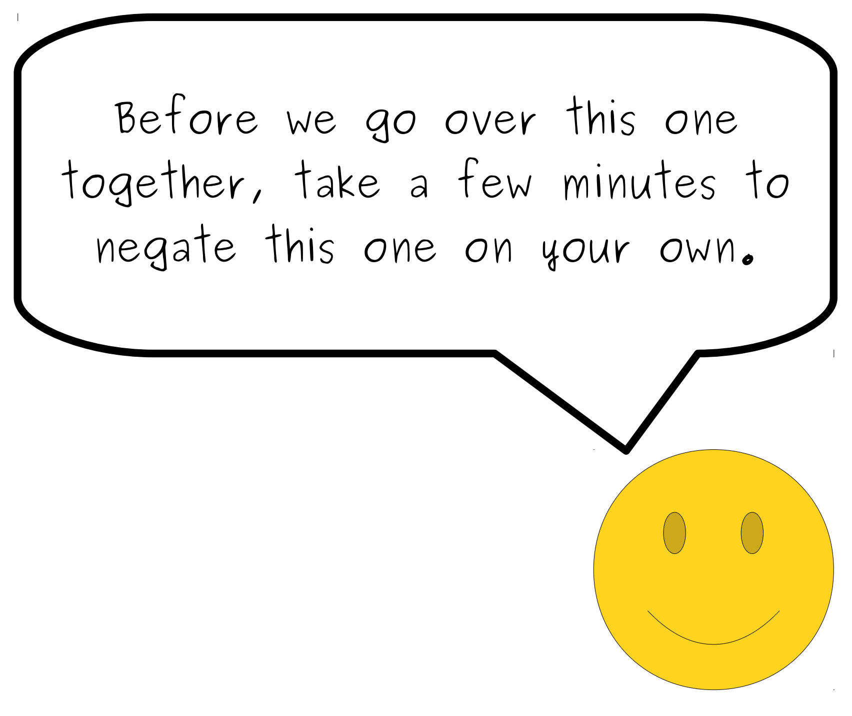
$$p \land q \land r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$p \land q \land r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \rightarrow B)}{A \wedge \neg B}$$

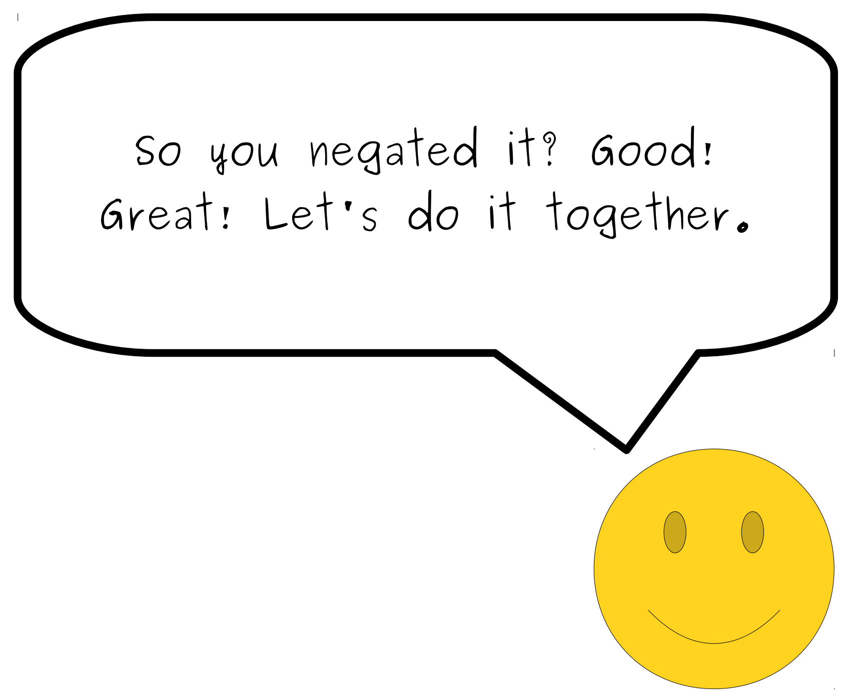
$$\frac{\neg(A \wedge B)}{A \rightarrow \neg B}$$

$$p \wedge q \wedge r$$

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

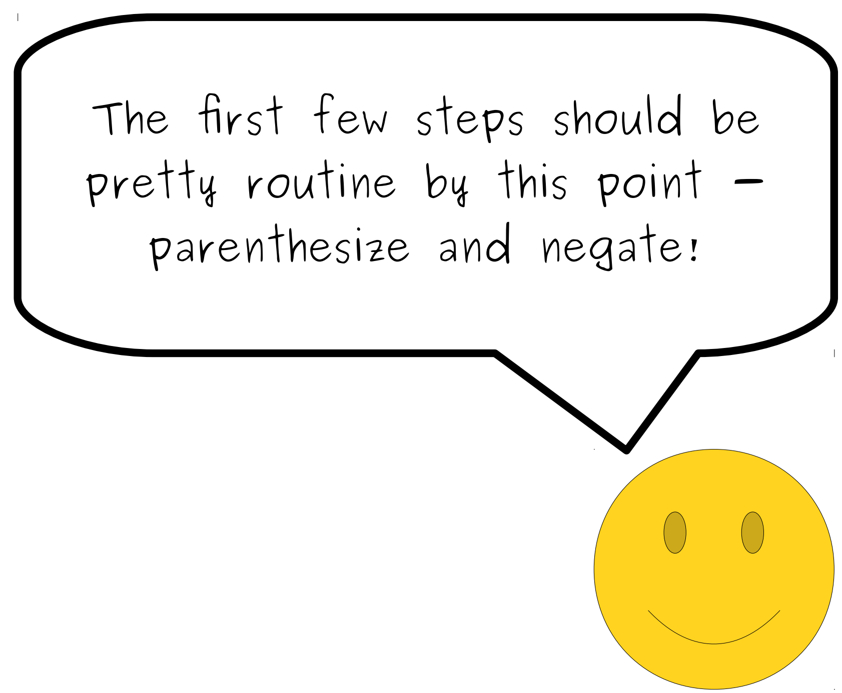
$$p \land q \land r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg(p \land q \land r)$$



$$\frac{\neg\neg A}{A}$$

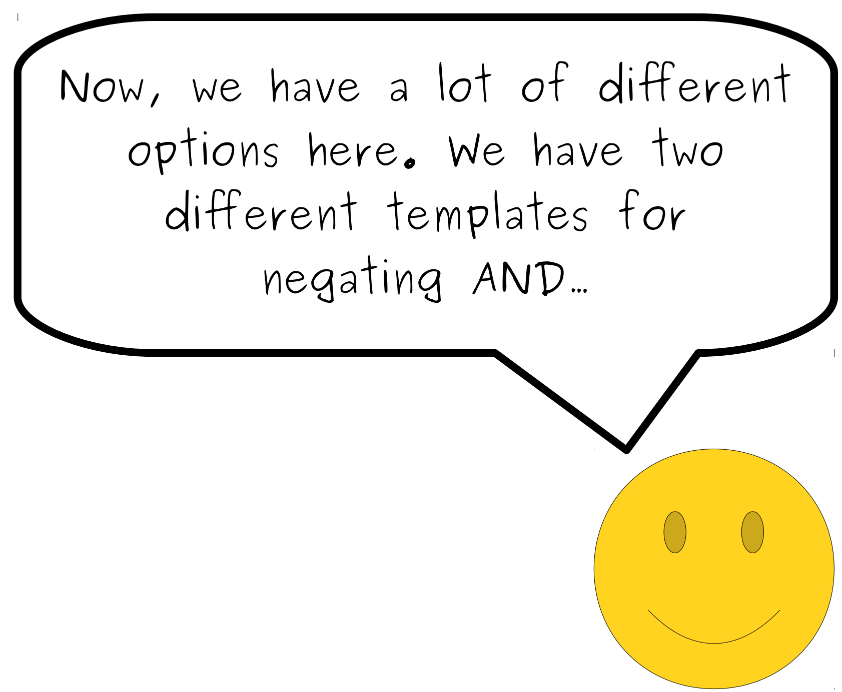
$$\frac{\neg(A \lor B)}{\neg A \land \neg B}$$

$$\frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\frac{\neg(A \to B)}{A \land \neg B}$$

$$\neg(p \land q \land r)$$

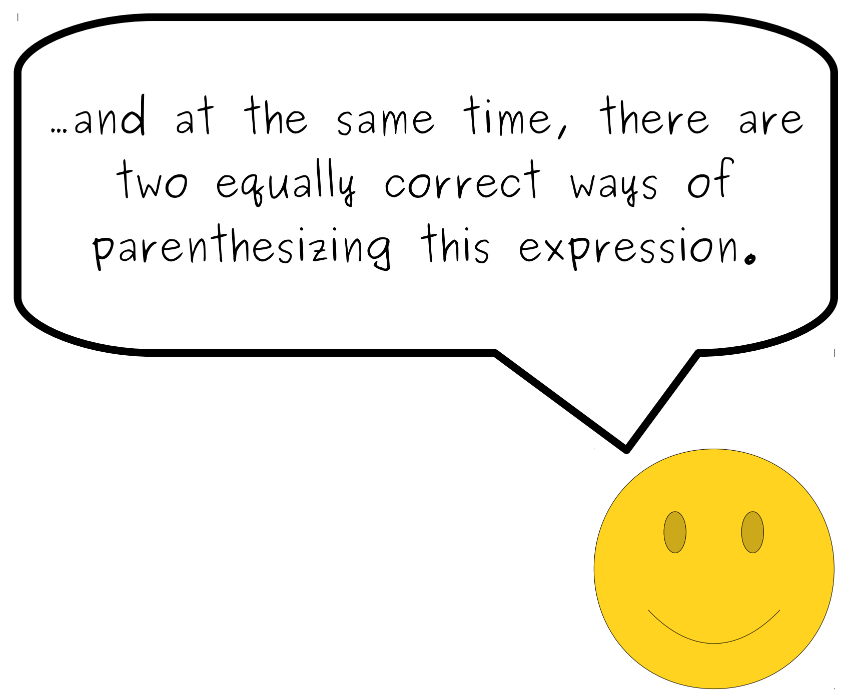


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg(p \land q \land r)$$

$$\neg((p \land q) \land r)$$
$$\neg(p \land (q \land r))$$



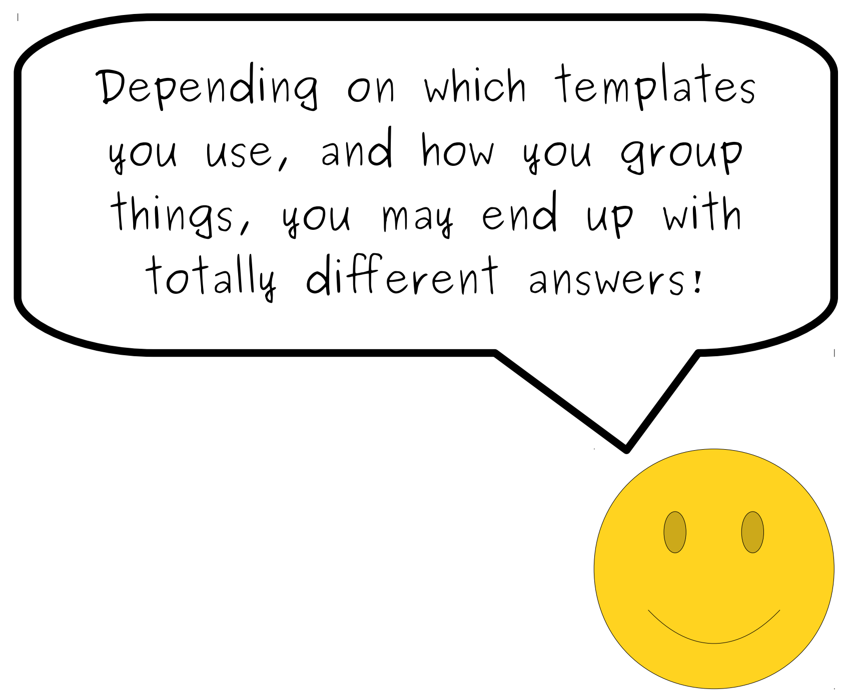
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg(p \land q \land r)$$

$$\neg((p \land q) \land r)$$

$$\neg(p \land (q \land r))$$

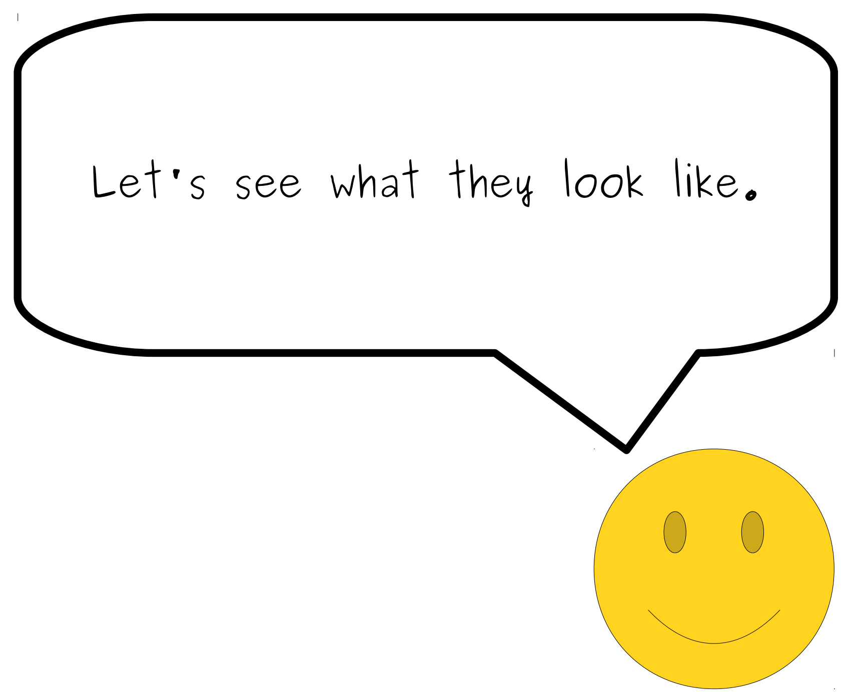


$$\frac{\neg\neg \boldsymbol{A}}{\boldsymbol{A}} \qquad \frac{\neg(\boldsymbol{A} \lor \boldsymbol{B})}{\neg \boldsymbol{A} \land \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\neg \boldsymbol{A} \lor \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \to \boldsymbol{B})}{\boldsymbol{A} \land \neg \boldsymbol{B}}$$

$$\frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\boldsymbol{A} \to \neg \boldsymbol{B}}$$

$$\neg(p \land q \land r)$$

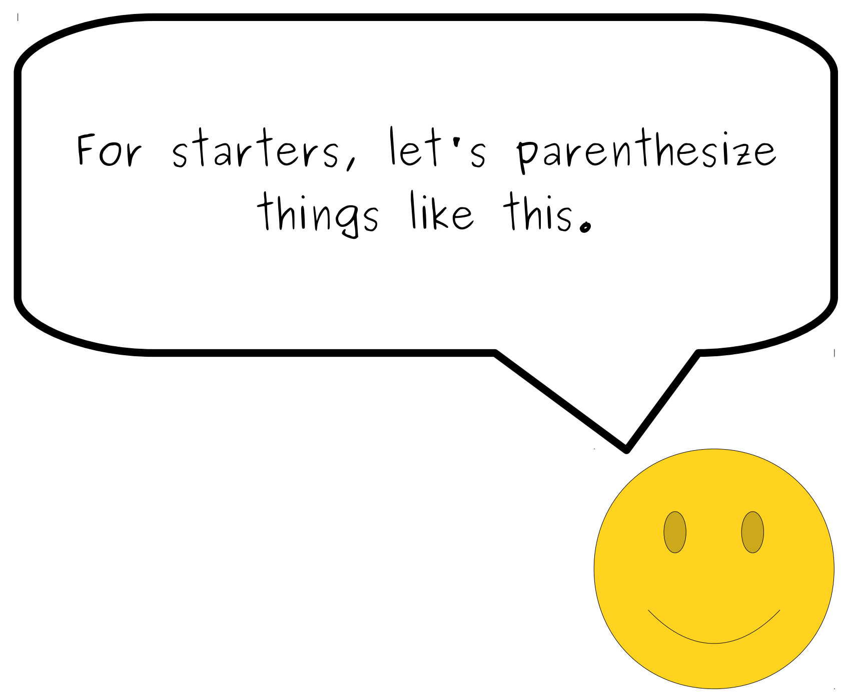
$$\neg((p \land q) \land r)$$
$$\neg(p \land (q \land r))$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

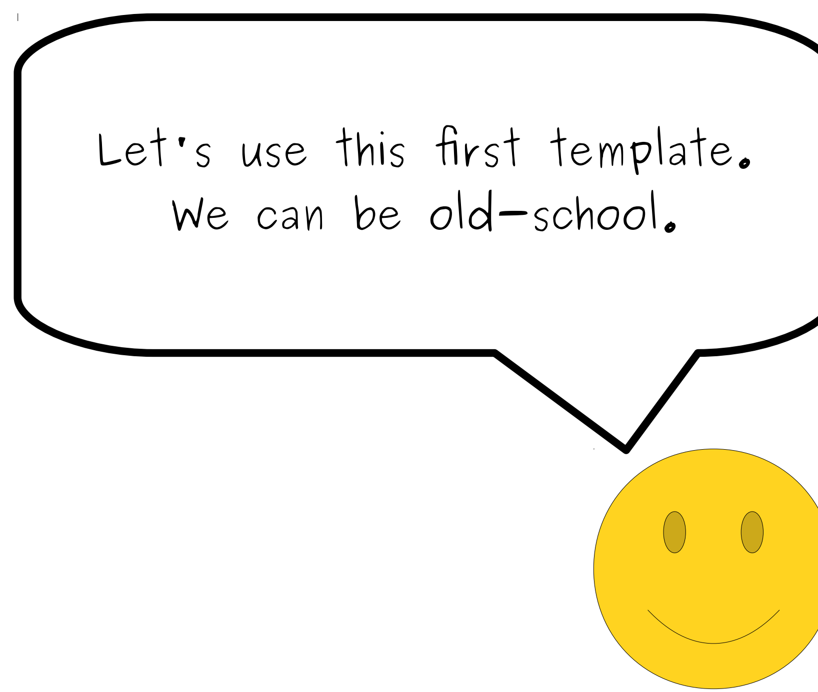
$$\neg(p \land (q \land r))$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

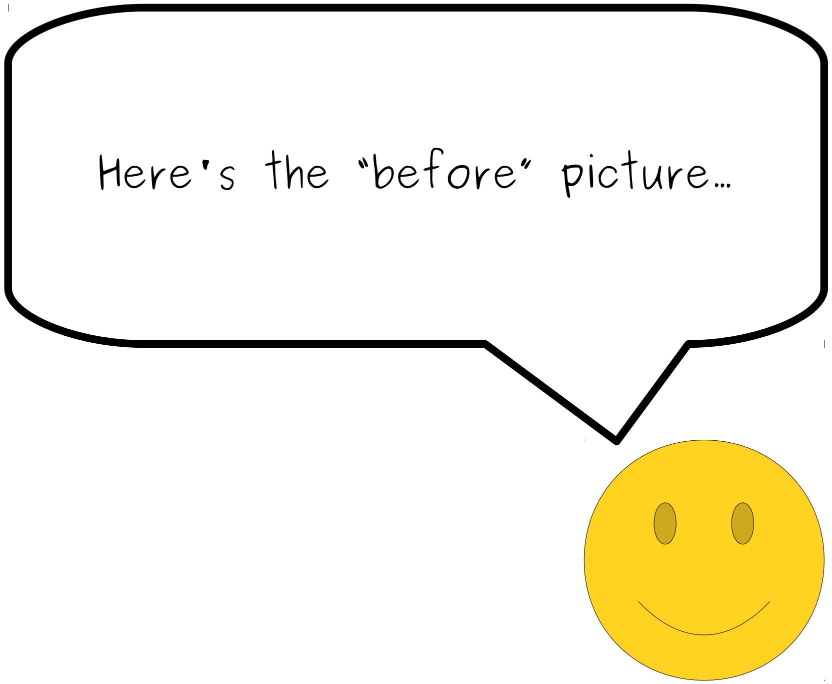
$$\neg(p \land (q \land r))$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

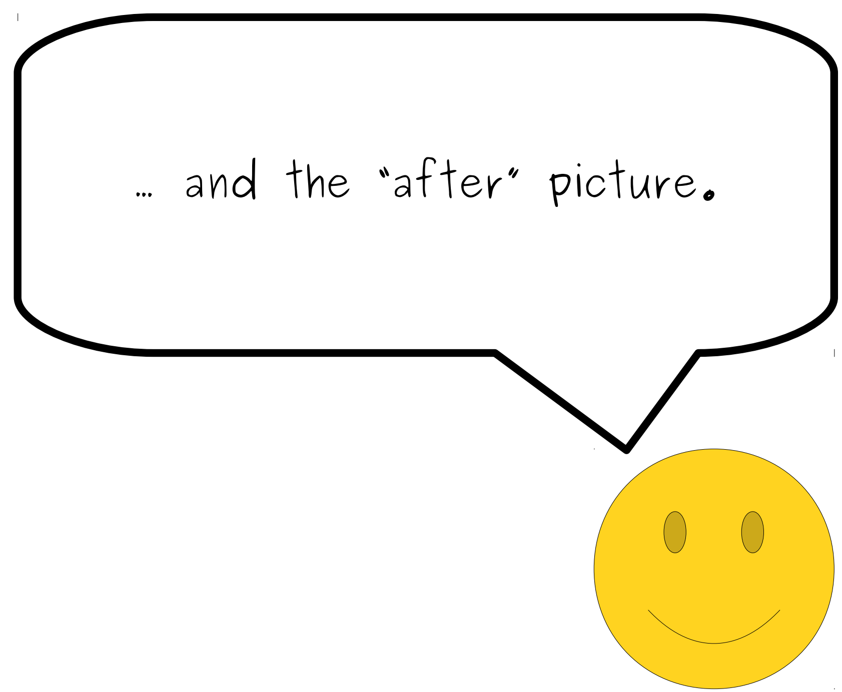
$$\neg(p \land (q \land r))$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

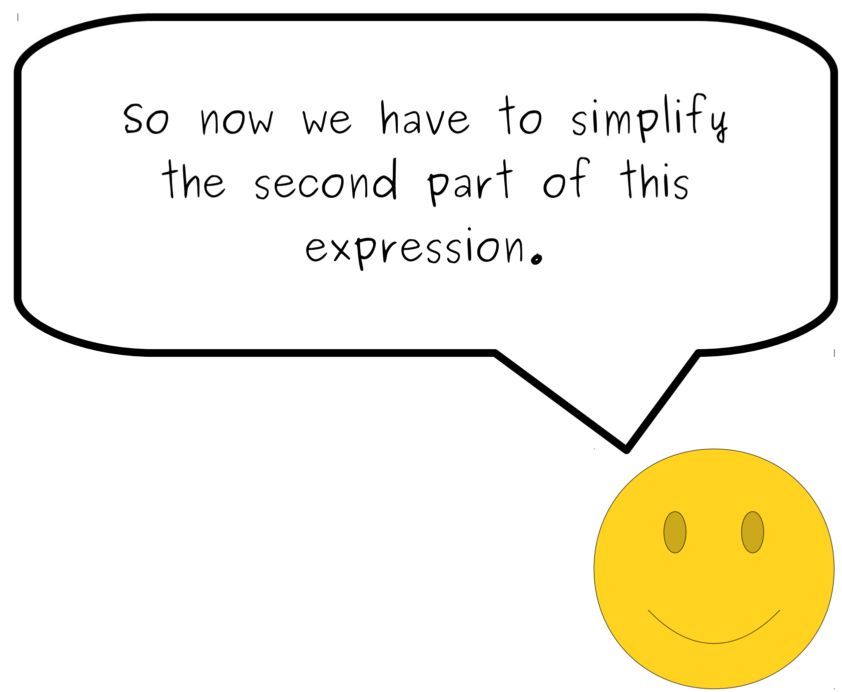
$$\neg p \lor \neg(q \land r)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

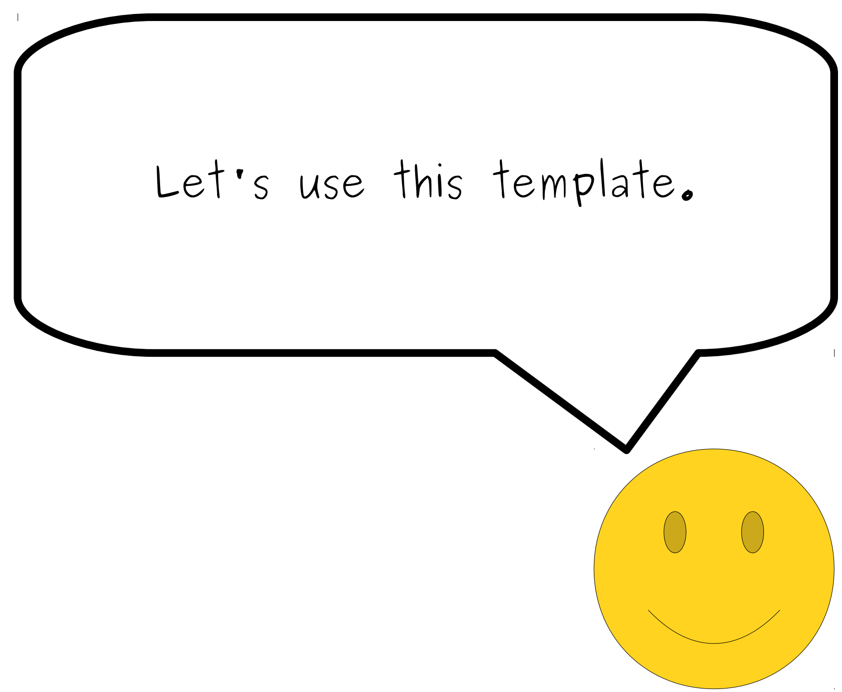
$$\neg p \lor \neg(q \land r)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

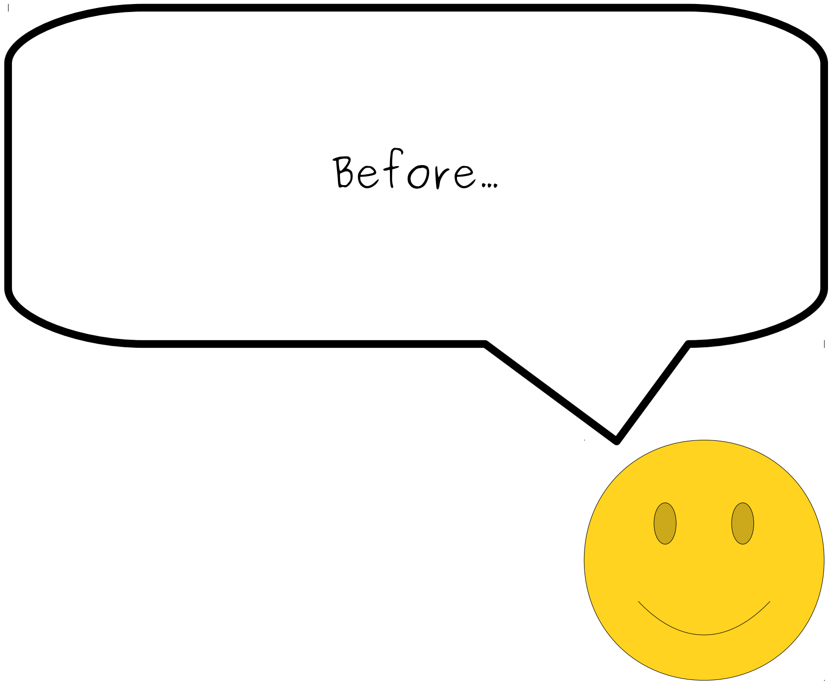
$$\neg p \lor \neg(q \land r)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \to B)}{A \wedge \neg B}$$

$$\frac{\neg(A \wedge B)}{A \to \neg B}$$

$$\neg p \vee \neg(q \wedge r)$$



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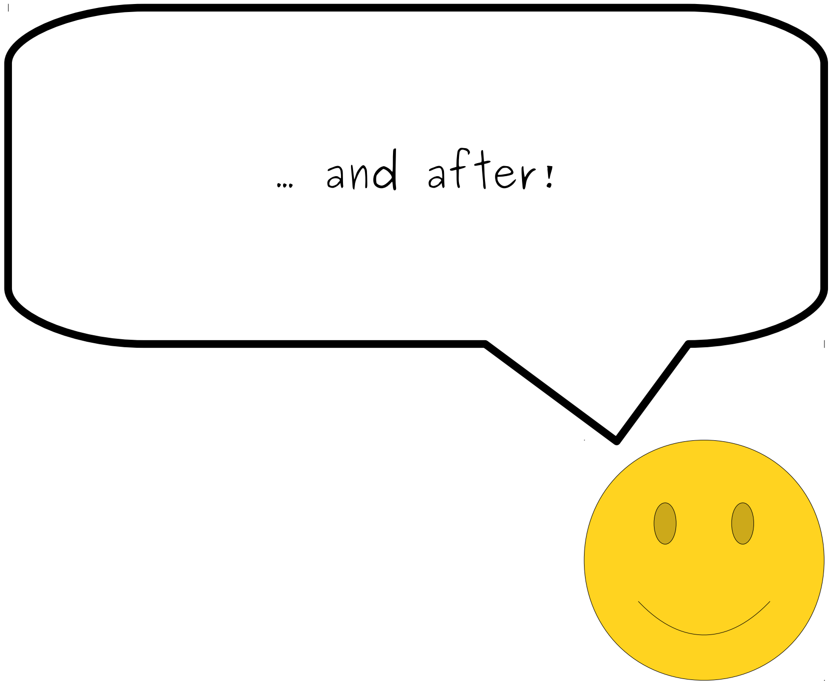
$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$



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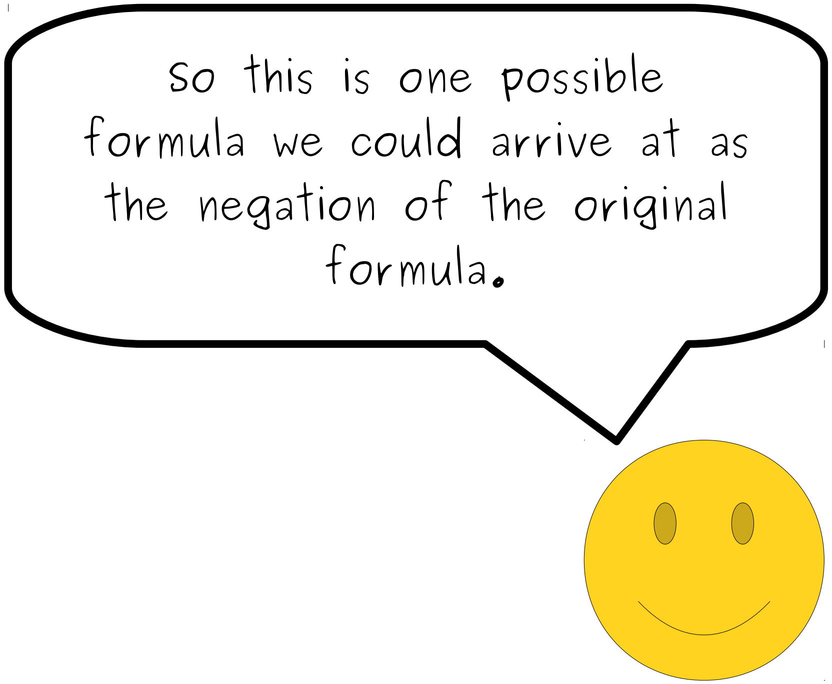
$$\frac{\neg(A \wedge B)}{A \rightarrow \neg B}$$

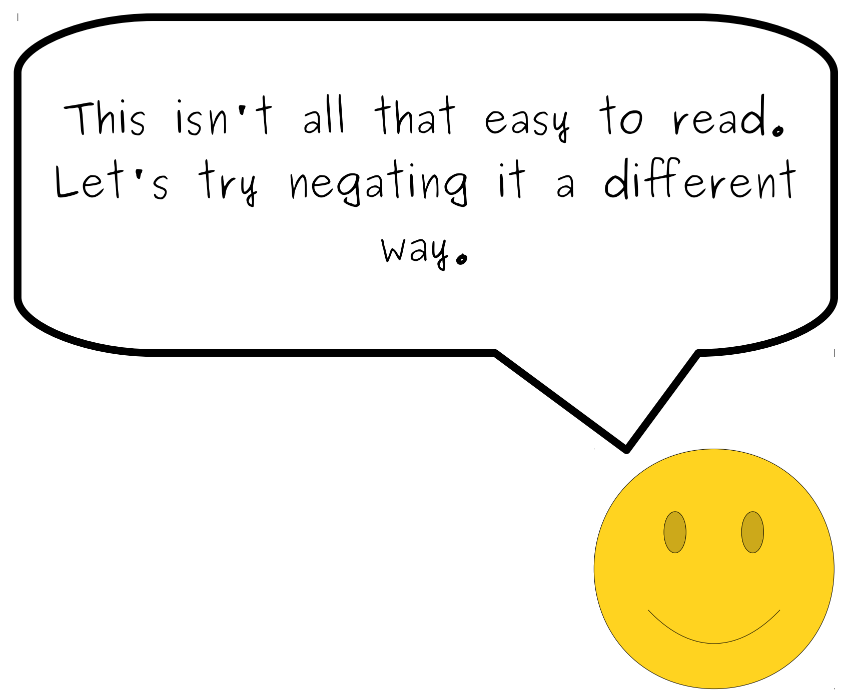
$$\neg p \vee (q \rightarrow \neg r)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$



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$$\frac{\neg(A \wedge B)}{A \to \neg B}$$

$$\neg p \vee (q \to \neg r)$$

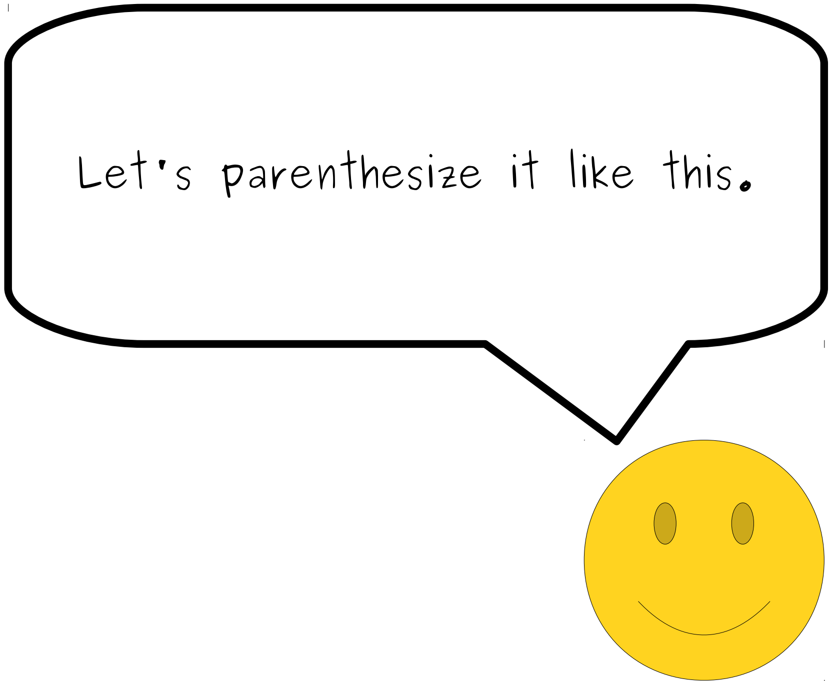
$$\neg(p \wedge q \wedge r)$$

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

$\neg p \lor (q \to \neg r)$

$\neg((p \land q) \land r)$



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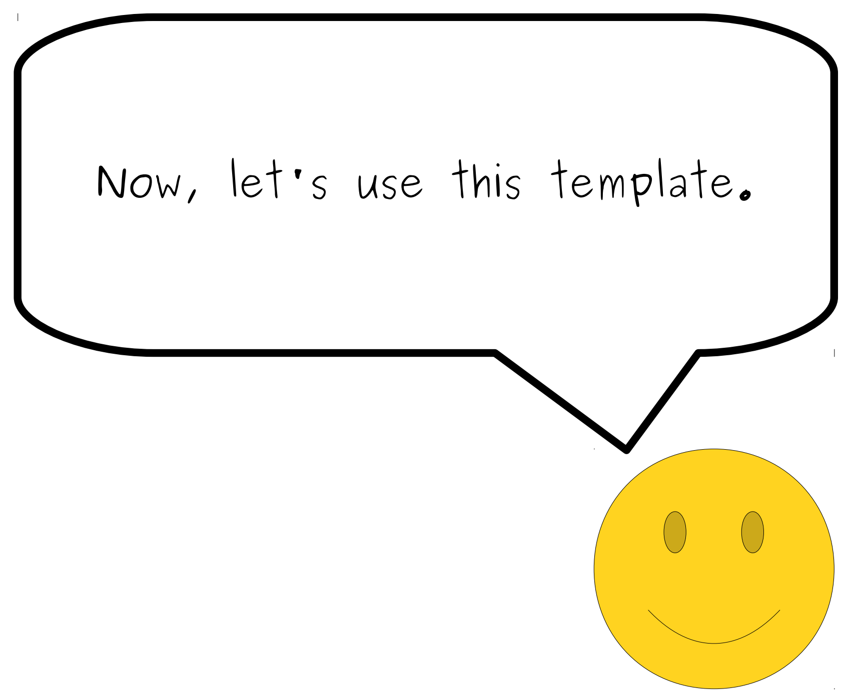
$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$

$$\neg((p \land q) \land r)$$

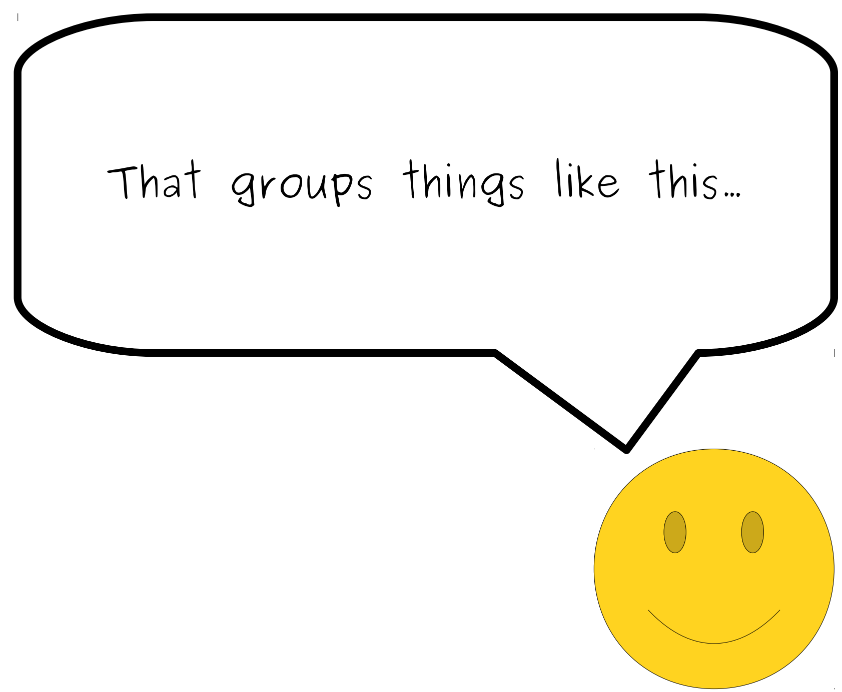


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \rightarrow B)}{A \wedge \neg B}$$

$$\frac{\neg(A \wedge B)}{A \rightarrow \neg B}$$

$$\neg p \vee (q \rightarrow \neg r)$$

$$\neg((p \wedge q) \wedge r)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$

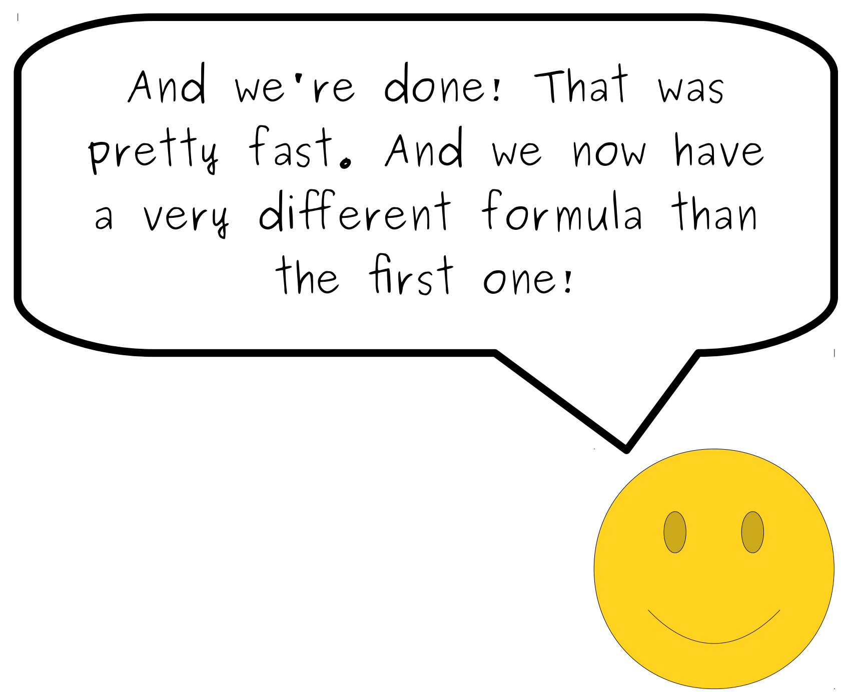
$$p \land q \to \neg r$$

$$\frac{\neg\neg \boldsymbol{A}}{\boldsymbol{A}} \qquad \frac{\neg(\boldsymbol{A} \lor \boldsymbol{B})}{\neg \boldsymbol{A} \land \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\neg \boldsymbol{A} \lor \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \to \boldsymbol{B})}{\boldsymbol{A} \land \neg \boldsymbol{B}}$$

$$\frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\boldsymbol{A} \to \neg \boldsymbol{B}}$$

$$\neg p \lor (q \to \neg r)$$

$$p \land q \to \neg r$$

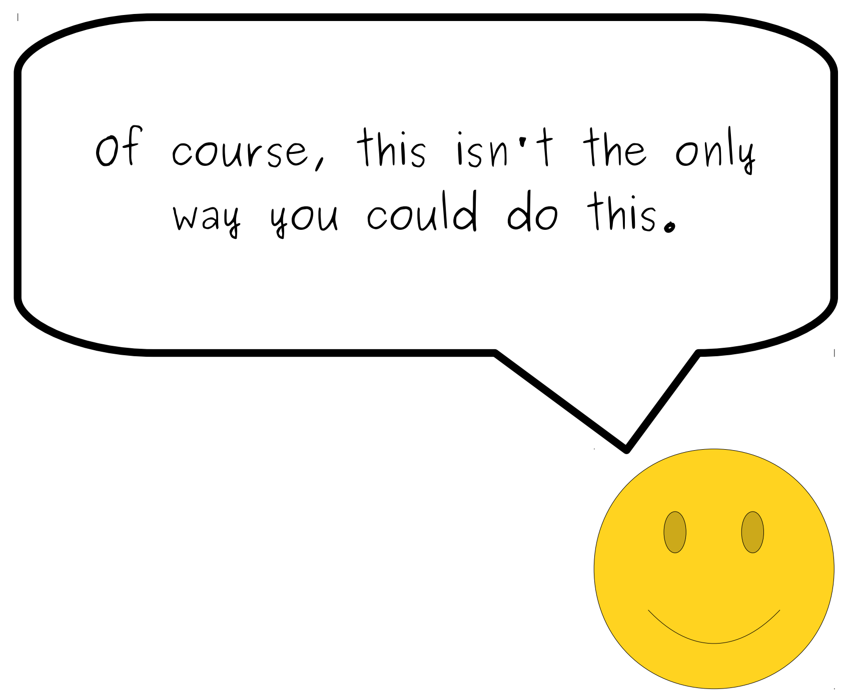


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$

$$p \land q \to \neg r$$



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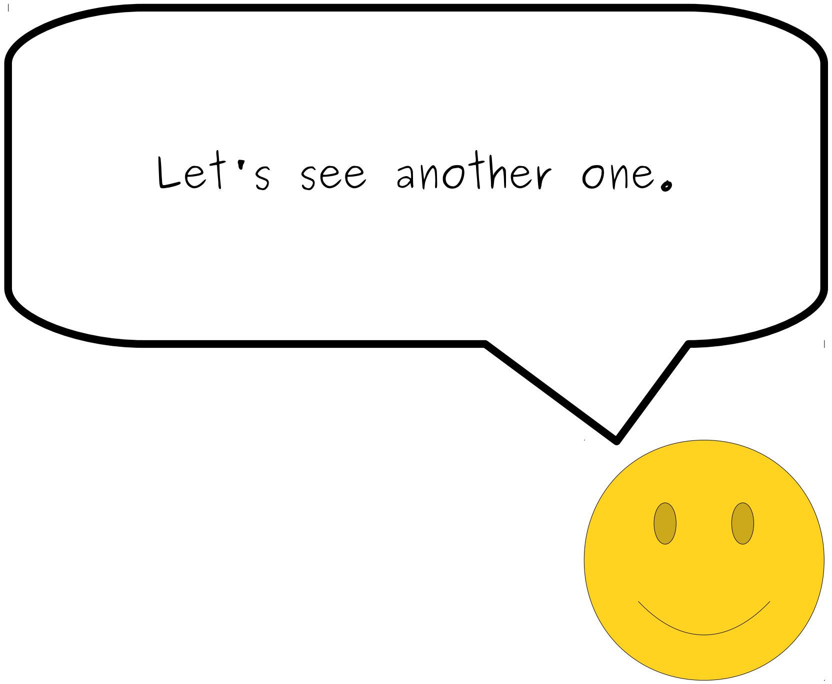
$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$

$$p \land q \to \neg r$$

$$\neg(p \land q \land r)$$



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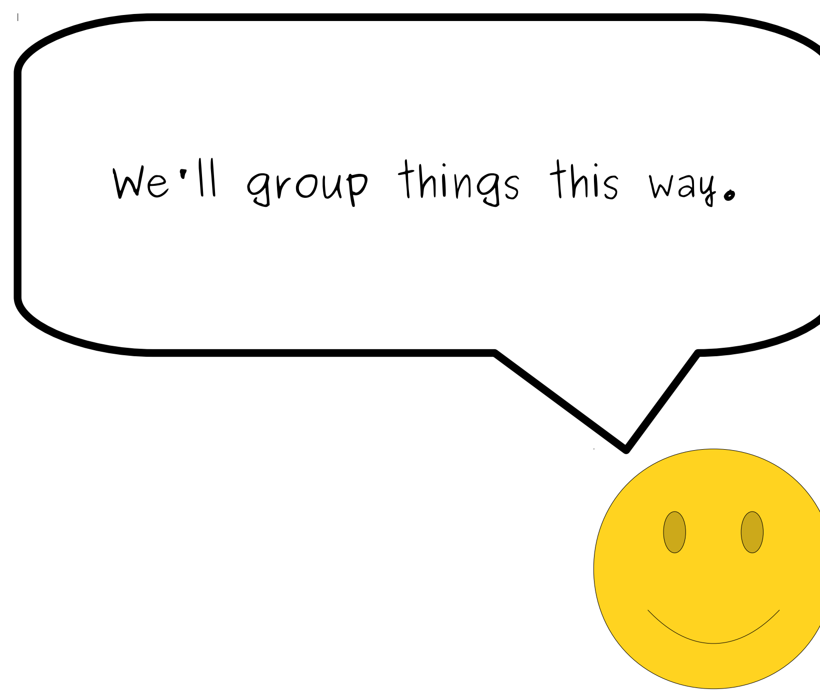
$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$
$$p \land q \to \neg r$$
$$\neg(p \land (q \land r))$$



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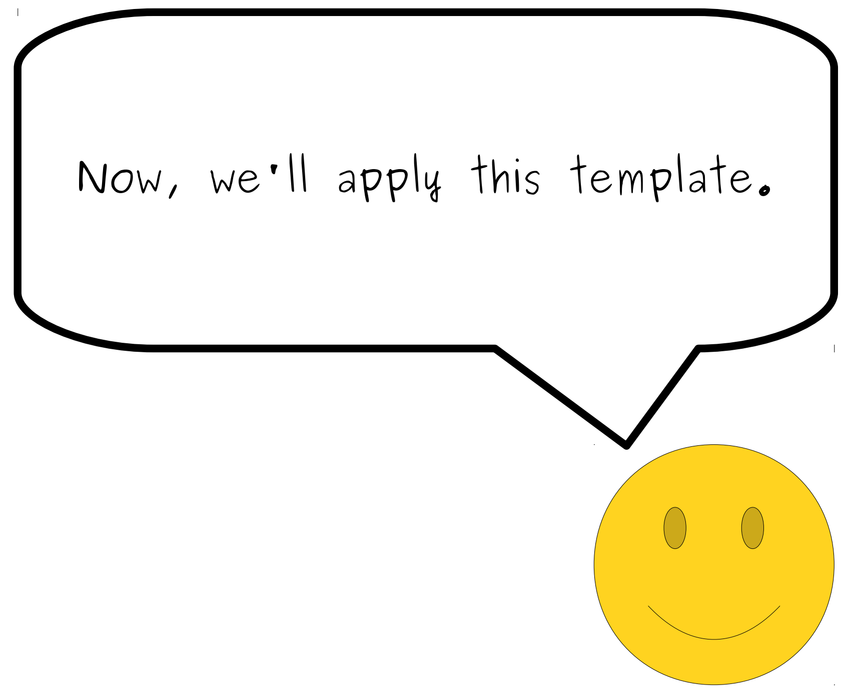
$$\frac{\neg(A \land B)}{A \to \neg B}$$

$\neg p \lor (q \to \neg r)$

$p \land q \to \neg r$

$\neg(p \land (q \land r))$



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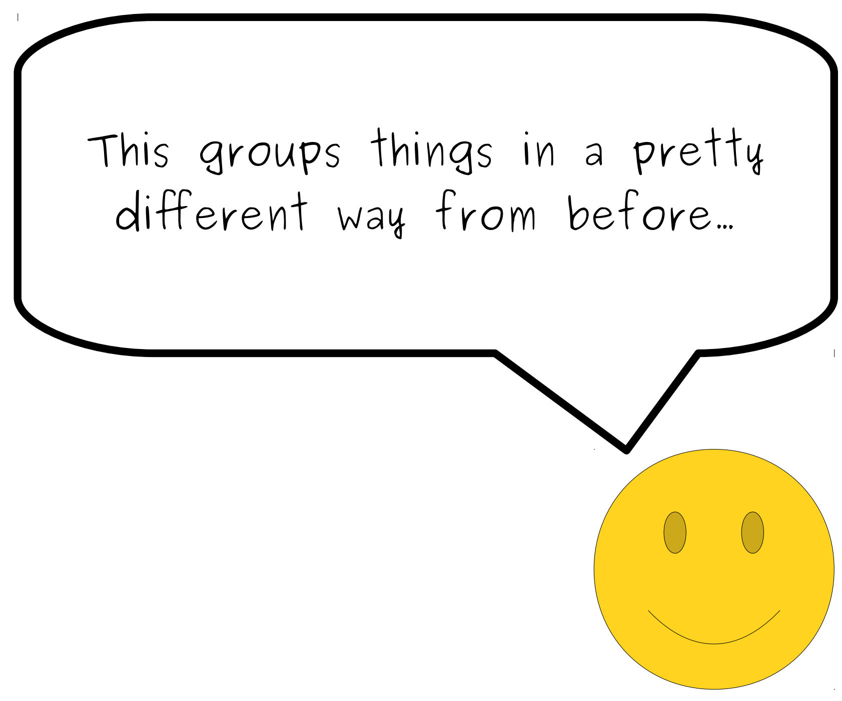
$$\frac{\neg(A \land B)}{A \to \neg B}$$

$\neg p \lor (q \to \neg r)$

$p \land q \to \neg r$

$\neg(p \land (q \land r))$



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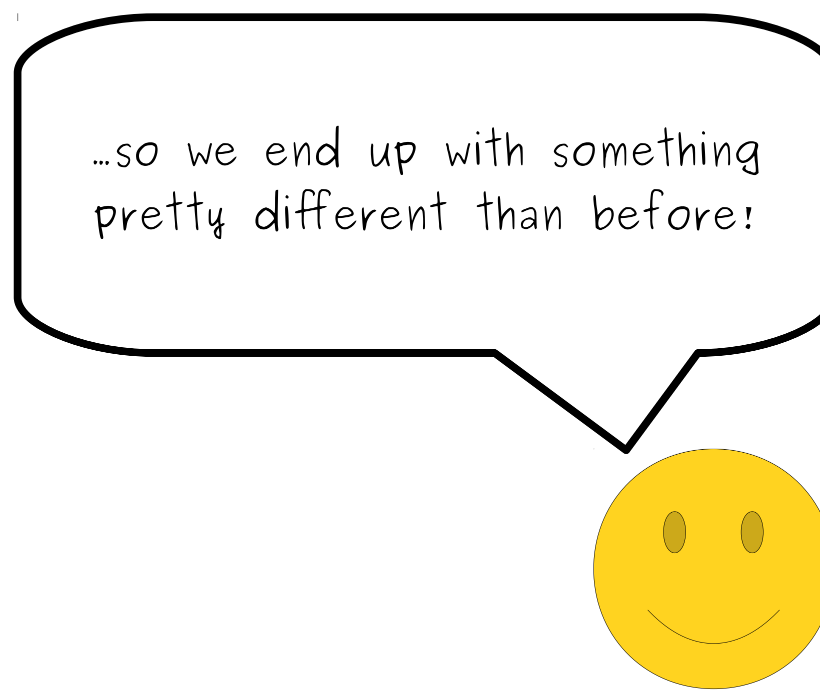
$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$

$$p \land q \to \neg r$$

$$p \to \neg(q \land r)$$



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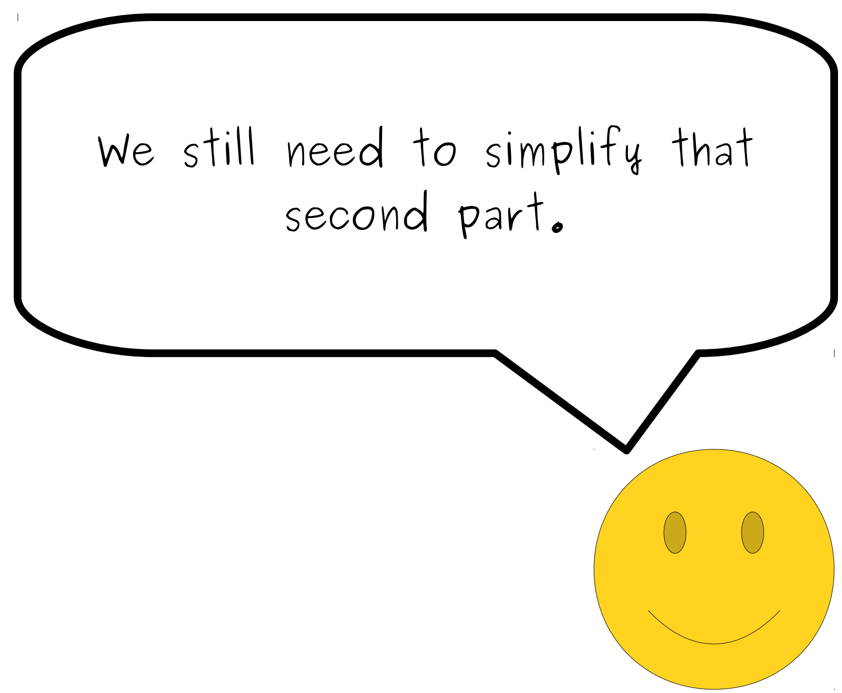
$$\frac{\neg(A \wedge B)}{A \to \neg B}$$

$$\neg p \vee (q \to \neg r)$$

$$p \wedge q \to \neg r$$

$$p \to \neg(q \wedge r)$$



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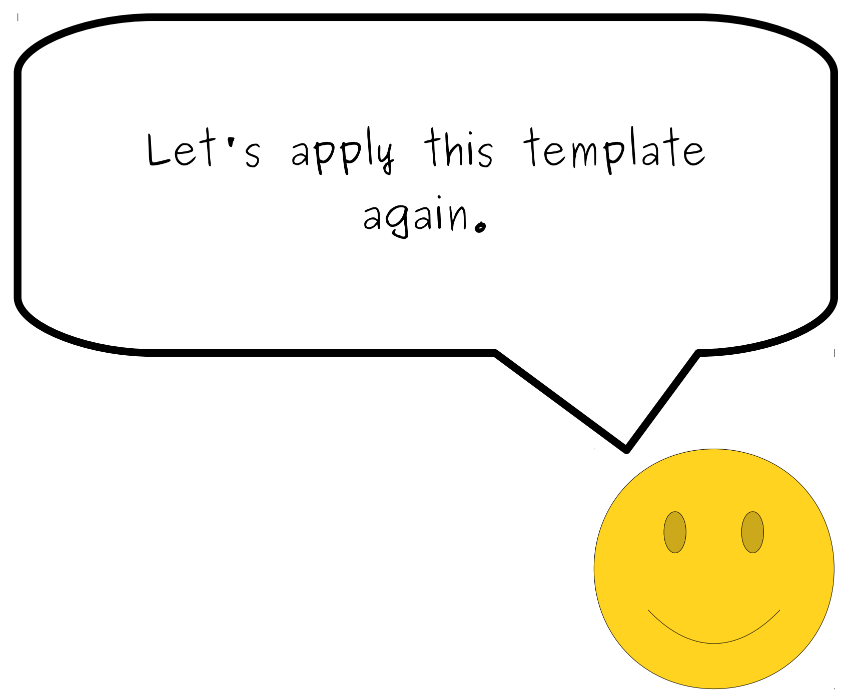
$$\frac{\neg(A \land B)}{A \to \neg B}$$

$\neg p \lor (q \to \neg r)$

$p \land q \to \neg r$

$p \to \neg(q \land r)$



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$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$

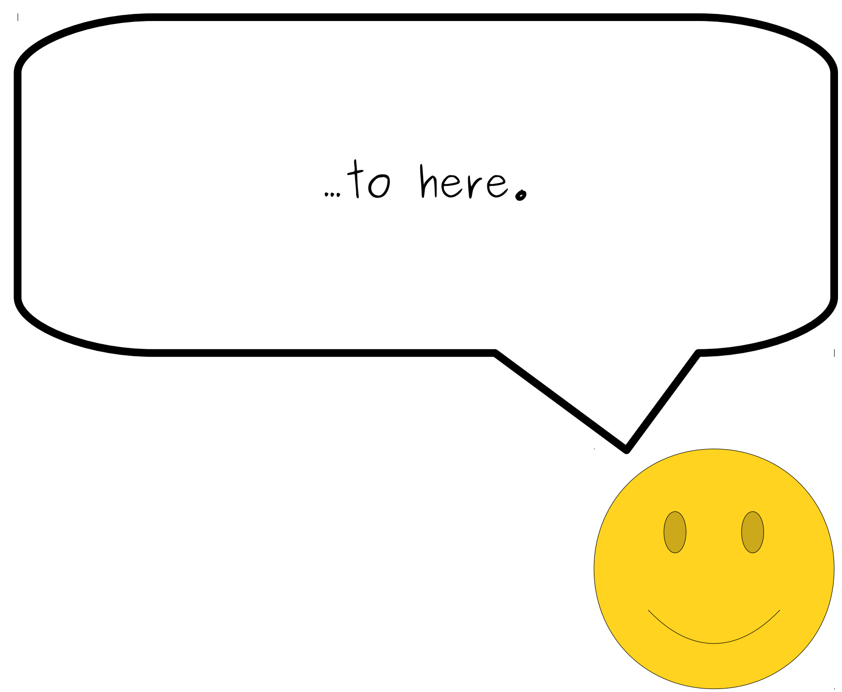
$$p \land q \to \neg r$$

$$p \to \neg(q \land r)$$

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \rightarrow B)}{A \wedge \neg B}$$

$$\frac{\neg(A \wedge B)}{A \rightarrow \neg B}$$

$$\neg p \vee (q \rightarrow \neg r)$$
$$p \wedge q \rightarrow \neg r$$
$$p \rightarrow q \rightarrow \neg r$$



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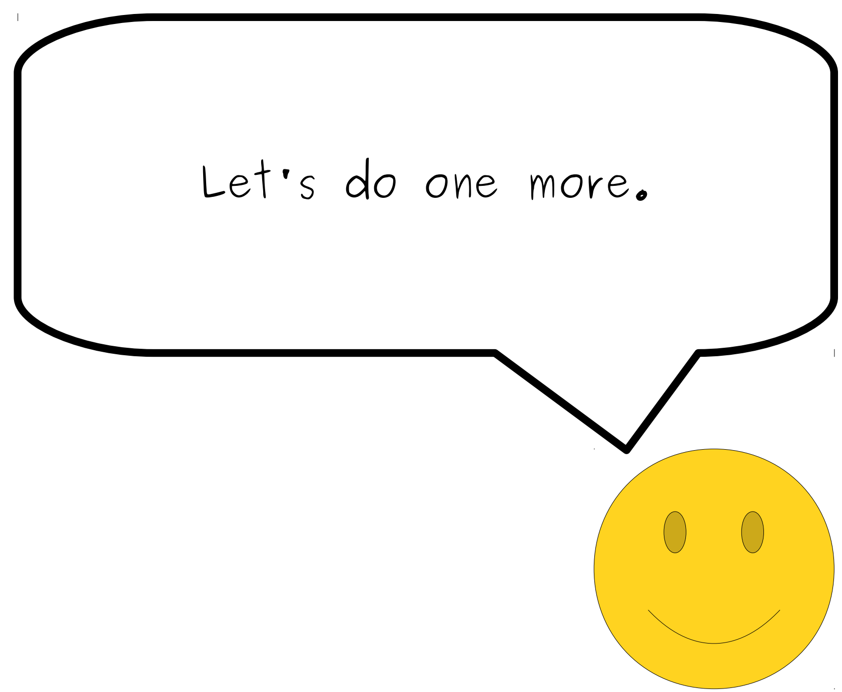
$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$
$$p \land q \to \neg r$$
$$p \to q \to \neg r$$

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$
$$p \land q \to \neg r$$
$$p \to q \to \neg r$$



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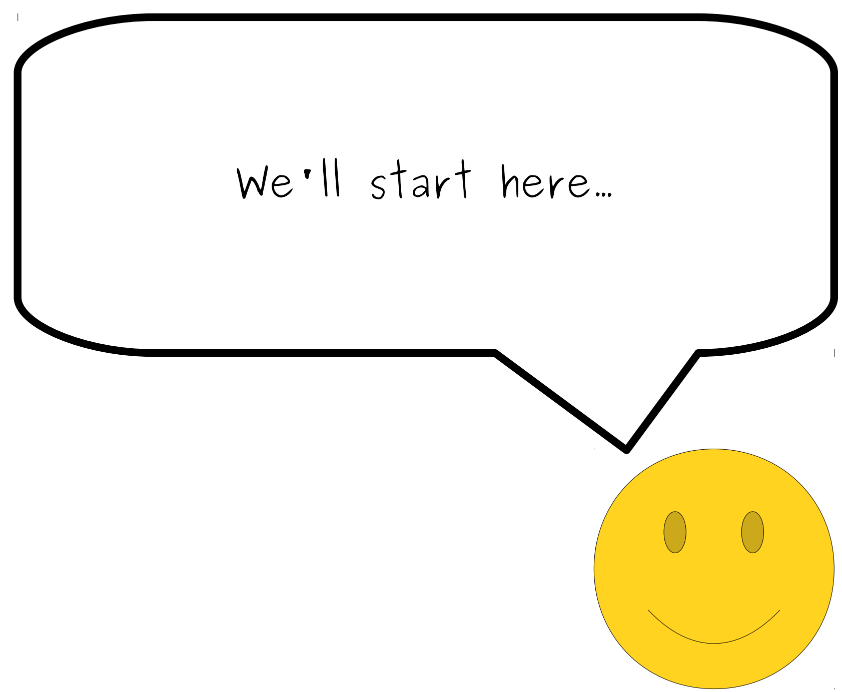
$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$
$$p \land q \to \neg r$$
$$p \to q \to \neg r$$
$$\neg(p \land q \land r)$$



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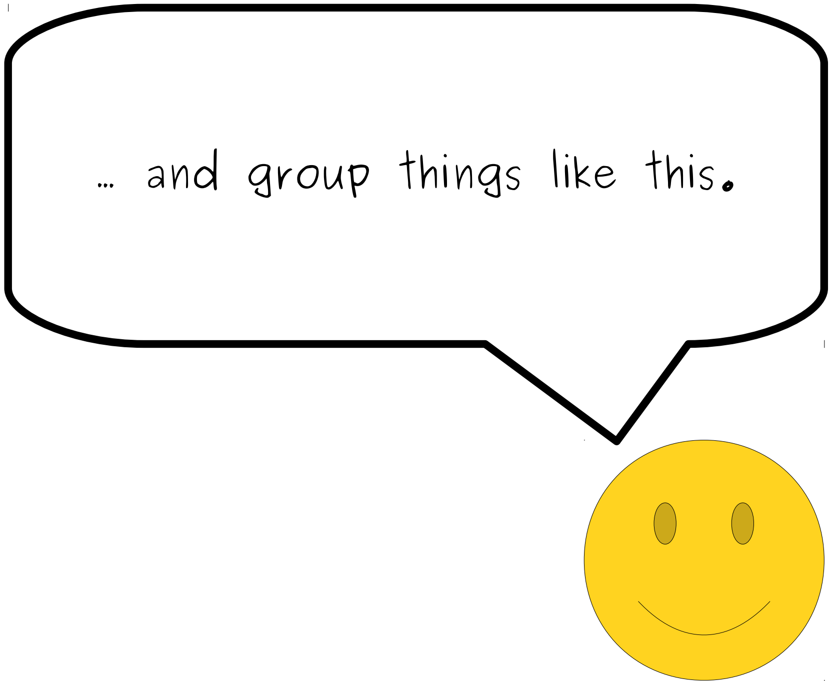
$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$
$$p \land q \to \neg r$$
$$p \to q \to \neg r$$
$$\neg((p \land q) \land r)$$



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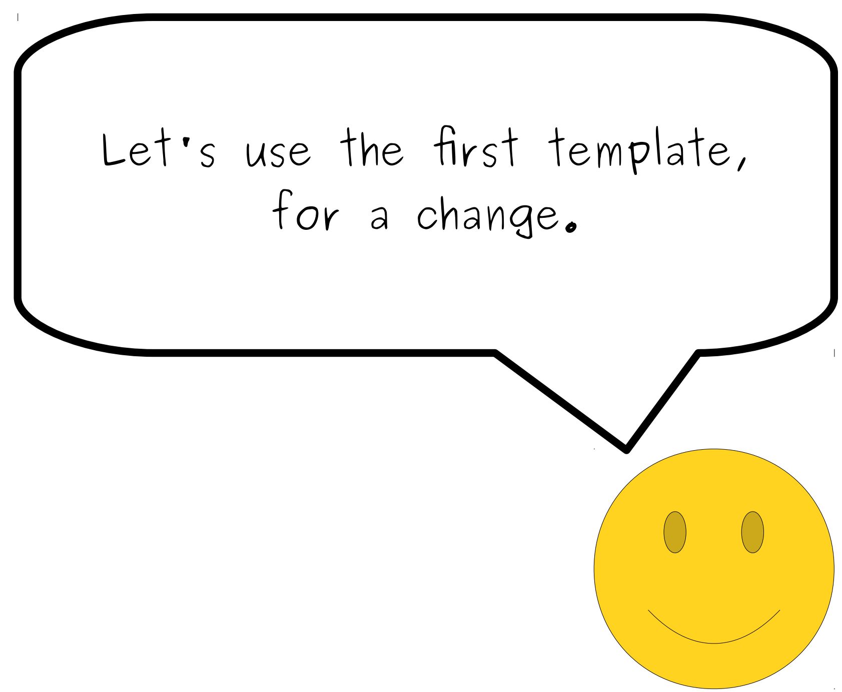
$$\frac{\neg(A \wedge B)}{A \to \neg B}$$

$$\neg p \vee (q \to \neg r)$$
$$p \wedge q \to \neg r$$
$$p \to q \to \neg r$$
$$\neg((p \wedge q) \wedge r)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

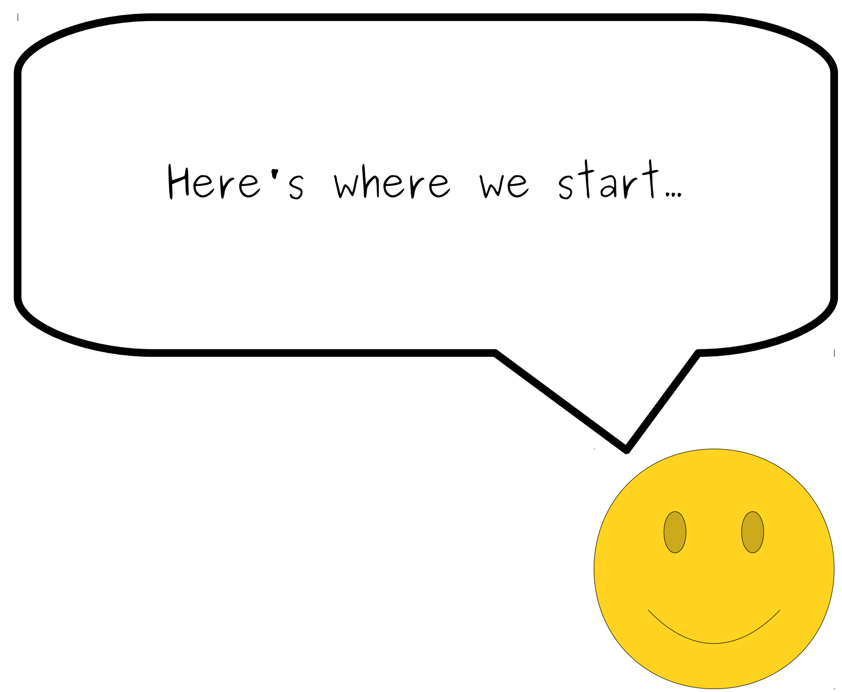
$$\frac{\neg(A \land B)}{A \to \neg B}$$

$\neg p \lor (q \to \neg r)$

$p \land q \to \neg r$

$p \to q \to \neg r$

$\neg((p \land q) \land r)$



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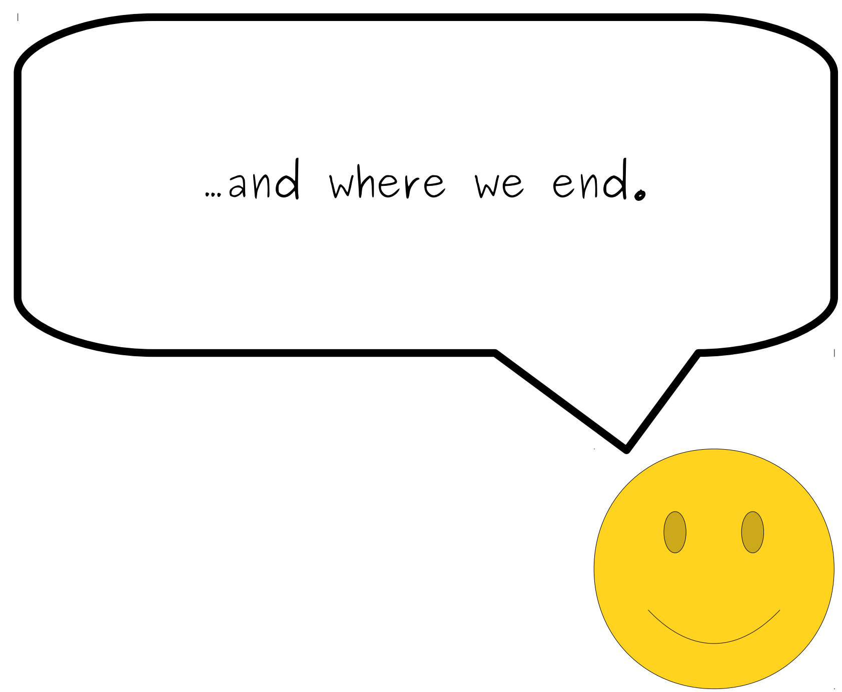
$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$

$$p \land q \to \neg r$$

$$p \to q \to \neg r$$

$$\neg(p \land q) \lor \neg r$$



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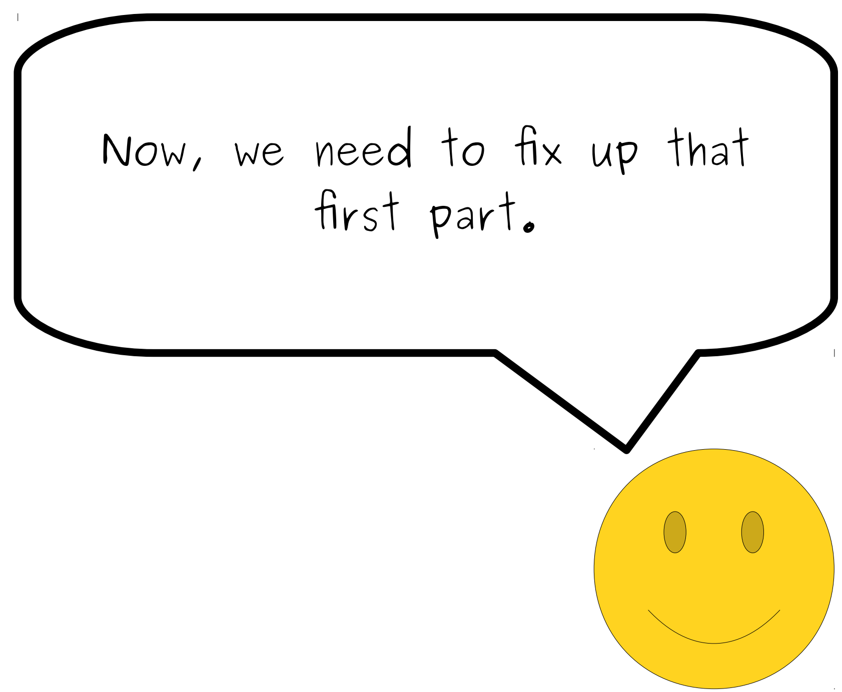
$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$
$$p \land q \to \neg r$$
$$p \to q \to \neg r$$
$$\neg(p \land q) \lor \neg r$$



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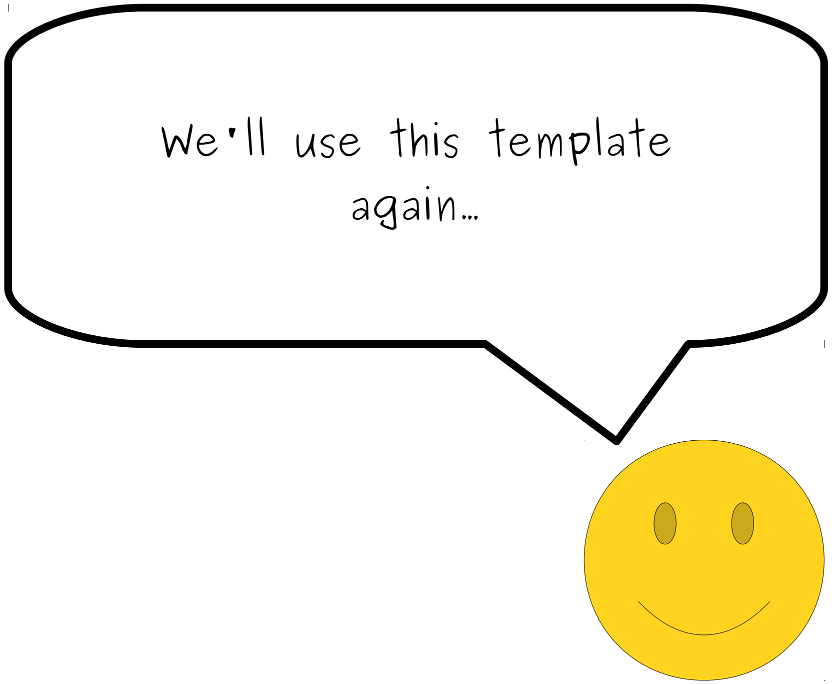
$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$
$$p \land q \to \neg r$$
$$p \to q \to \neg r$$
$$\neg(p \land q) \lor \neg r$$



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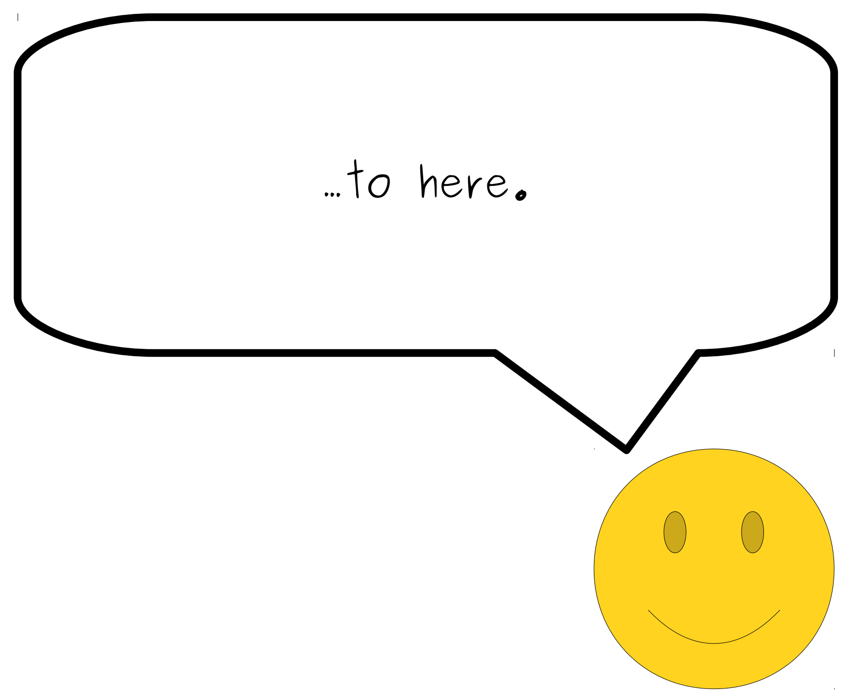
$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$
$$p \land q \to \neg r$$
$$p \to q \to \neg r$$
$$\neg(p \land q) \lor \neg r$$

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

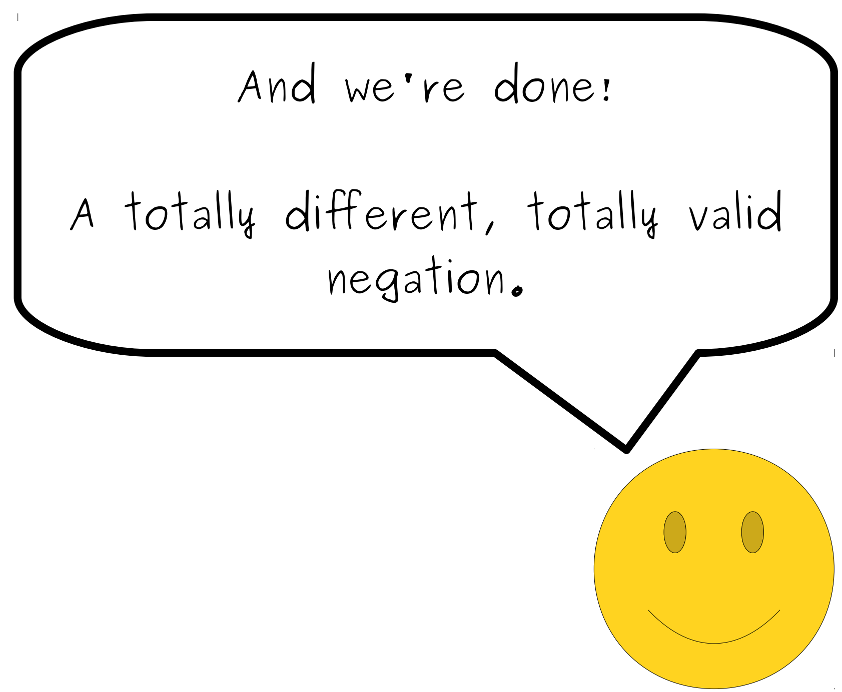
$$\neg p \lor (q \to \neg r)$$
$$p \land q \to \neg r$$
$$p \to q \to \neg r$$
$$\neg p \lor \neg q \lor \neg r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$
$$p \land q \to \neg r$$
$$p \to q \to \neg r$$
$$\neg p \lor \neg q \lor \neg r$$



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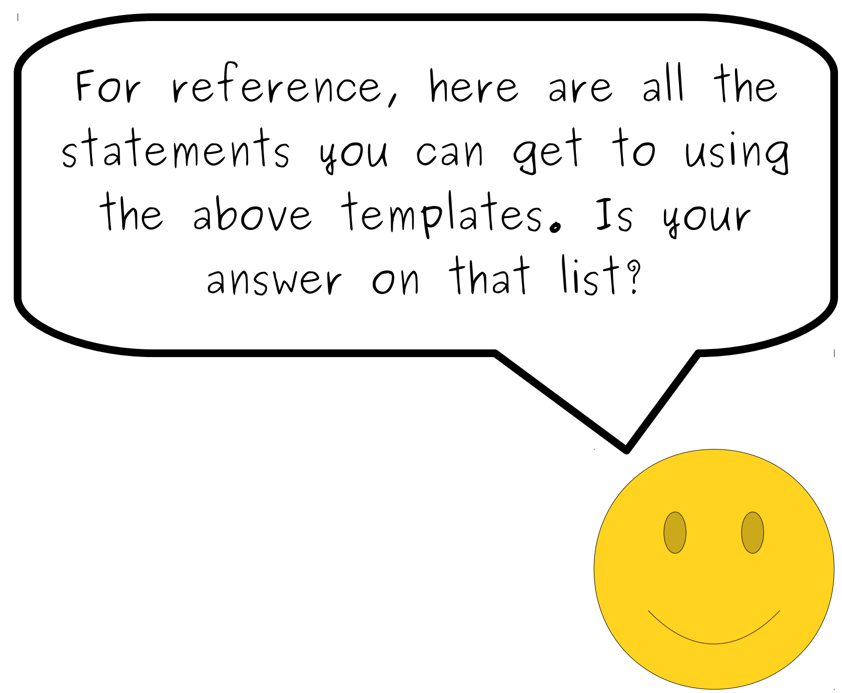
$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\neg p \lor (q \to \neg r)$$
$$p \land q \to \neg r$$
$$p \to q \to \neg r$$
$$\neg p \lor \neg q \lor \neg r$$
$$p \to (\neg q \lor \neg r)$$
$$(p \to \neg q) \lor \neg r$$



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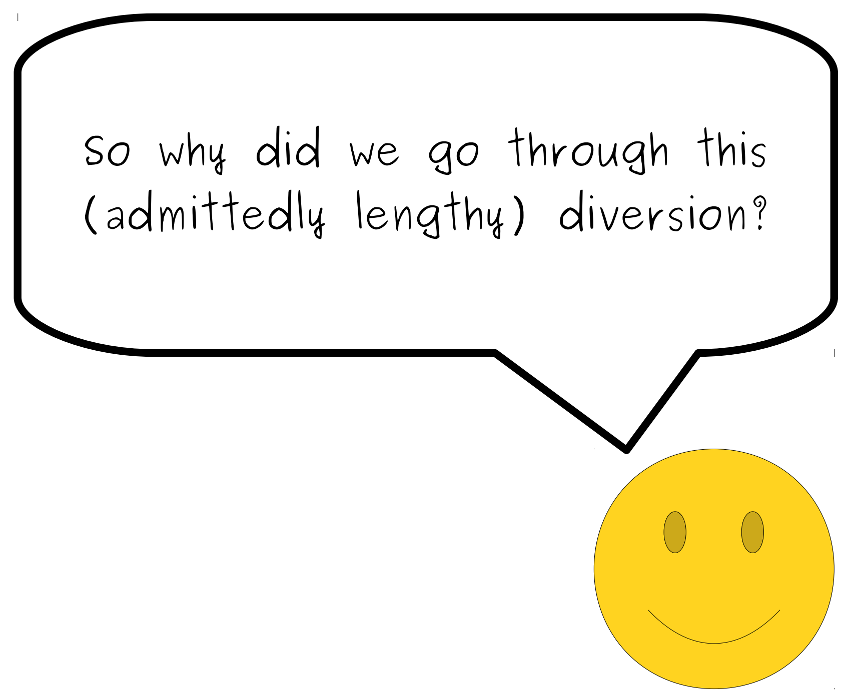
$$\frac{\neg(A \land B)}{A \to \neg B}$$

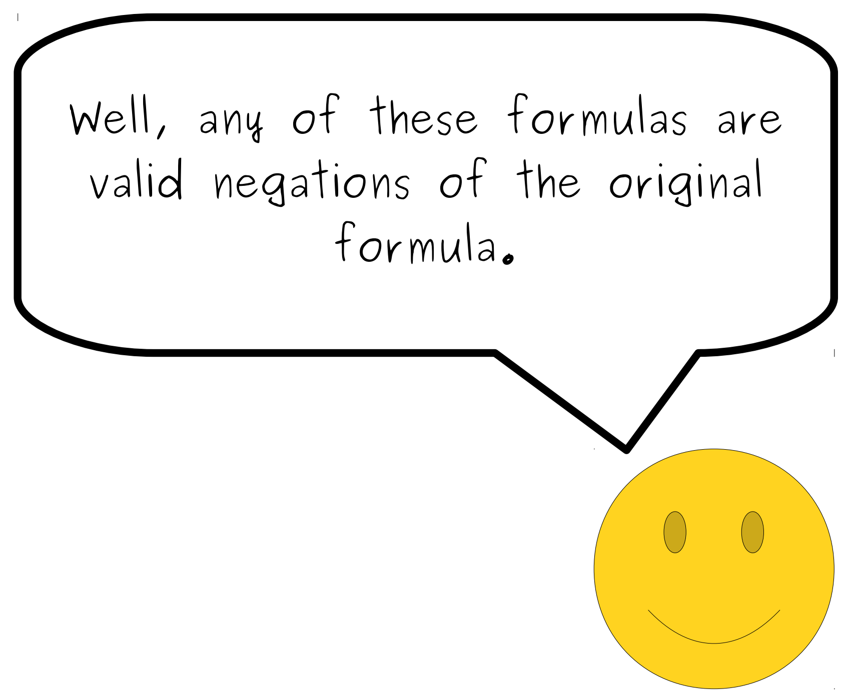
$$\neg p \lor (q \to \neg r)$$
$$p \land q \to \neg r$$
$$p \to q \to \neg r$$
$$\neg p \lor \neg q \lor \neg r$$
$$p \to (\neg q \lor \neg r)$$
$$(p \to \neg q) \lor \neg r$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \to B)}{A \wedge \neg B}$$

$$\frac{\neg(A \wedge B)}{A \to \neg B}$$

$$\neg p \vee (q \to \neg r)$$
$$p \wedge q \to \neg r$$
$$p \to q \to \neg r$$
$$\neg p \vee \neg q \vee \neg r$$
$$p \to (\neg q \vee \neg r)$$
$$(p \to \neg q) \vee \neg r$$



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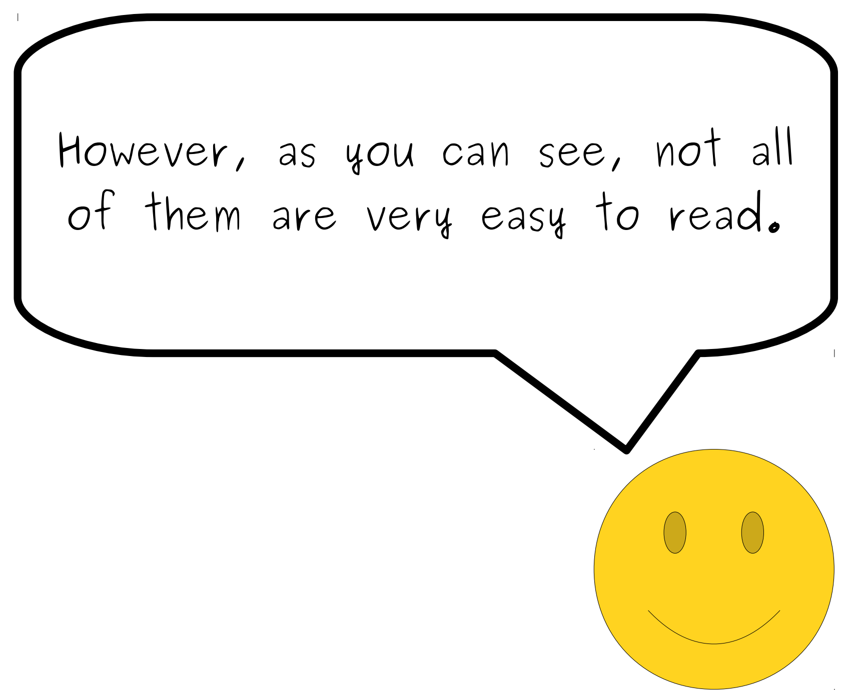


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

$\neg p \lor (q \to \neg r)$

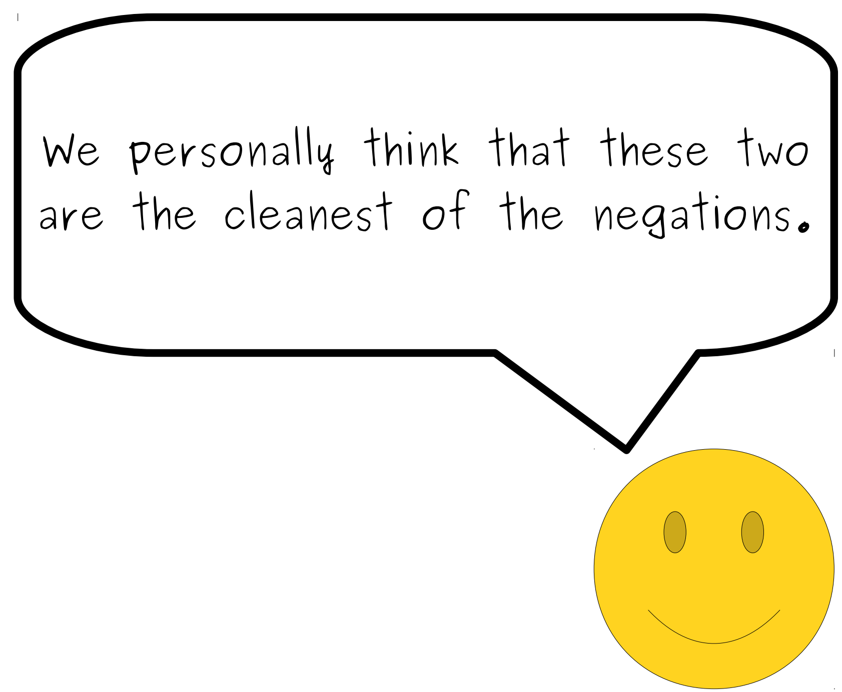
$p \land q \to \neg r$

$p \to q \to \neg r$

$\neg p \lor \neg q \lor \neg r$

$p \to (\neg q \lor \neg r)$

$(p \to \neg q) \lor \neg r$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \to B)}{A \wedge \neg B}$$

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$p \to (\neg q \vee \neg r)$

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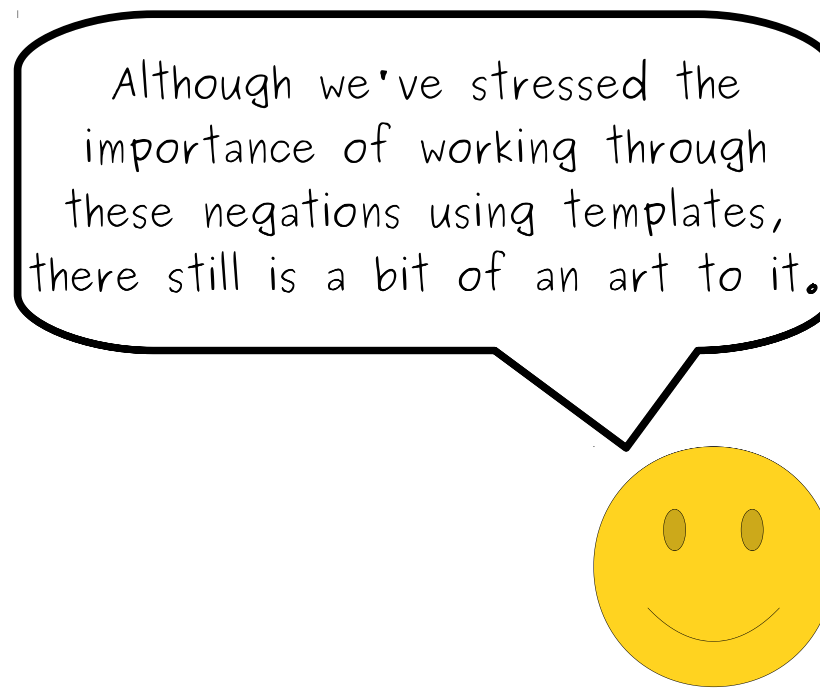

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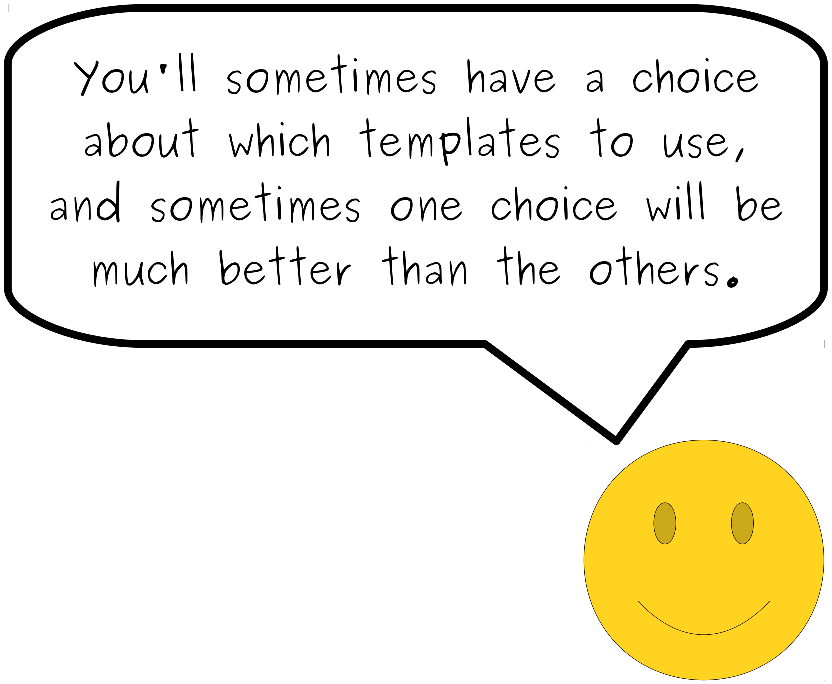

$$\frac{\neg\neg \mathbf{A}}{\mathbf{A}} \qquad \frac{\neg(\mathbf{A} \lor \mathbf{B})}{\neg \mathbf{A} \land \neg \mathbf{B}} \qquad \frac{\neg(\mathbf{A} \land \mathbf{B})}{\neg \mathbf{A} \lor \neg \mathbf{B}} \qquad \frac{\neg(\mathbf{A} \to \mathbf{B})}{\mathbf{A} \land \neg \mathbf{B}}$$

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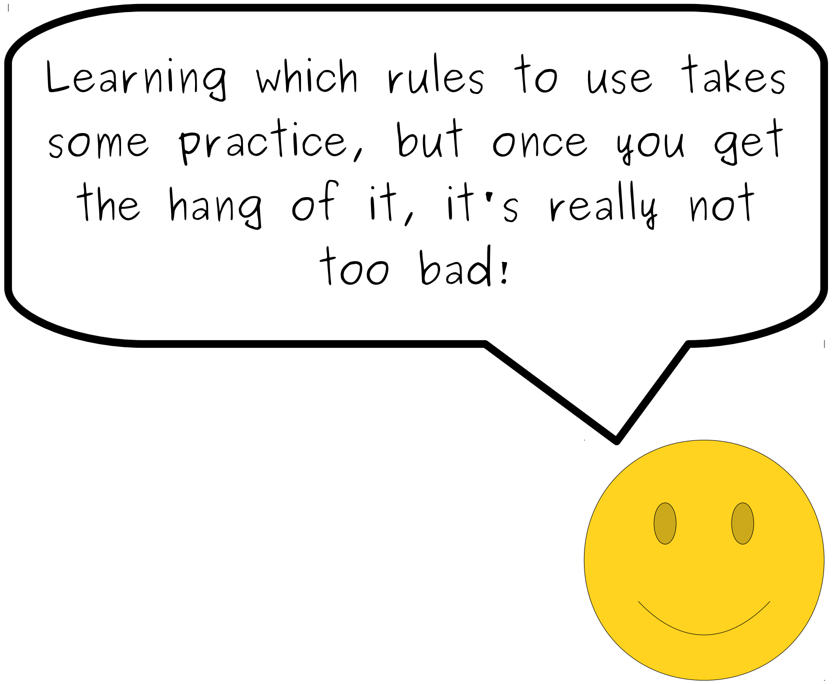

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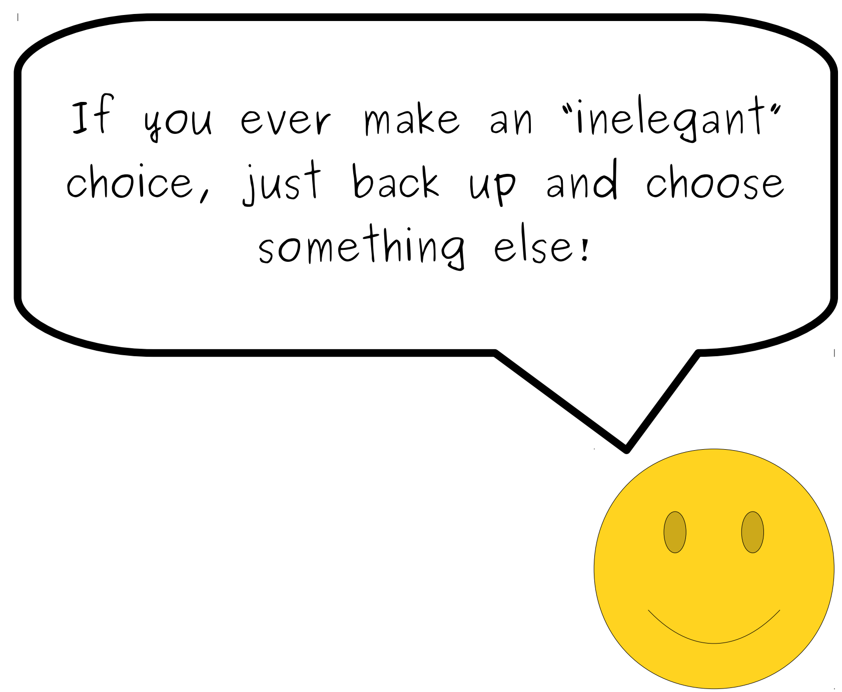

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

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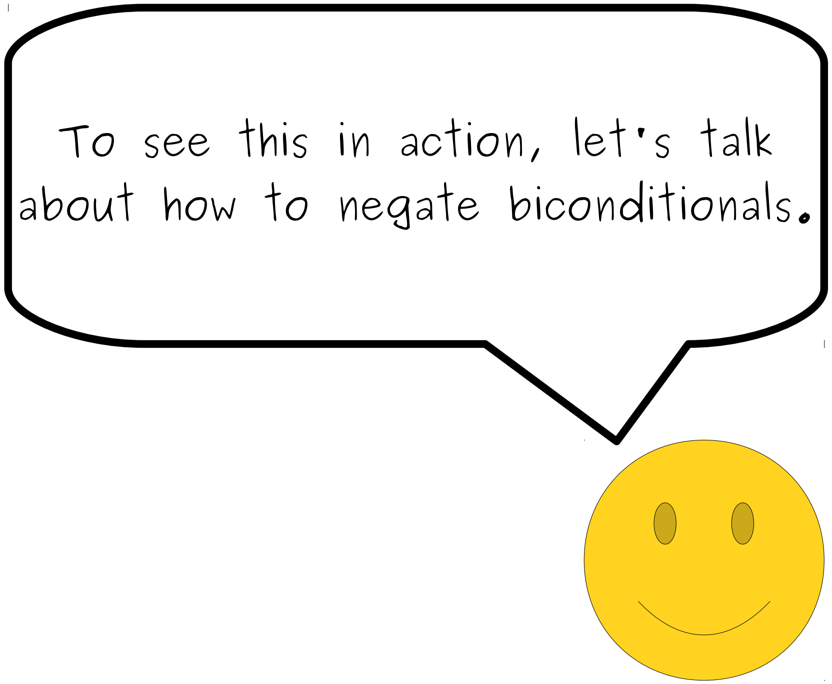

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B}$$

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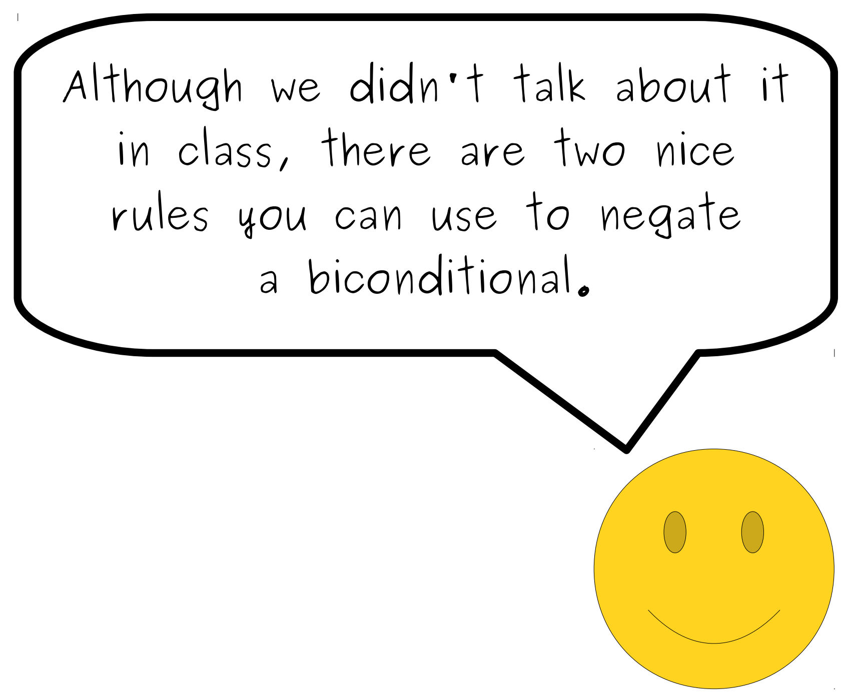


$$\frac{\neg\neg A}{A}$$

$$\frac{\neg(A \lor B)}{\neg A \land \neg B}$$

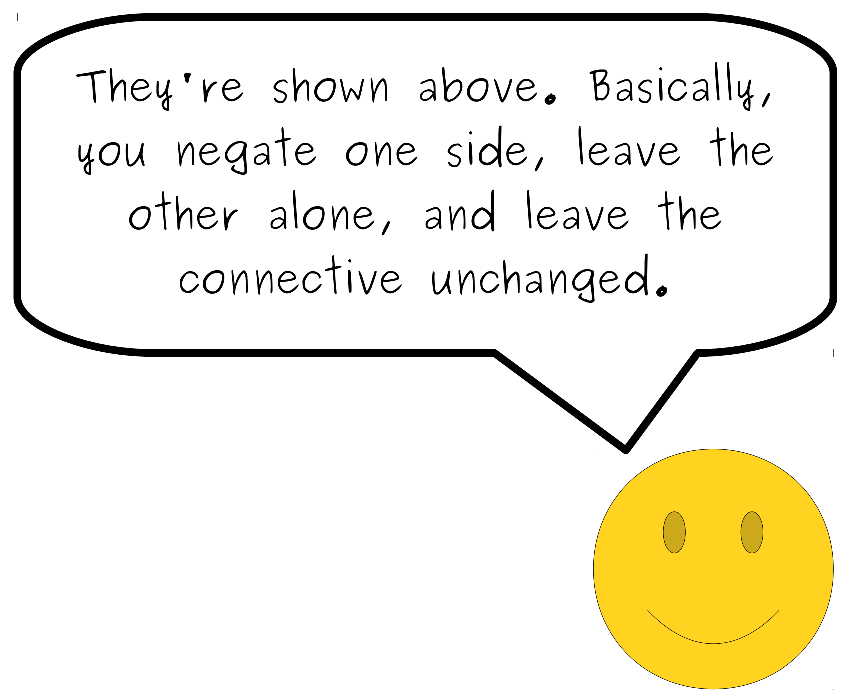
$$\frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$



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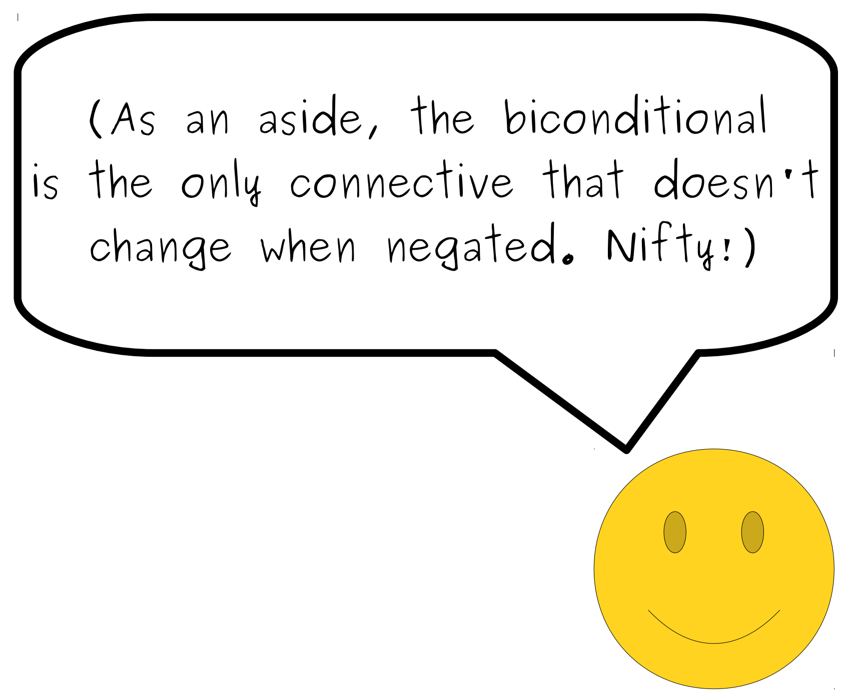
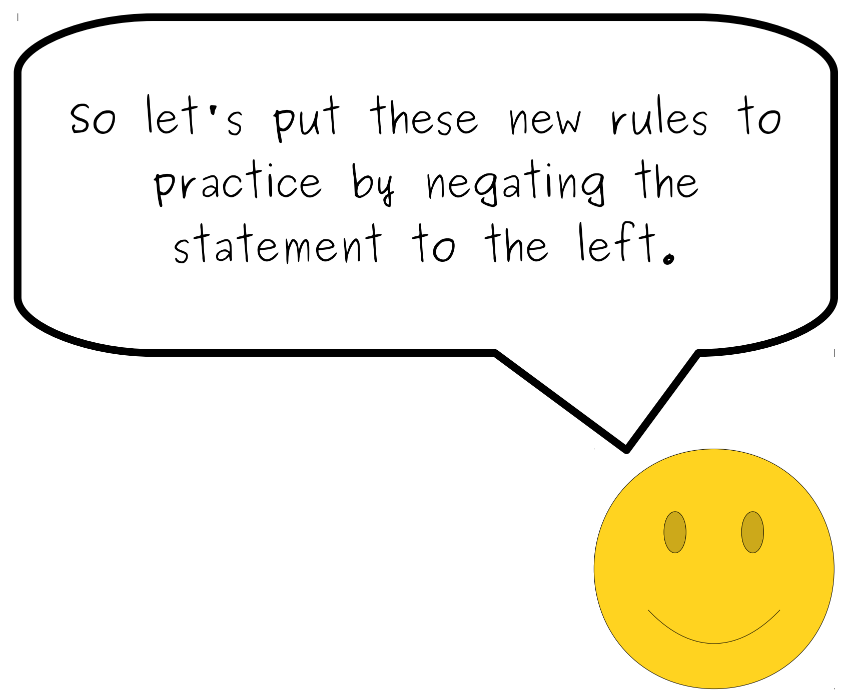


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

$$\frac{\neg(A \land B)}{A \to \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$

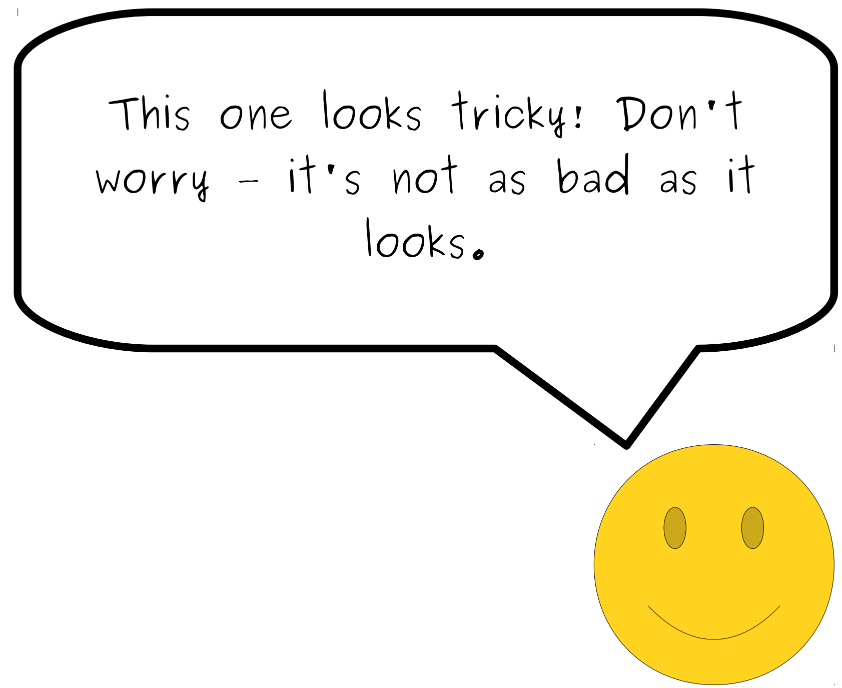
$$((p \land q) \lor r) \leftrightarrow ((q \lor r) \to p)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \to B)}{A \wedge \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

$$\frac{\neg(A \wedge B)}{A \to \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$

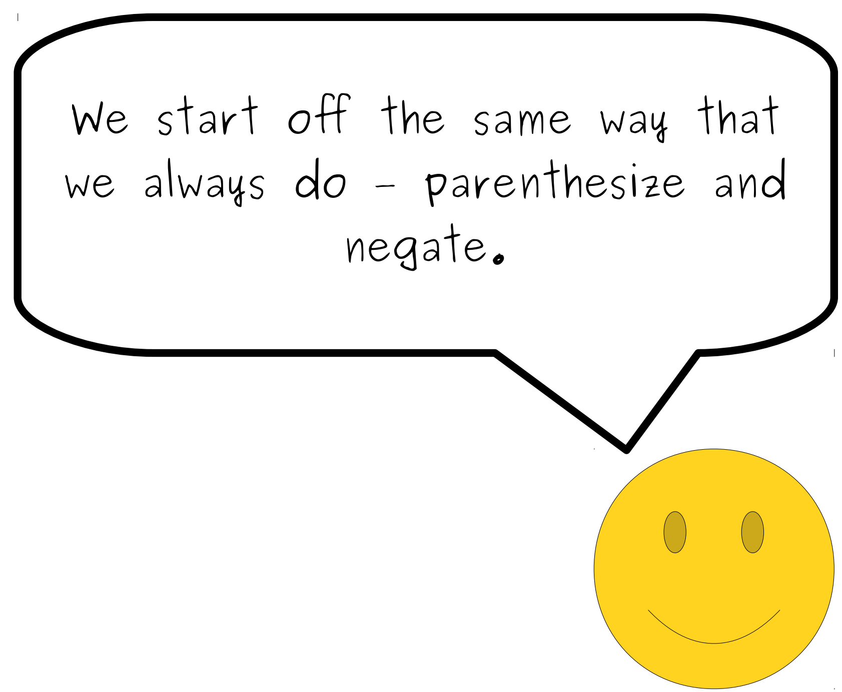
$$((p \wedge q) \vee r) \leftrightarrow ((q \vee r) \to p)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

$$\frac{\neg(A \land B)}{A \to \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$

$$\neg(((p \land q) \lor r) \leftrightarrow ((q \lor r) \to p))$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \to B)}{A \wedge \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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$$\neg(((p \wedge q) \vee r) \leftrightarrow ((q \vee r) \to p))$$

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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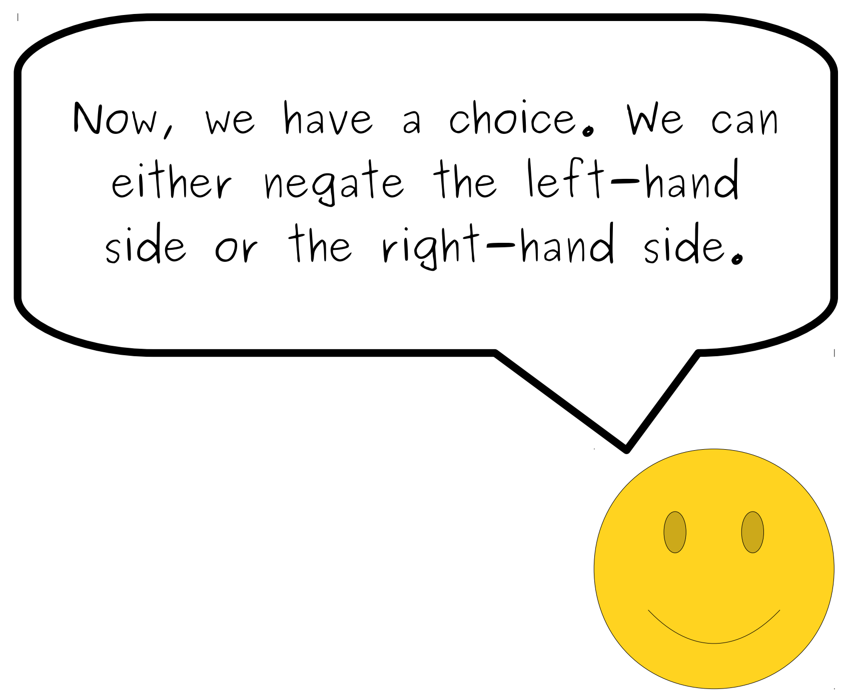
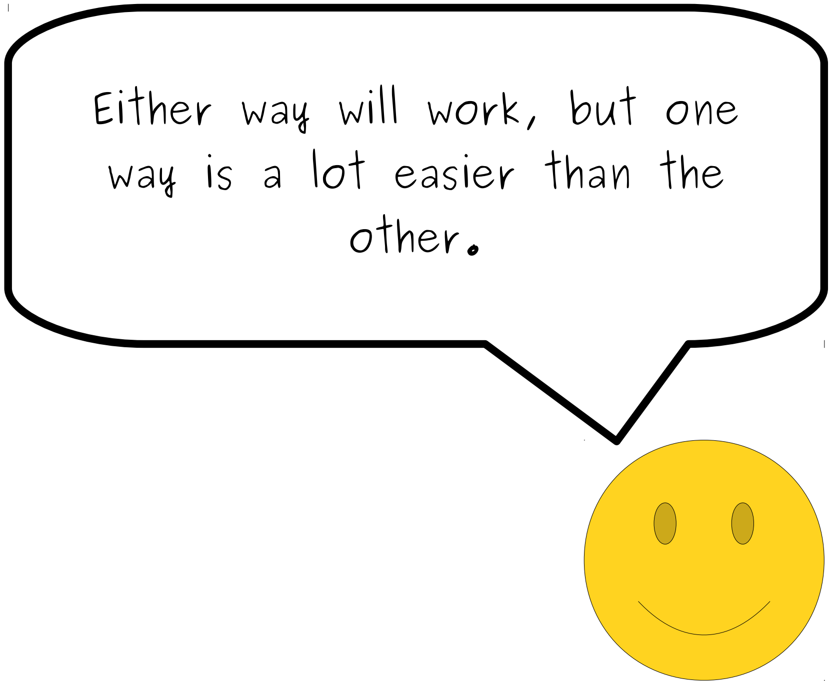


$$\frac{\neg\neg \boldsymbol{A}}{\boldsymbol{A}} \qquad \frac{\neg(\boldsymbol{A} \lor \boldsymbol{B})}{\neg \boldsymbol{A} \land \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\neg \boldsymbol{A} \lor \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \to \boldsymbol{B})}{\boldsymbol{A} \land \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \leftrightarrow \boldsymbol{B})}{\neg \boldsymbol{A} \leftrightarrow \boldsymbol{B}}$$

$$\frac{\neg(\boldsymbol{A} \land \boldsymbol{B})}{\boldsymbol{A} \to \neg \boldsymbol{B}} \qquad \frac{\neg(\boldsymbol{A} \leftrightarrow \boldsymbol{B})}{\boldsymbol{A} \leftrightarrow \neg \boldsymbol{B}}$$

$$\neg(((p \land q) \lor r) \leftrightarrow ((q \lor r) \to p))$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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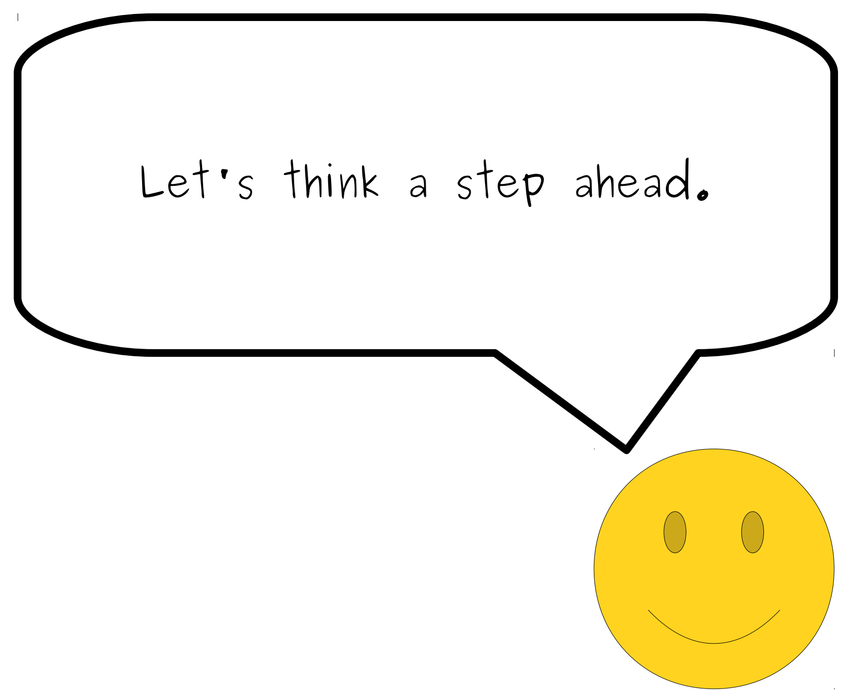
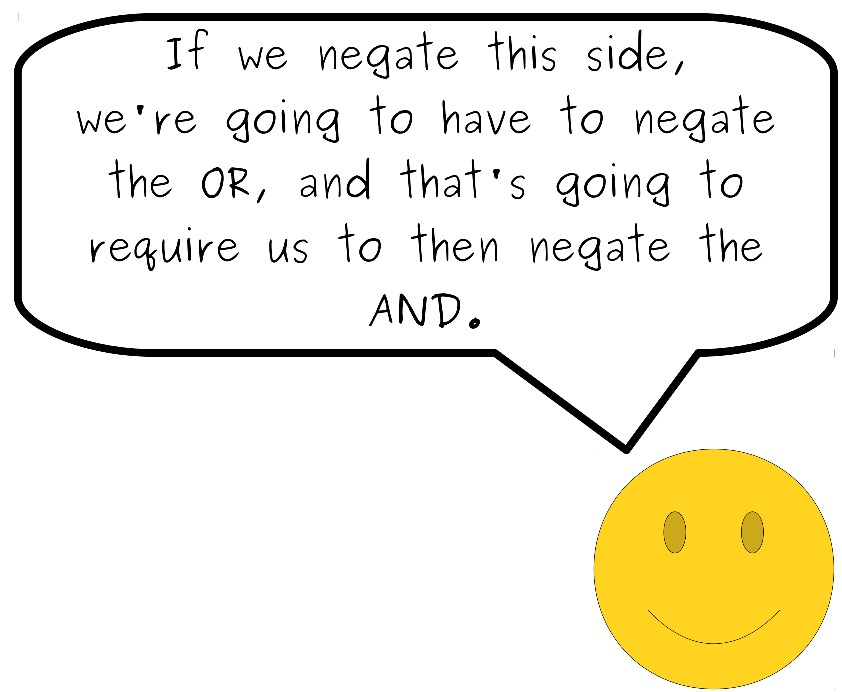


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \to B)}{A \wedge \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

$$\frac{\neg(A \wedge B)}{A \to \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$

$$\neg(((p \wedge q) \vee r) \leftrightarrow ((q \vee r) \to p))$$



$$\frac{\neg\neg A}{A}$$

$$\frac{\neg(A \lor B)}{\neg A \land \neg B}$$

$$\frac{\neg(A \land B)}{\neg A \lor \neg B}$$

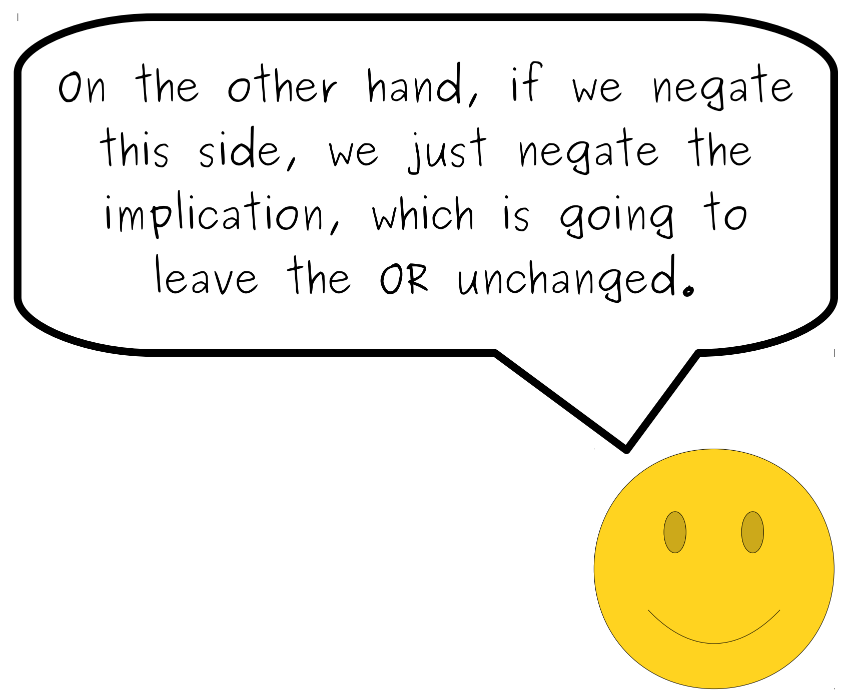
$$\frac{\neg(A \land B)}{A \to \neg B}$$

$$\frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

$$\frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$

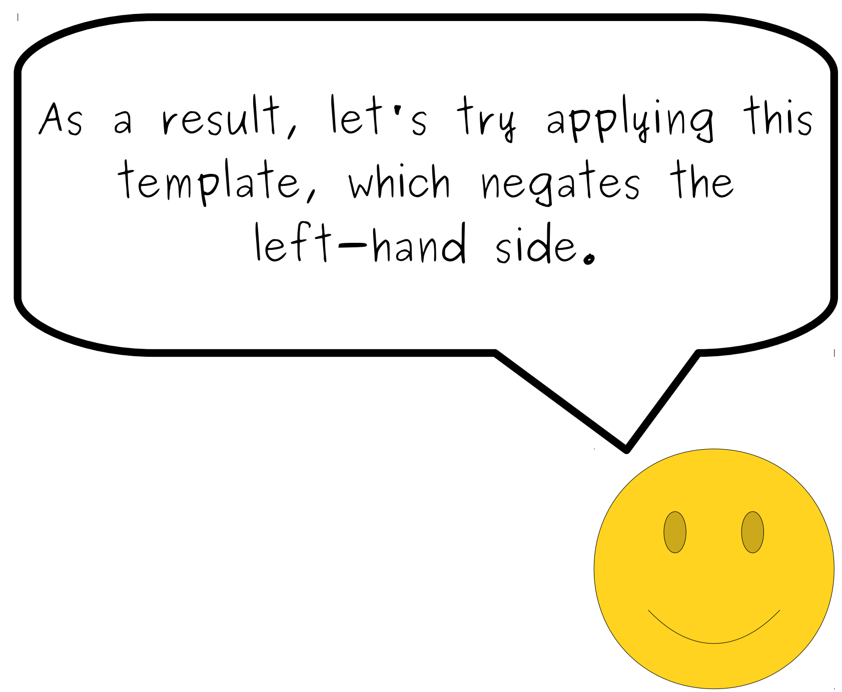
$$\neg(((p \land q) \lor r) \leftrightarrow ((q \lor r) \to p))$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

$$\frac{\neg(A \land B)}{A \to \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$

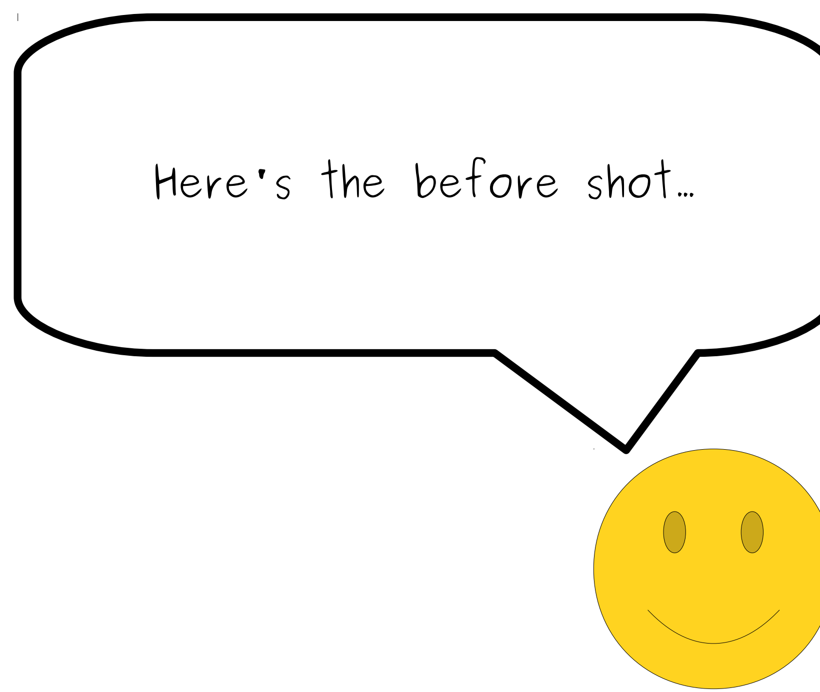
$$\neg(((p \land q) \lor r) \leftrightarrow ((q \lor r) \to p))$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

$$\frac{\neg(A \land B)}{A \to \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$

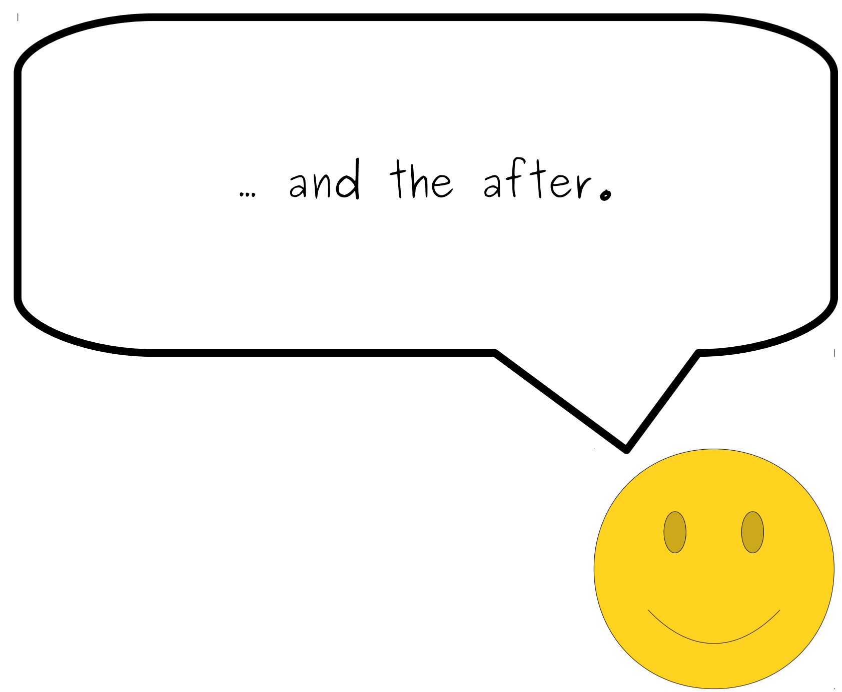
$$\neg(((p \land q) \lor r) \leftrightarrow ((q \lor r) \to p))$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

$$\frac{\neg(A \land B)}{A \to \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$

$$((p \land q) \lor r) \leftrightarrow \neg((q \lor r) \to p)$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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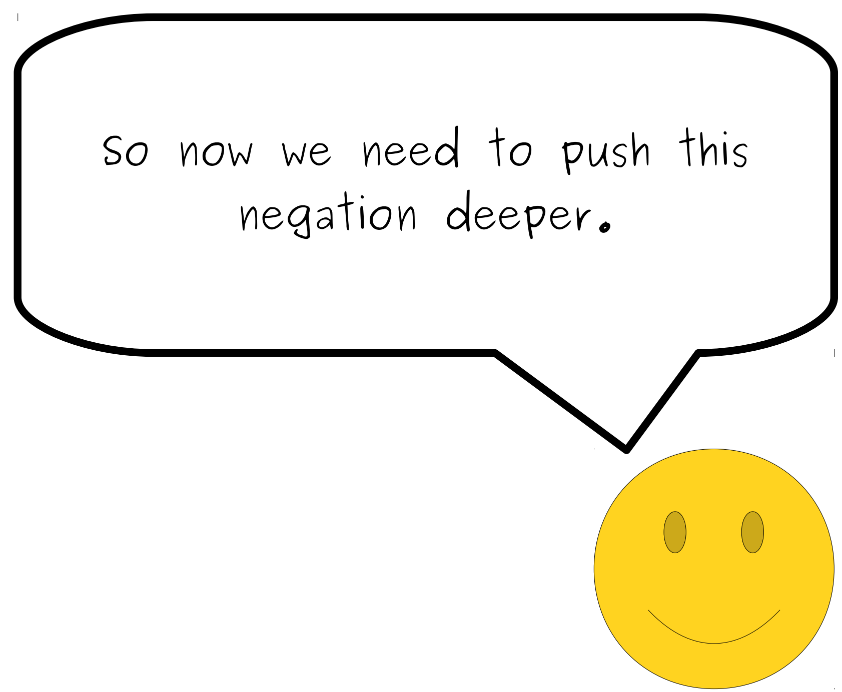

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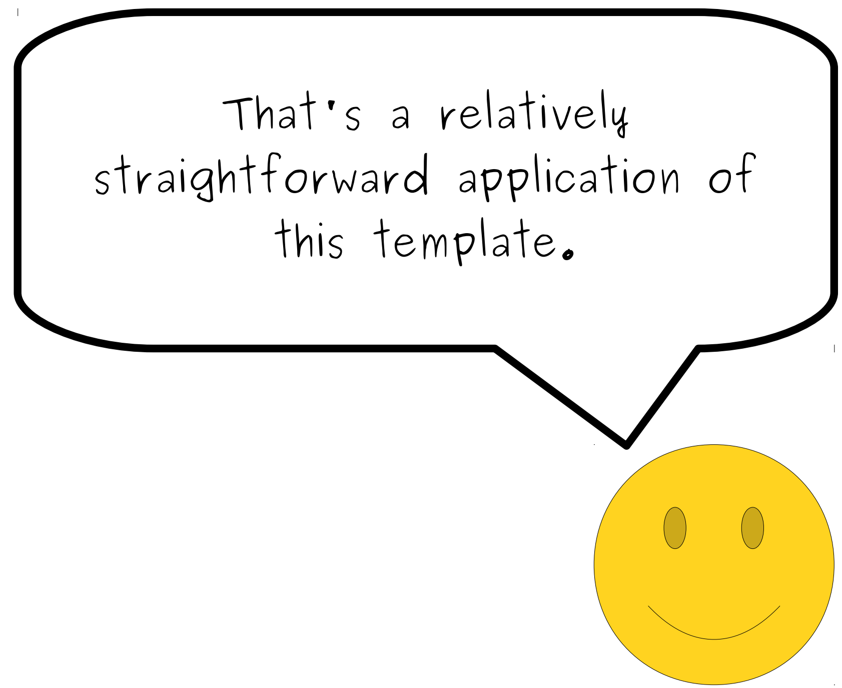

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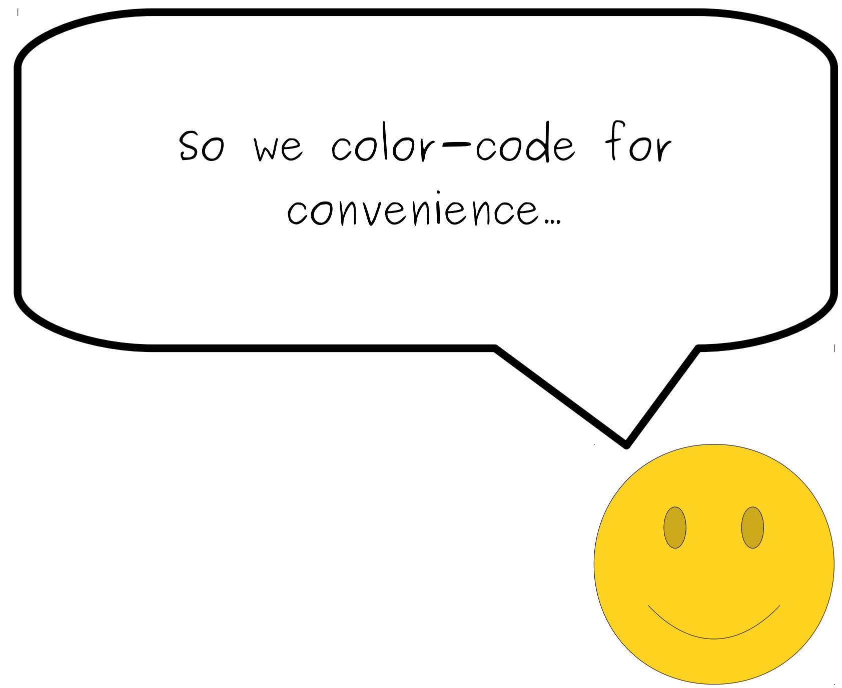

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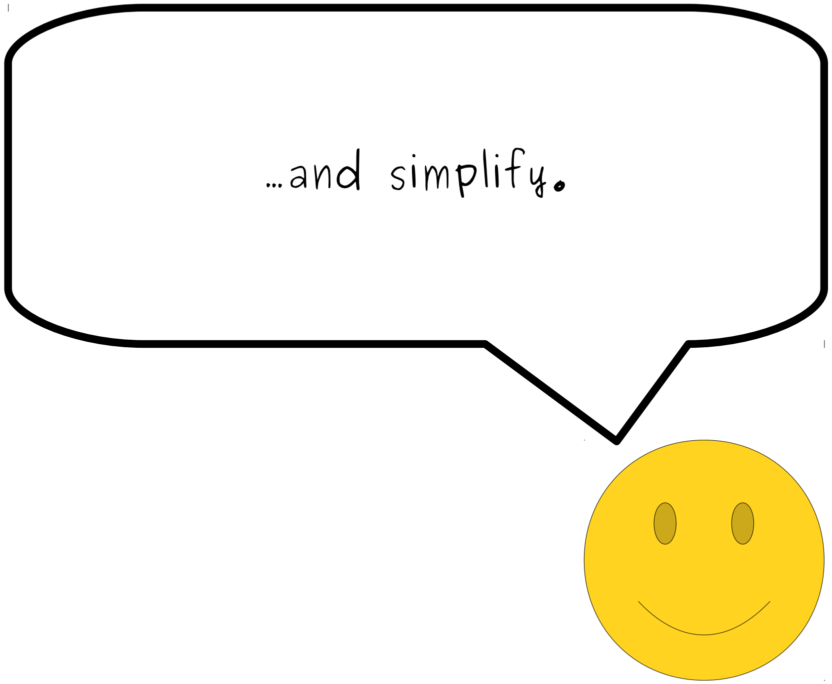

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$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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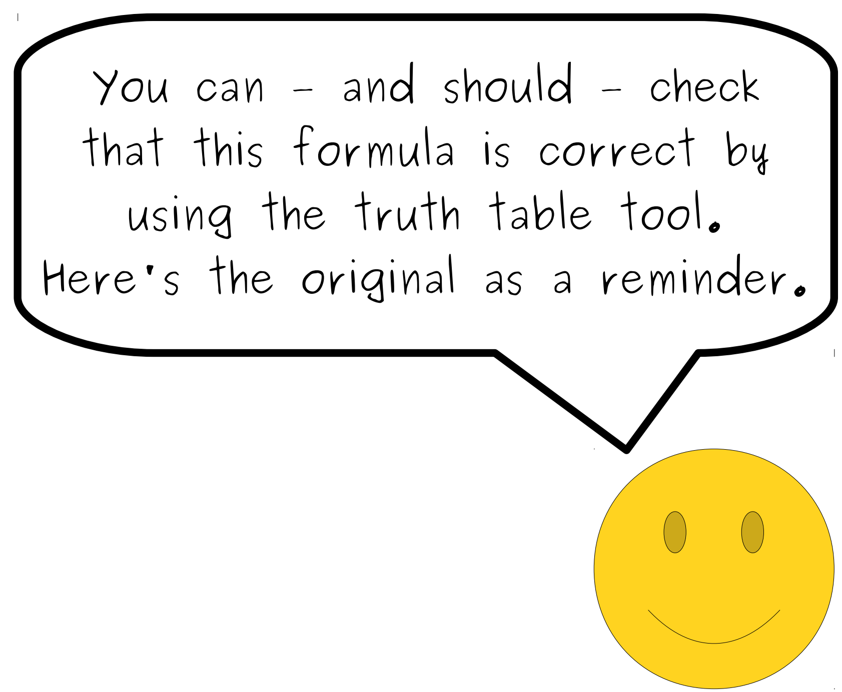
$$((p \land q) \lor r) \leftrightarrow ((q \lor r) \land \neg p)$$

$$((p \land q) \lor r) \leftrightarrow ((q \lor r) \to p)$$



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$$\frac{\neg(A \land B)}{A \to \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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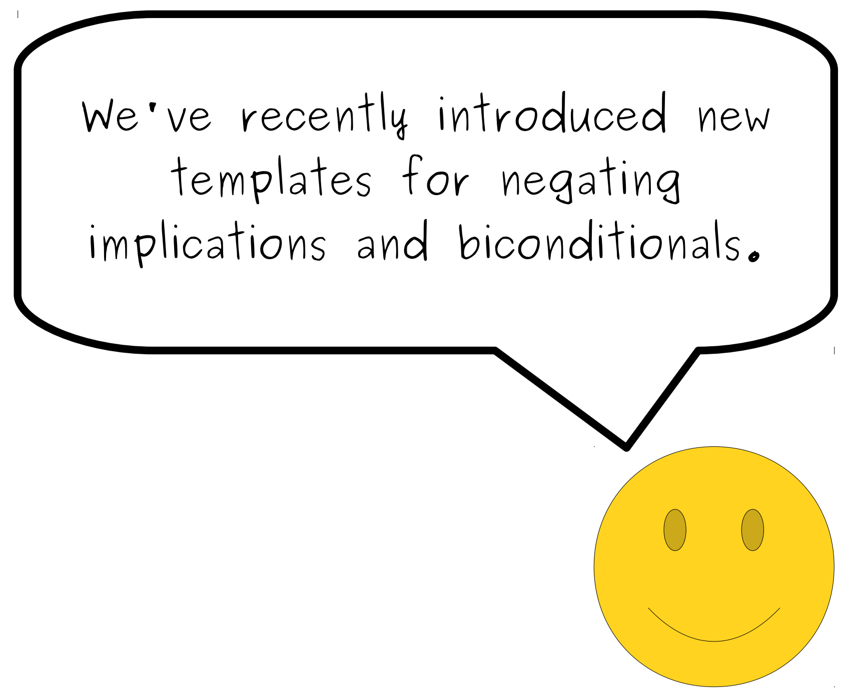

$$\frac{\neg\neg A}{A}$$

$$\frac{\neg(A \lor B)}{\neg A \land \neg B}$$

$$\frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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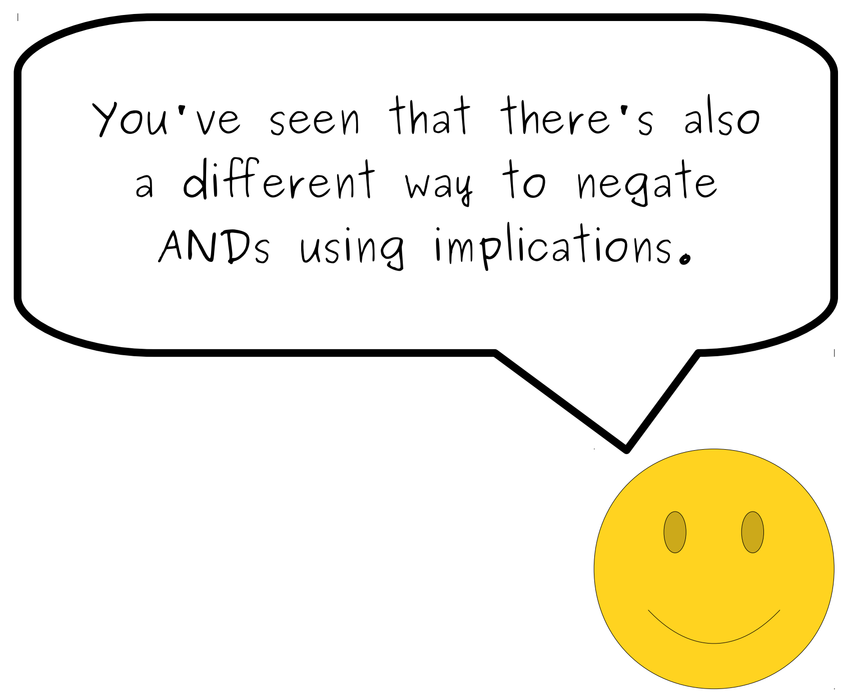


$$\frac{\neg\neg A}{A}$$

$$\frac{\neg(A \vee B)}{\neg A \wedge \neg B}$$

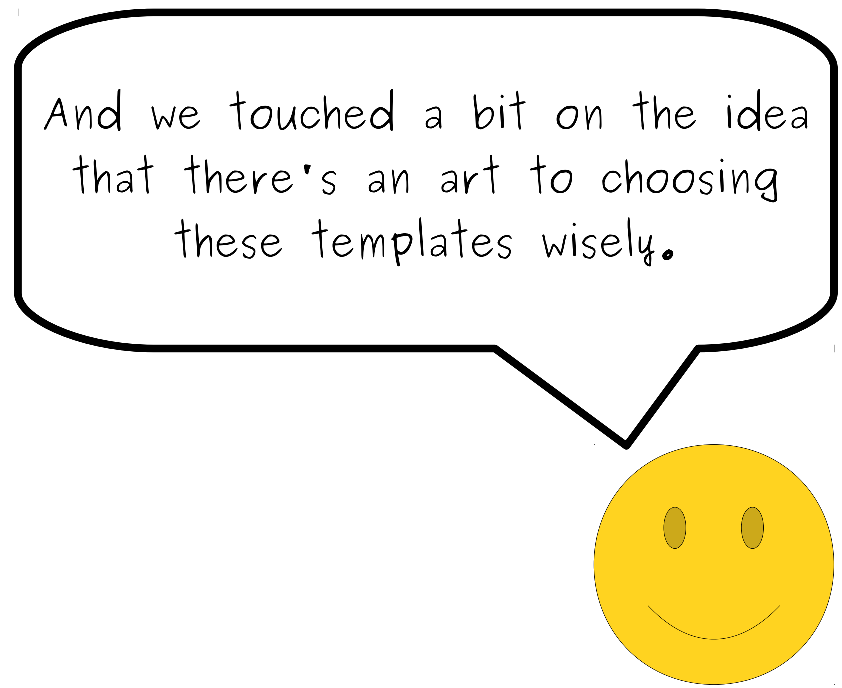
$$\frac{\neg(A \wedge B)}{\neg A \vee \neg B}$$

$$\frac{\neg(A \to B)}{A \wedge \neg B}$$

$$\frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

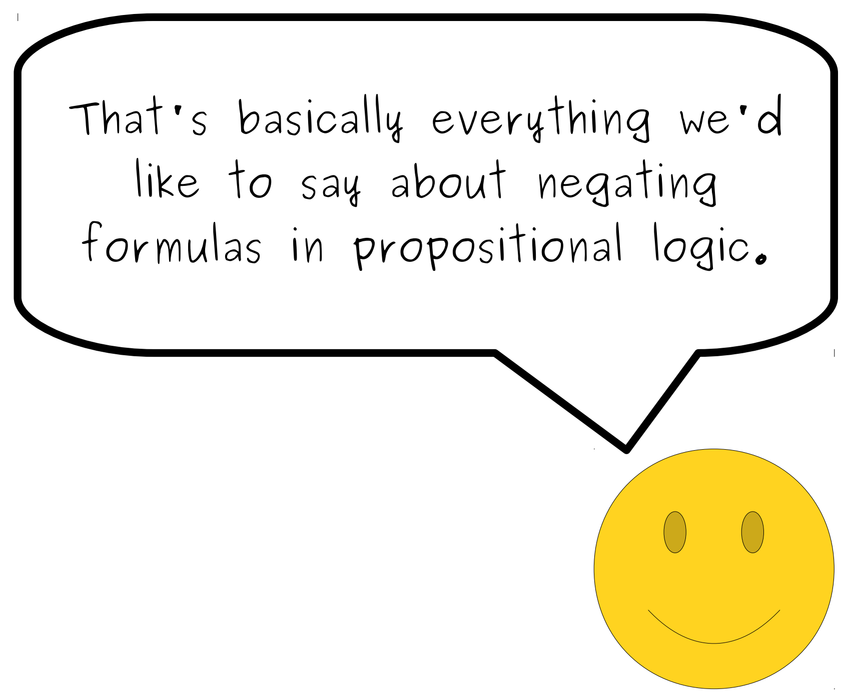
$$\frac{\neg(A \wedge B)}{A \to \neg B}$$

$$\frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

$$\frac{\neg(A \land B)}{A \to \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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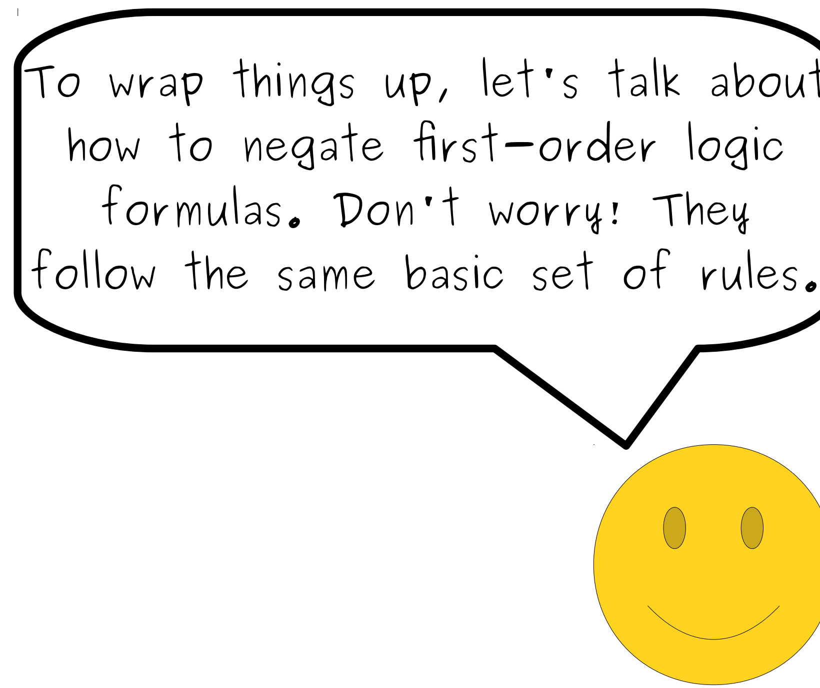

$$\frac{\neg\neg A}{A}$$

$$\frac{\neg(A \lor B)}{\neg A \land \neg B}$$

$$\frac{\neg(A \land B)}{\neg A \lor \neg B}$$

$$\frac{\neg(A \to B)}{A \land \neg B}$$

$$\frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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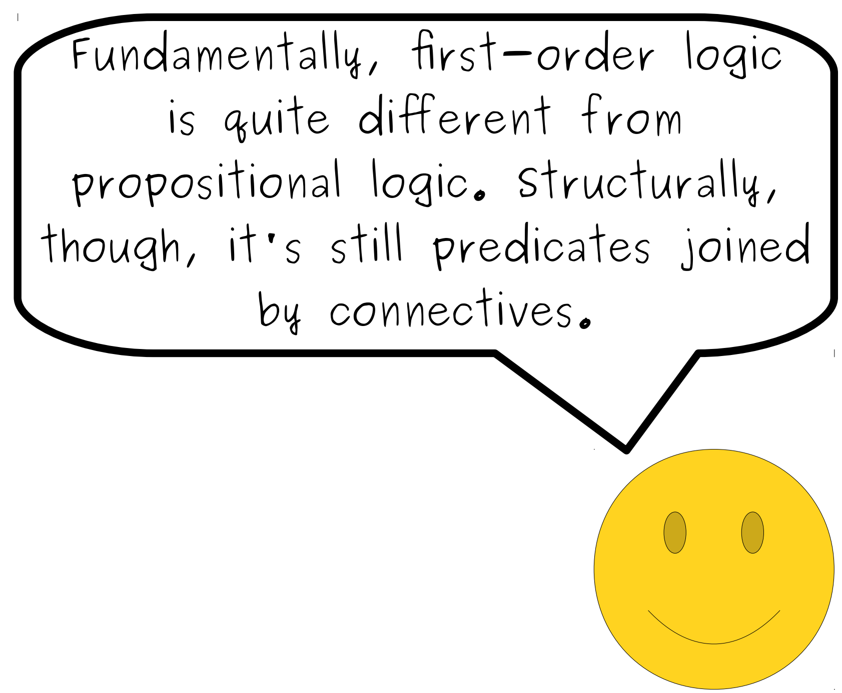
$$\frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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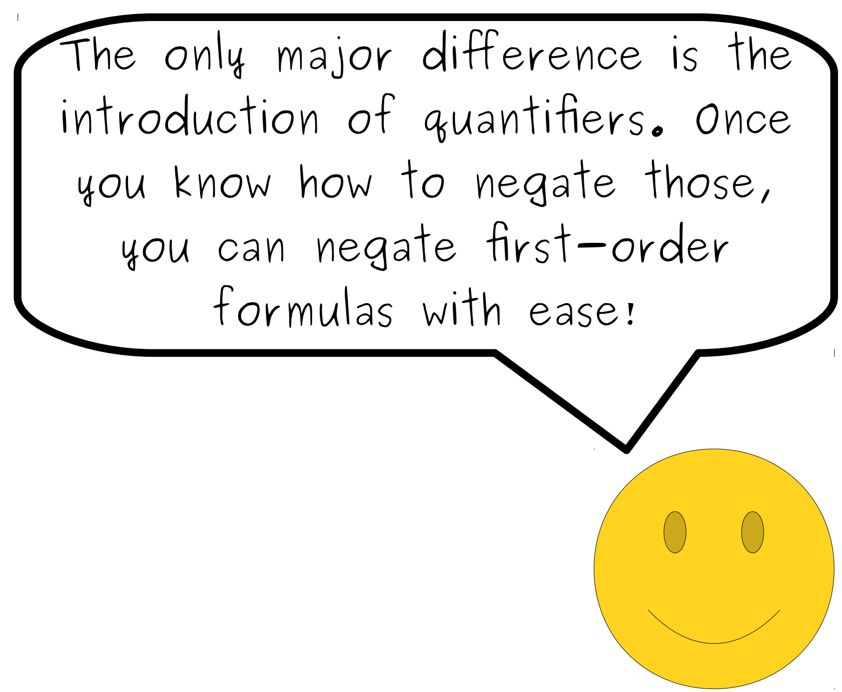


$$\frac{\neg\neg A}{A}$$

$$\frac{\neg(A \vee B)}{\neg A \wedge \neg B}$$

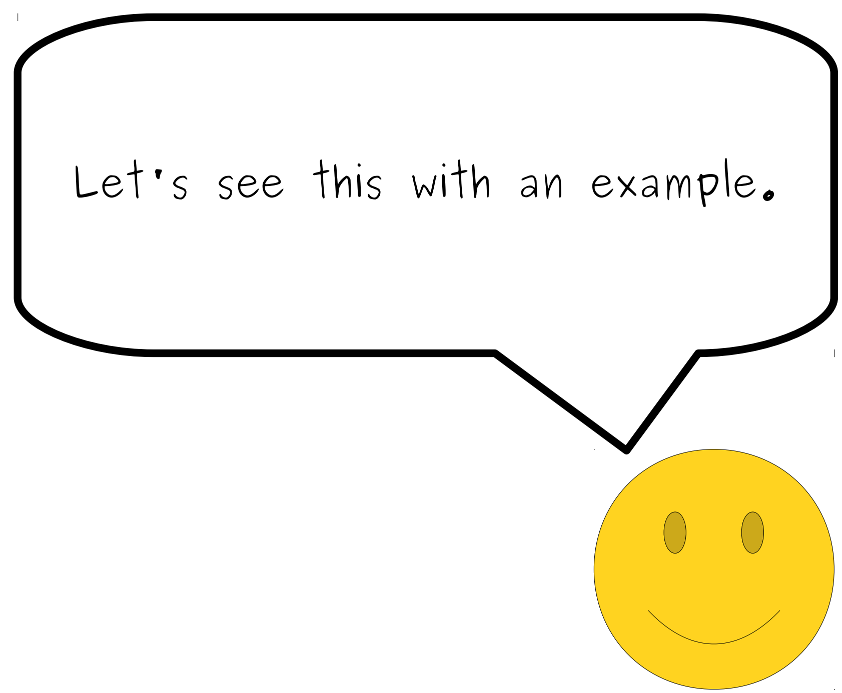
$$\frac{\neg(A \wedge B)}{\neg A \vee \neg B}$$

$$\frac{\neg(A \rightarrow B)}{A \wedge \neg B}$$

$$\frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

$$\frac{\neg(A \wedge B)}{A \rightarrow \neg B}$$

$$\frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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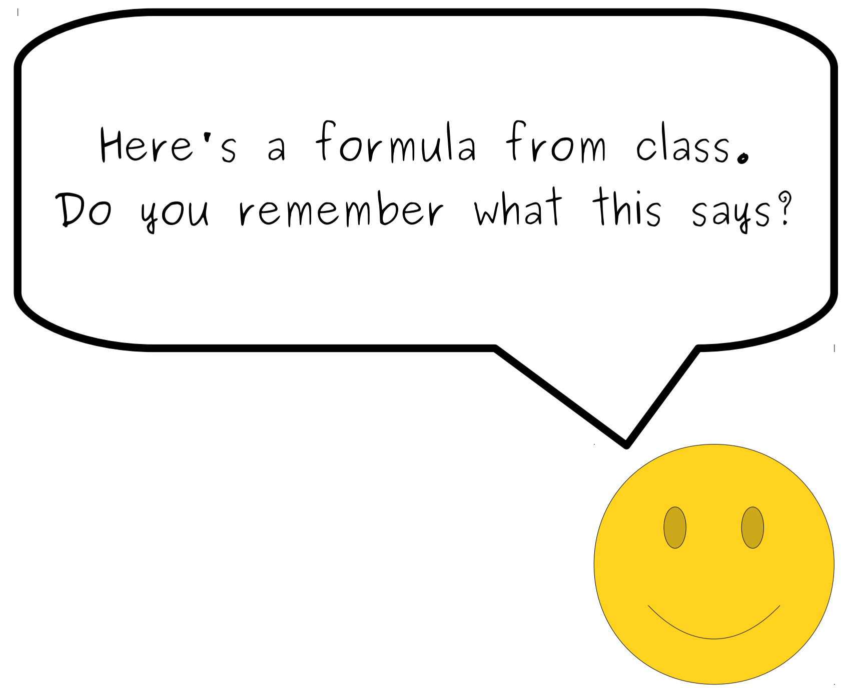
$$\forall x.\ (Person(x) \to \exists y.\ (Person(y) \land y \neq x \land Loves(x, y)))$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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&\quad) \\
&)
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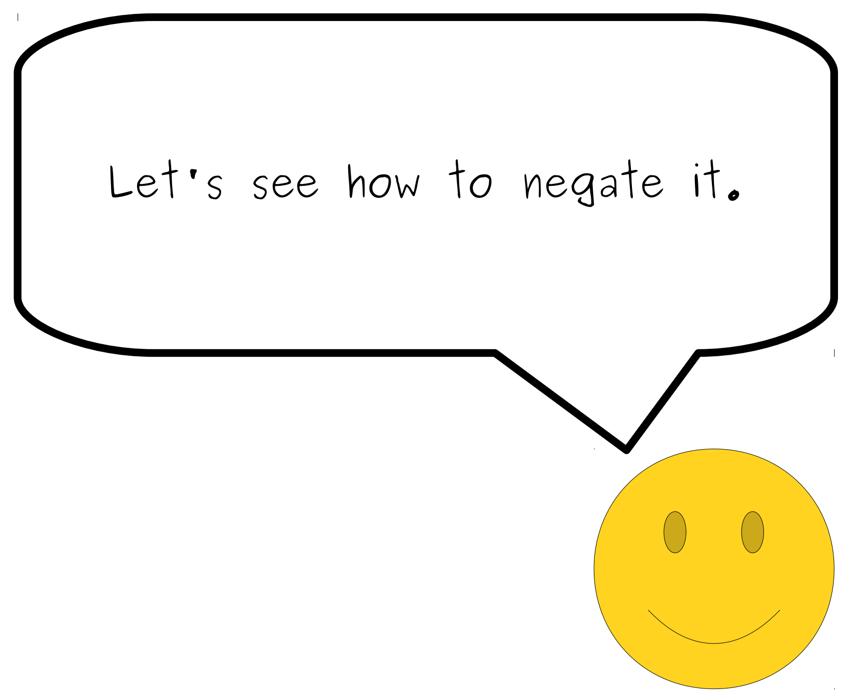

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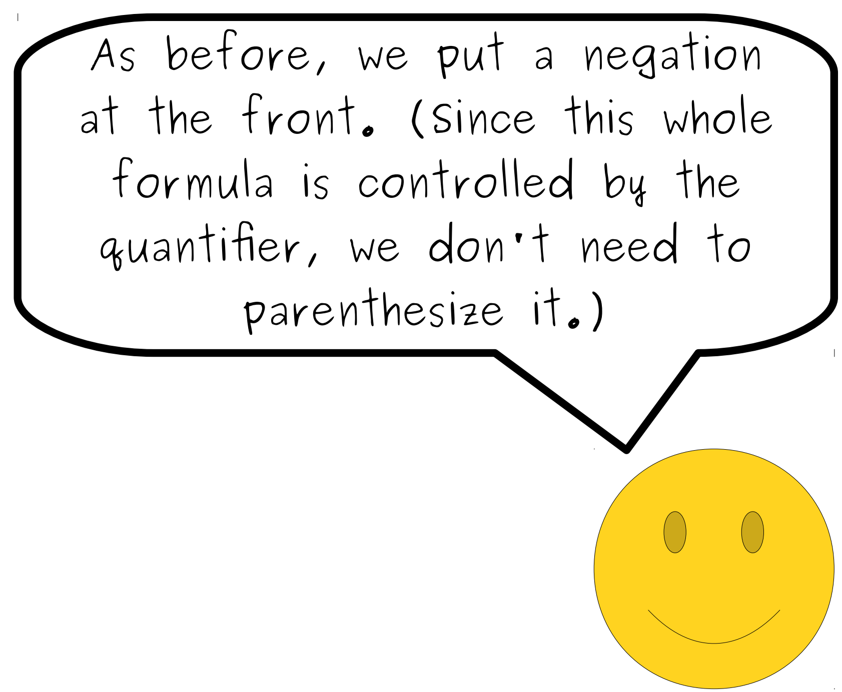

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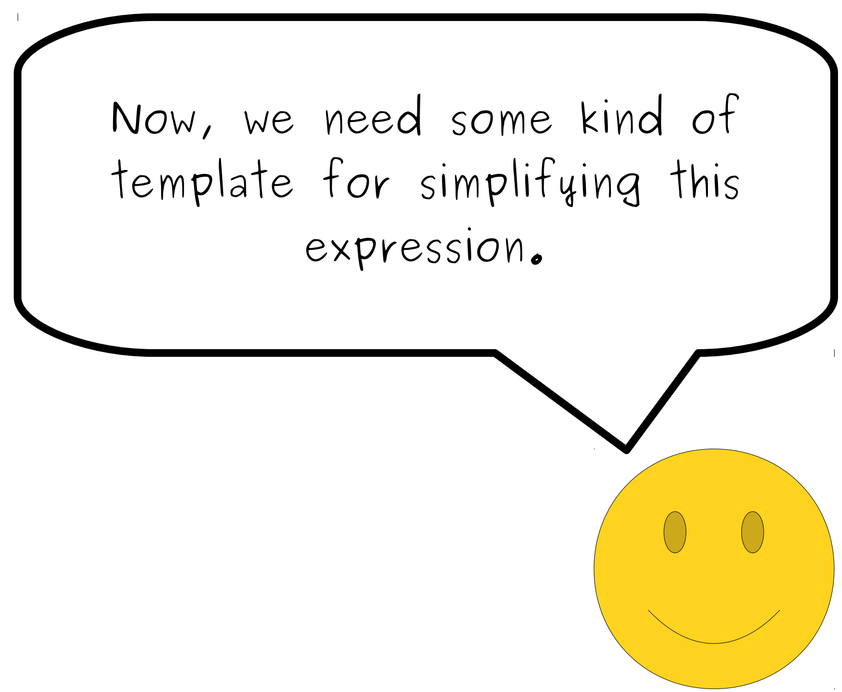

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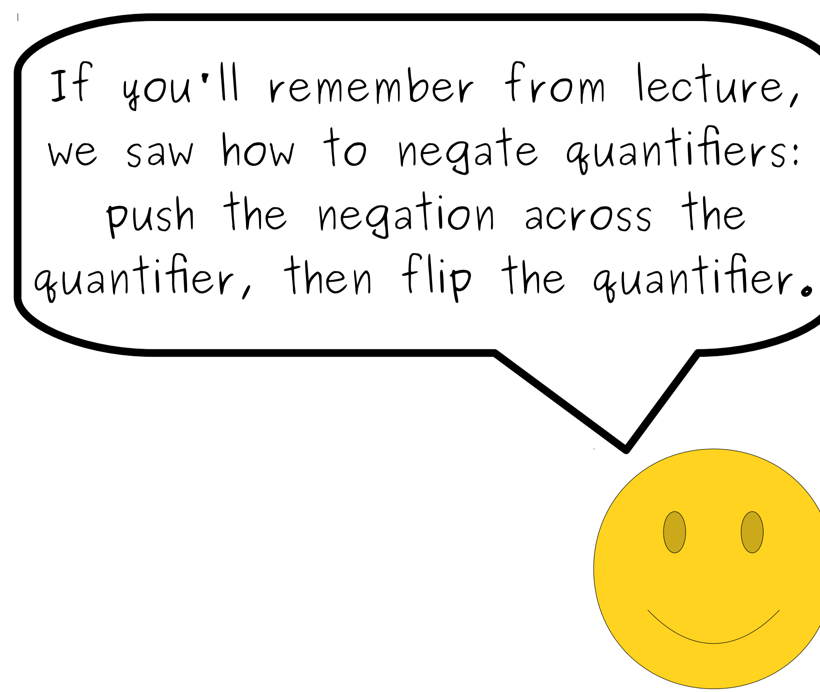

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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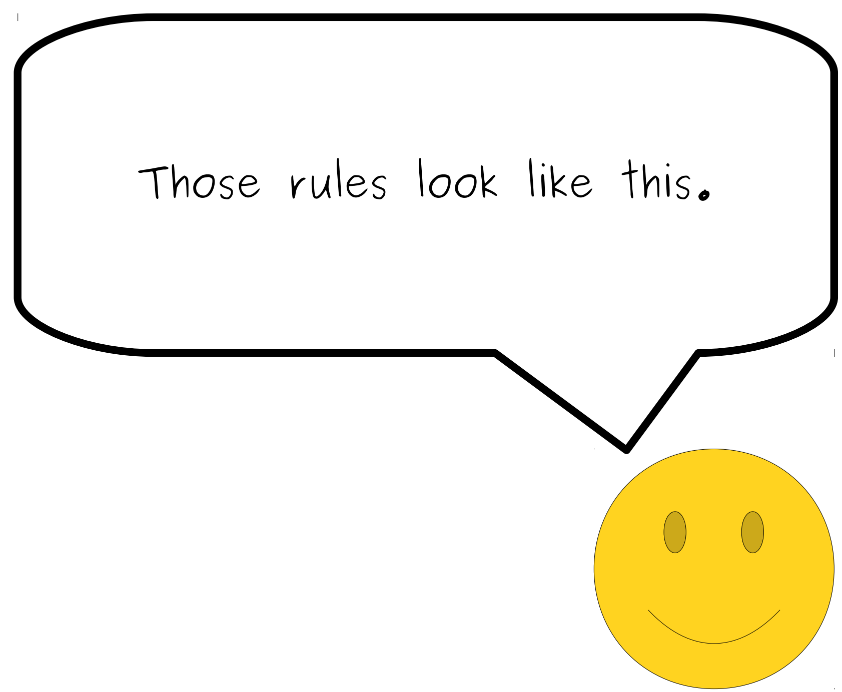

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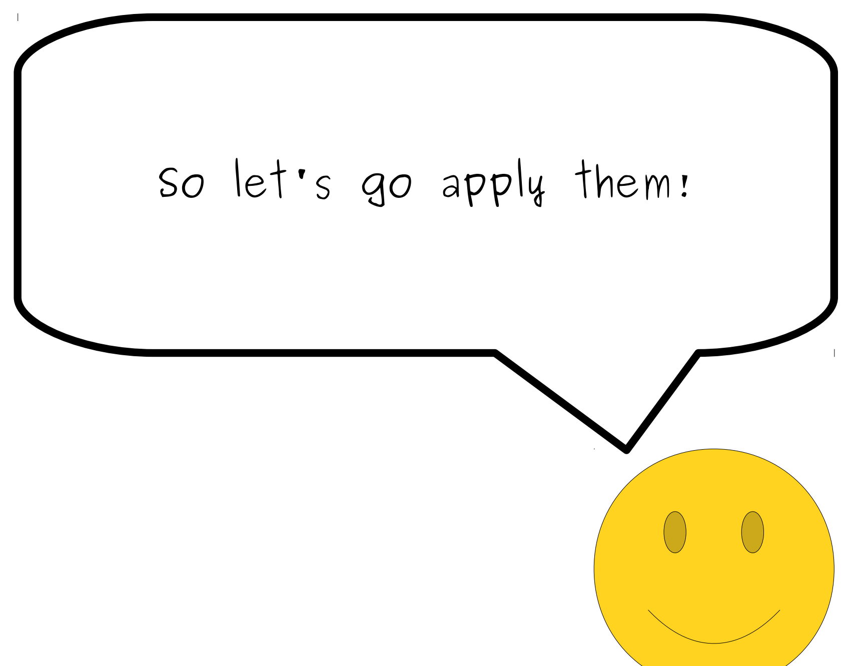

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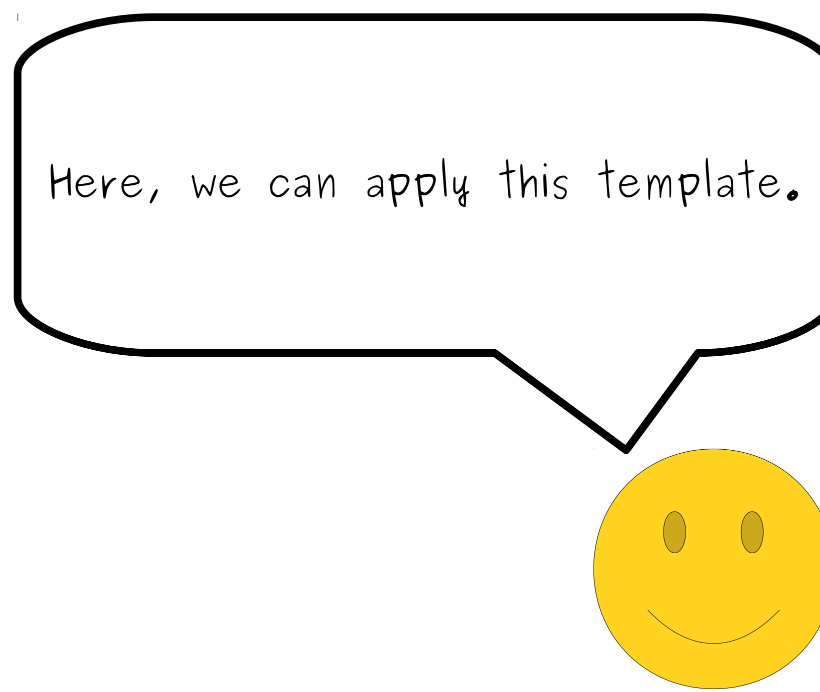
$$\boxed{\frac{\neg\forall x.\ A}{\exists x.\ \neg A}} \qquad \frac{\neg\exists x.\ A}{\forall x.\ \neg A}$$

$$\neg\forall x.\ (Person(x) \to \exists y.\ (Person(y) \land y \neq x \land Loves(x, y)))$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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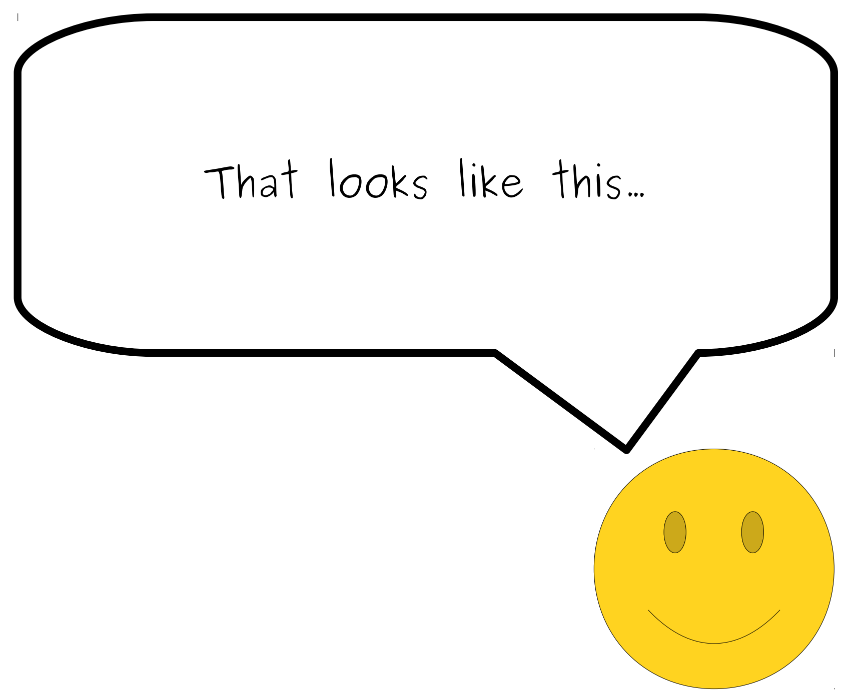

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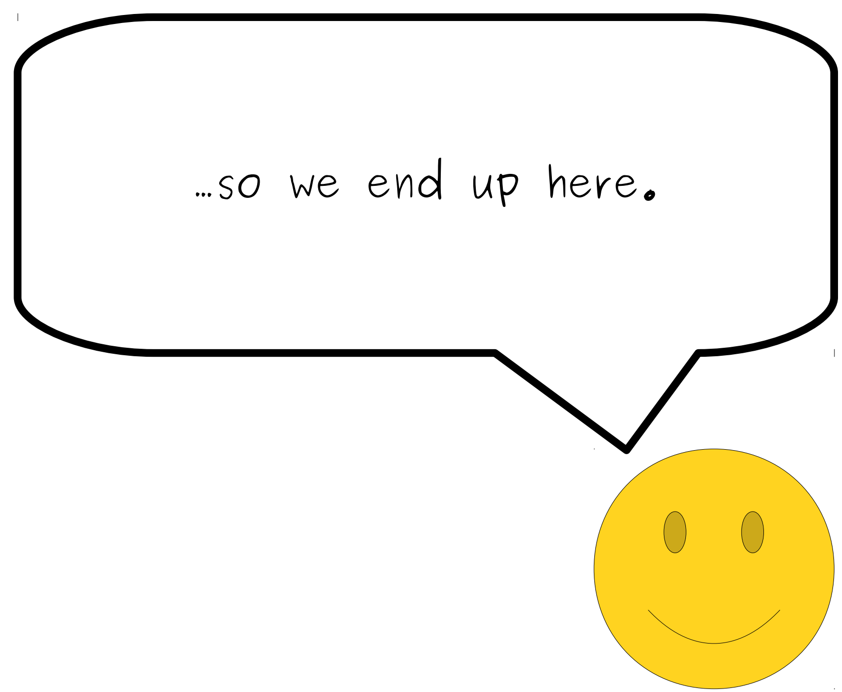

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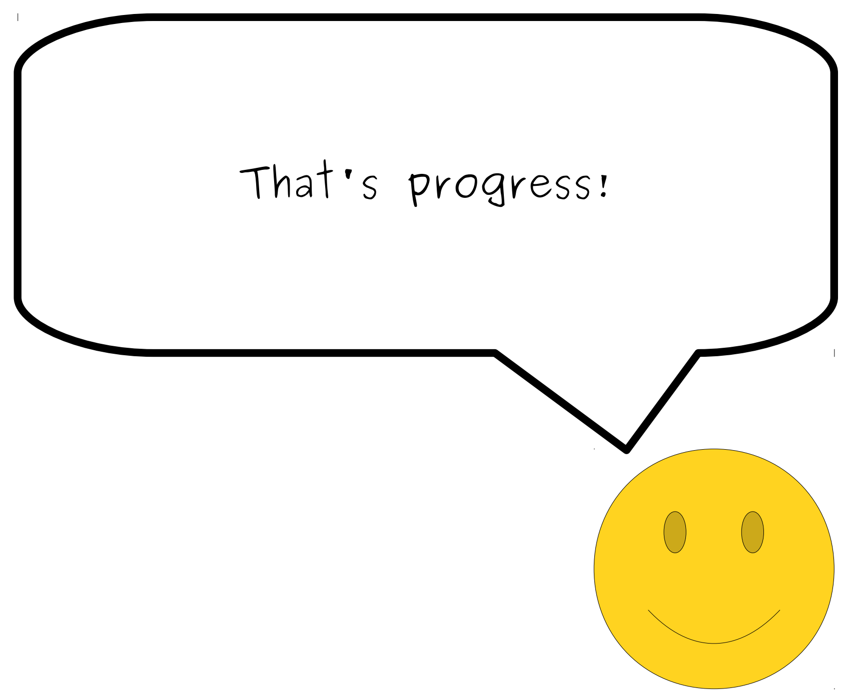

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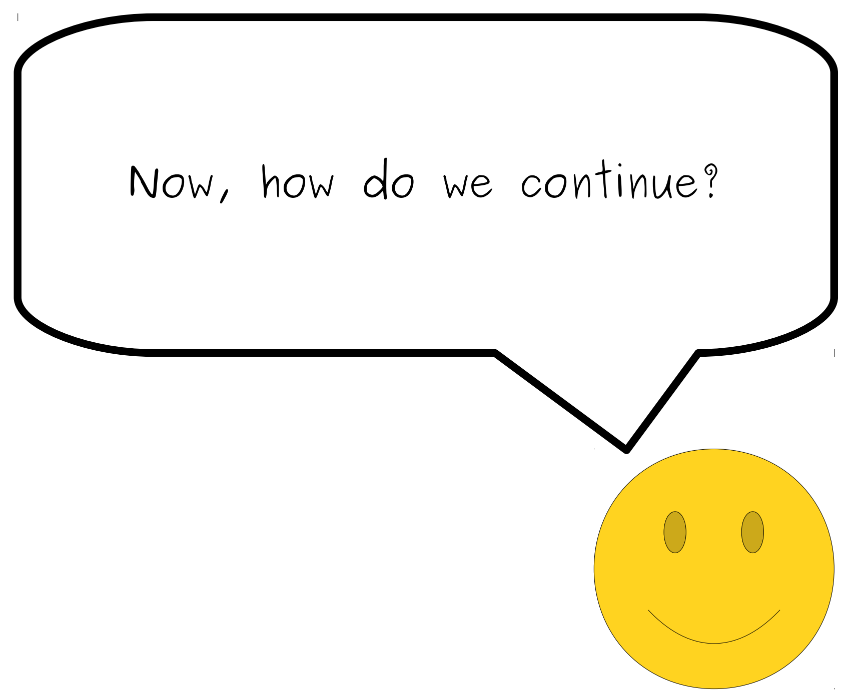
$$\begin{array}{l} \exists x.\ \neg(Person(x) \to \\ \quad \exists y.\ (Person(y) \land y \neq x \land \\ \qquad Loves(x, y) \\ \quad) \\ ) \end{array}$$



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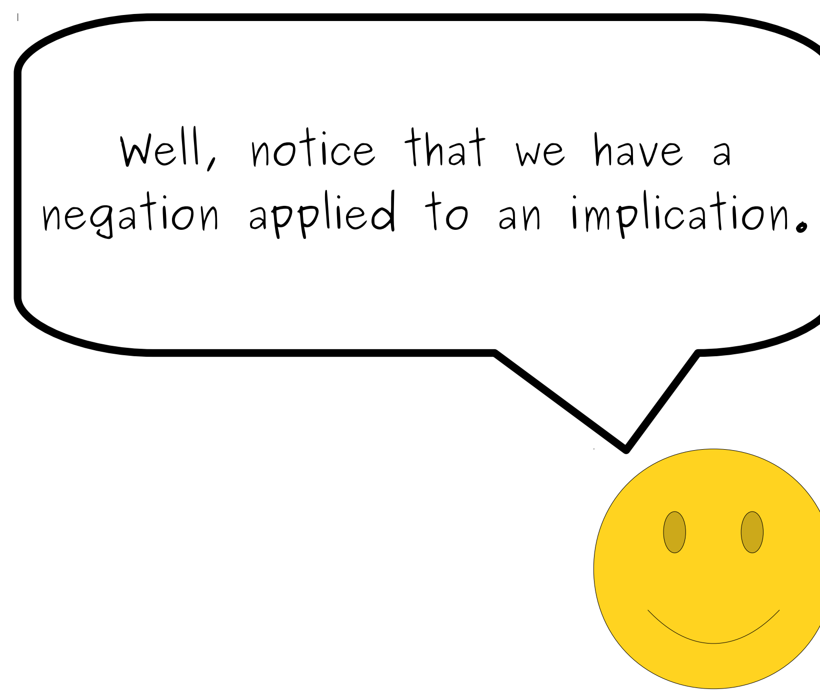

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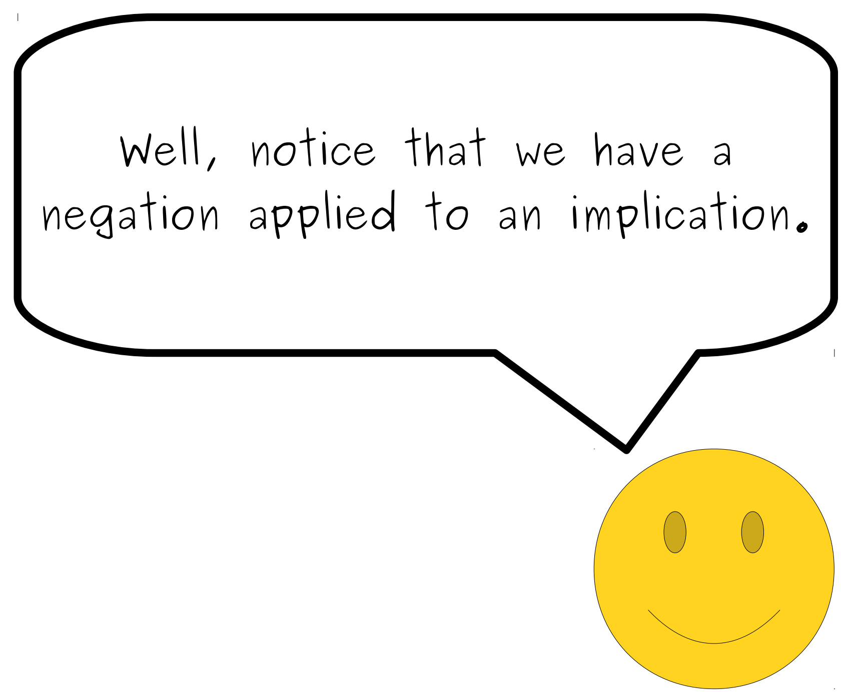

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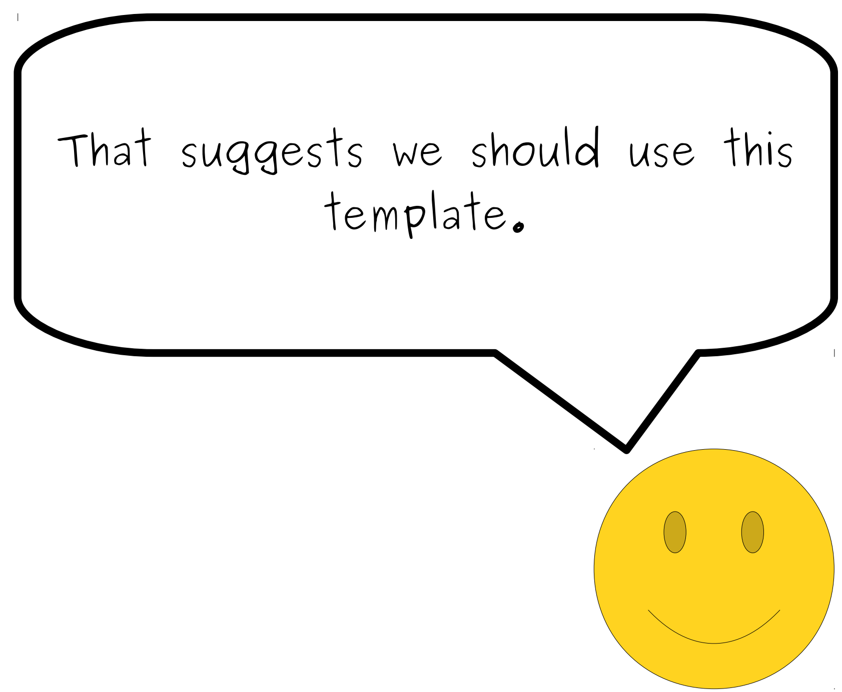
$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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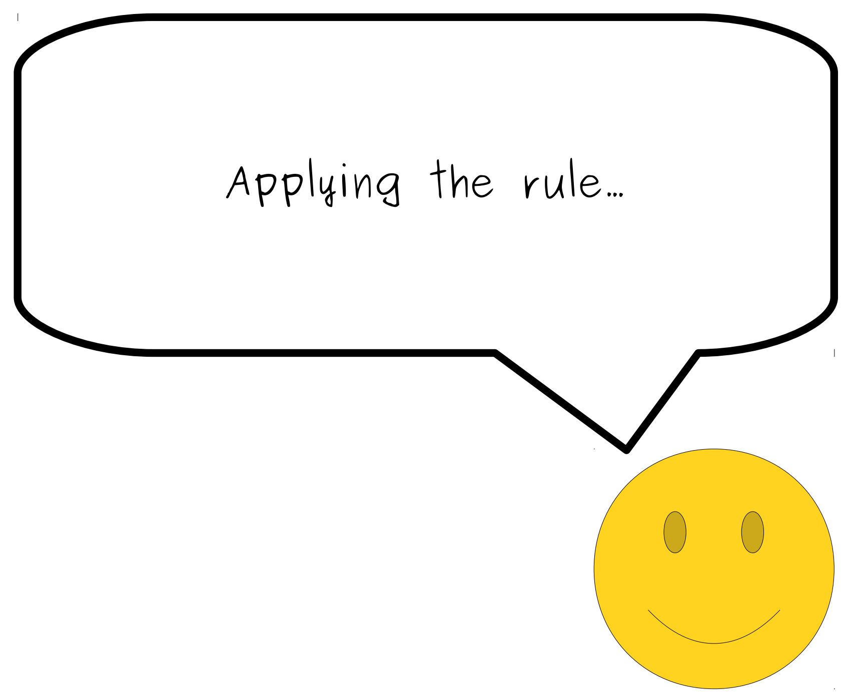

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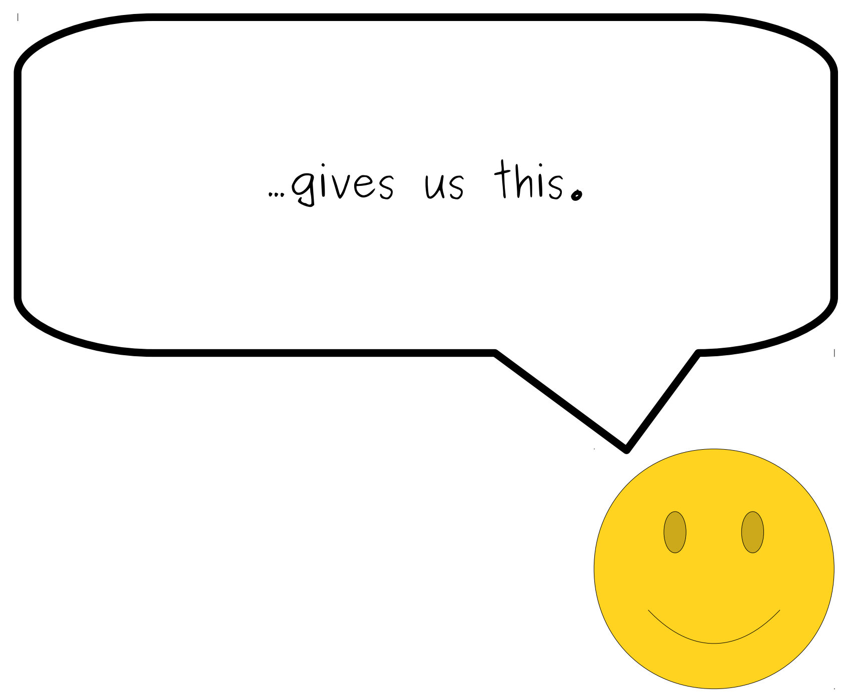

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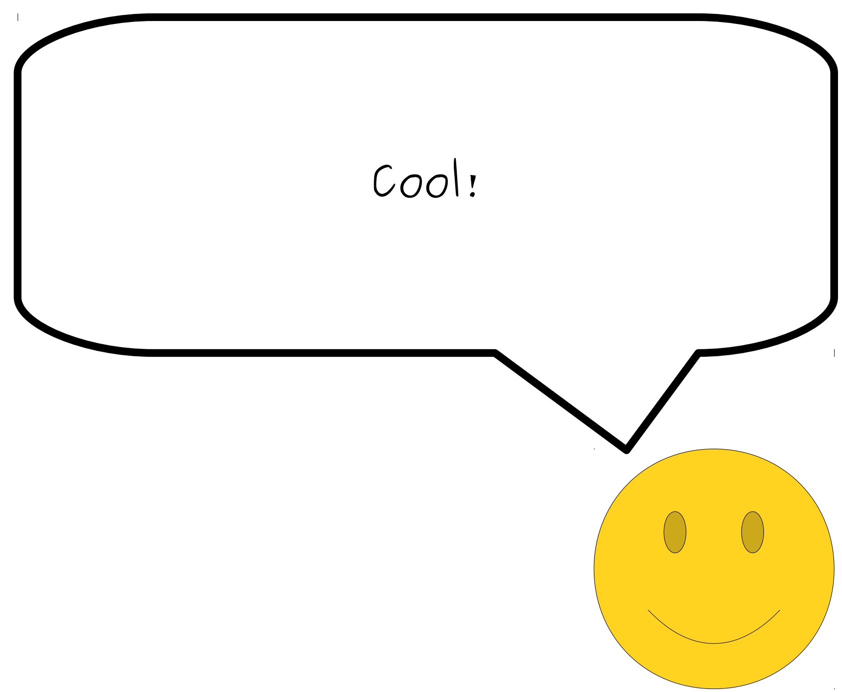

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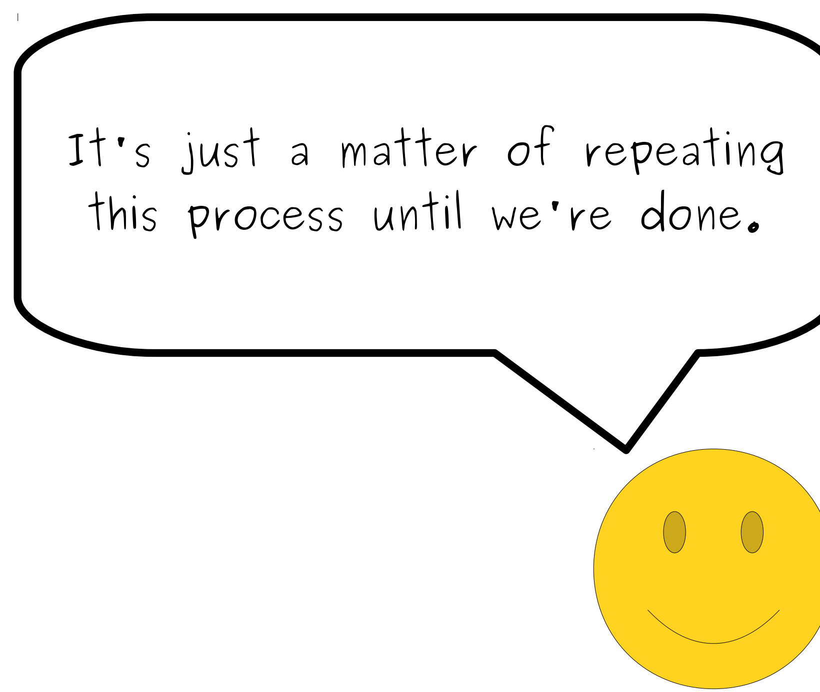

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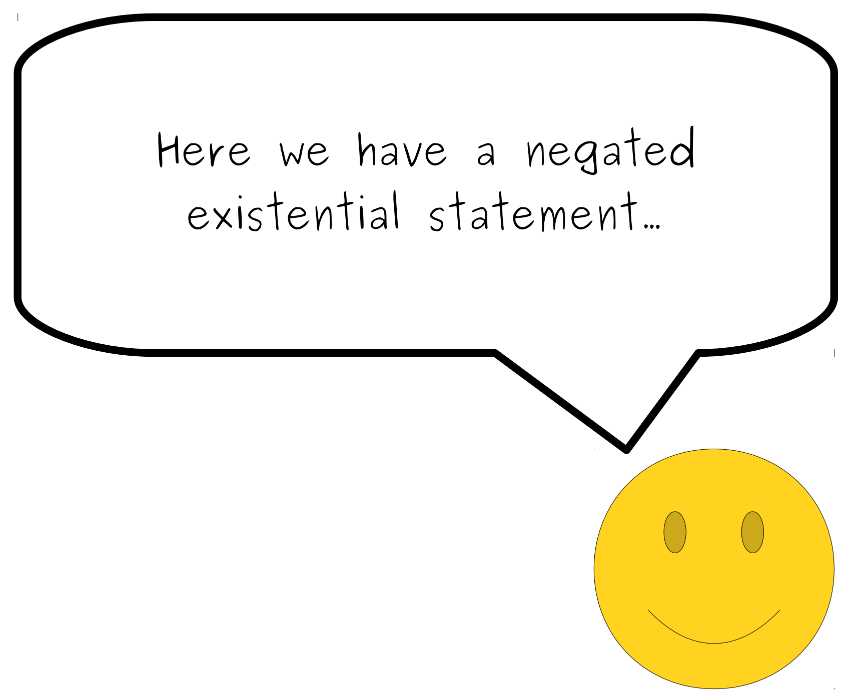

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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&)
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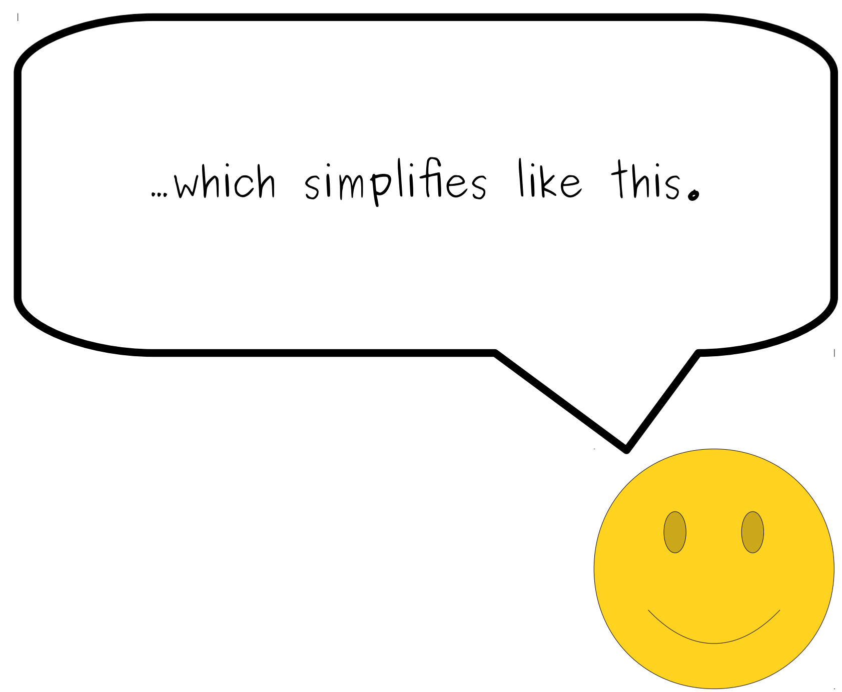

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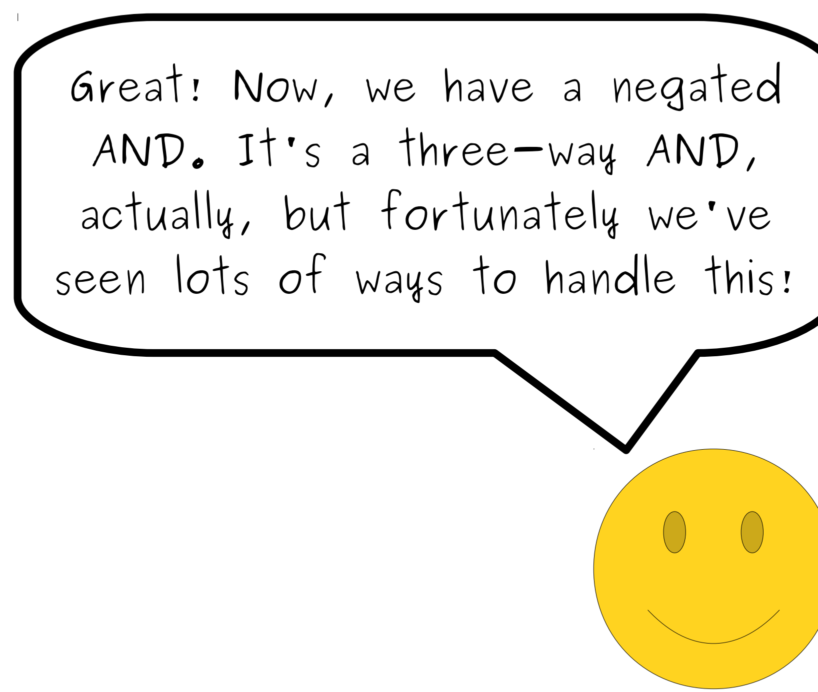

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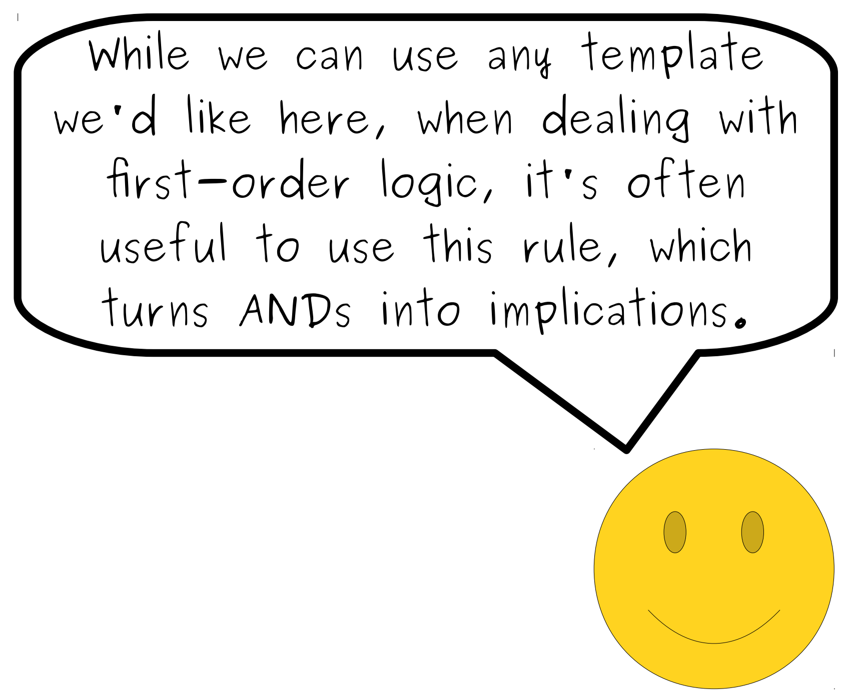

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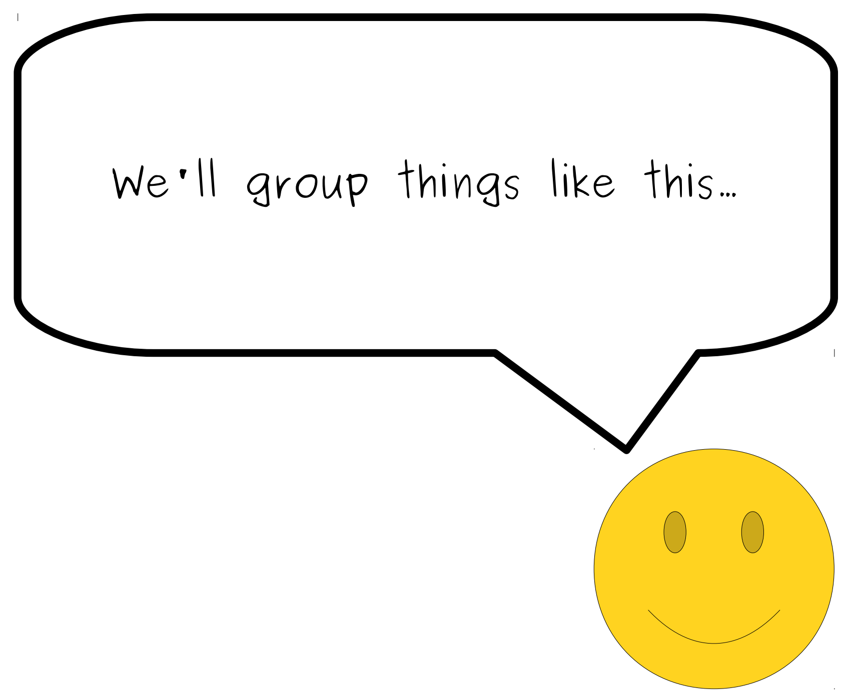

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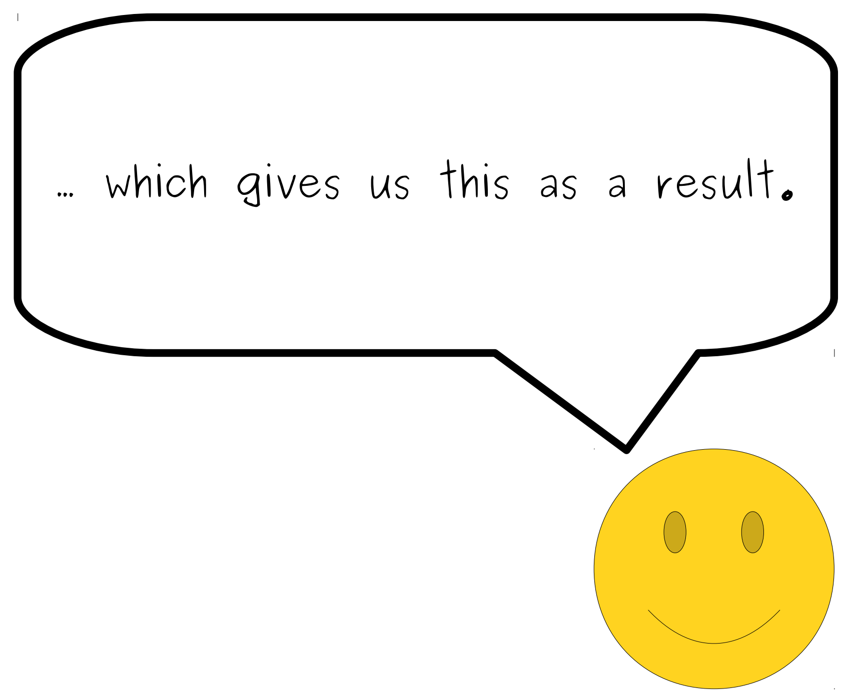

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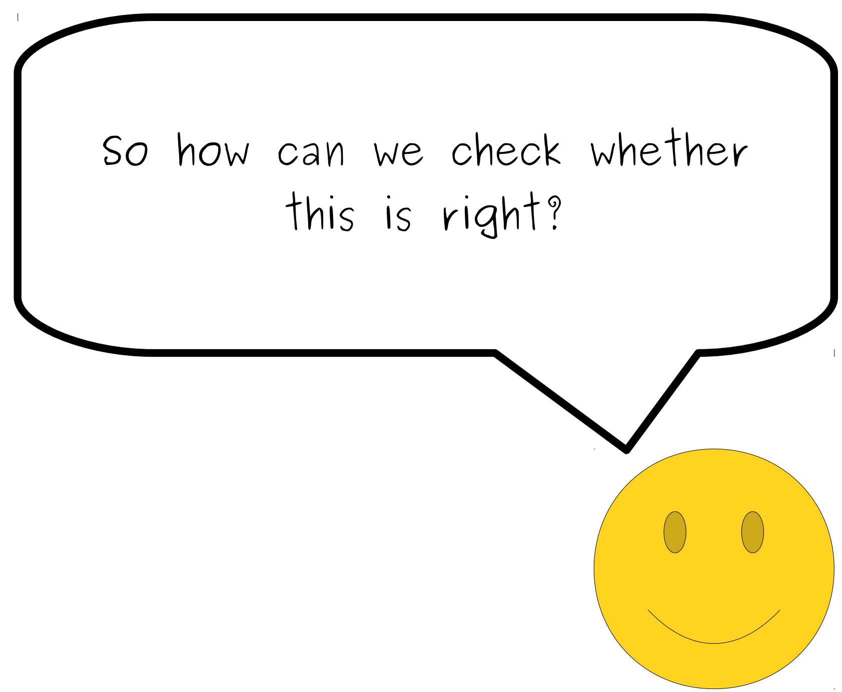

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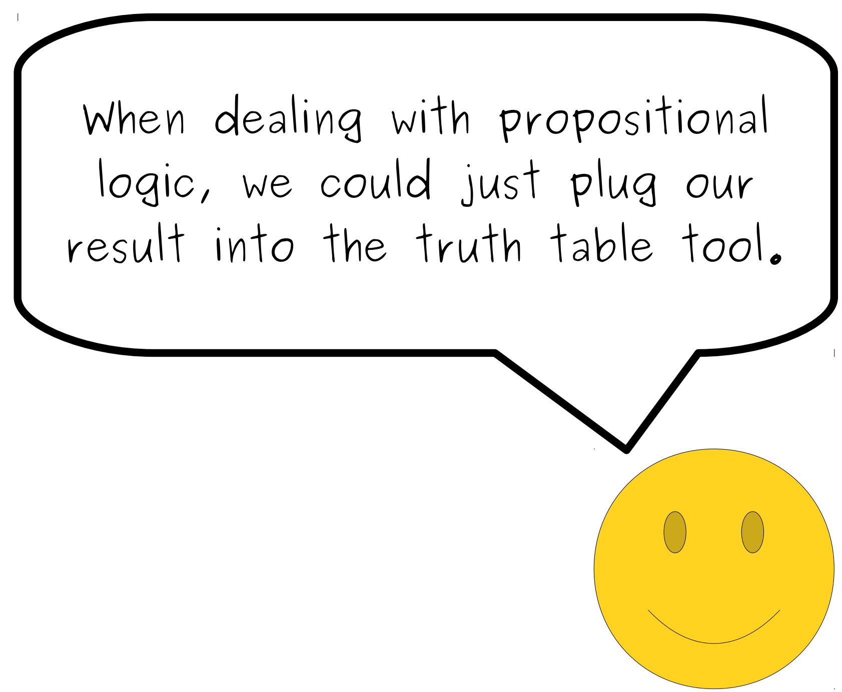

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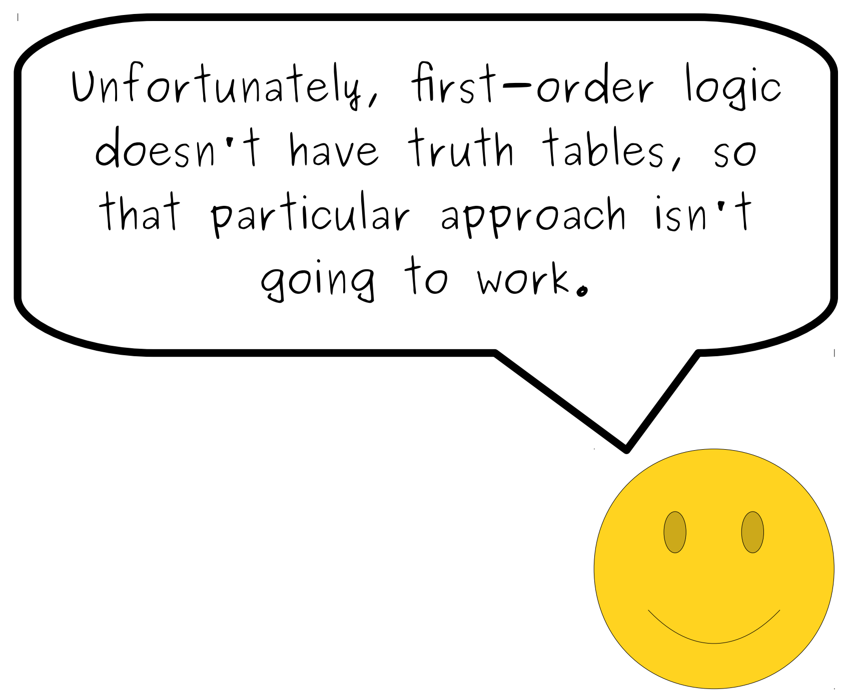

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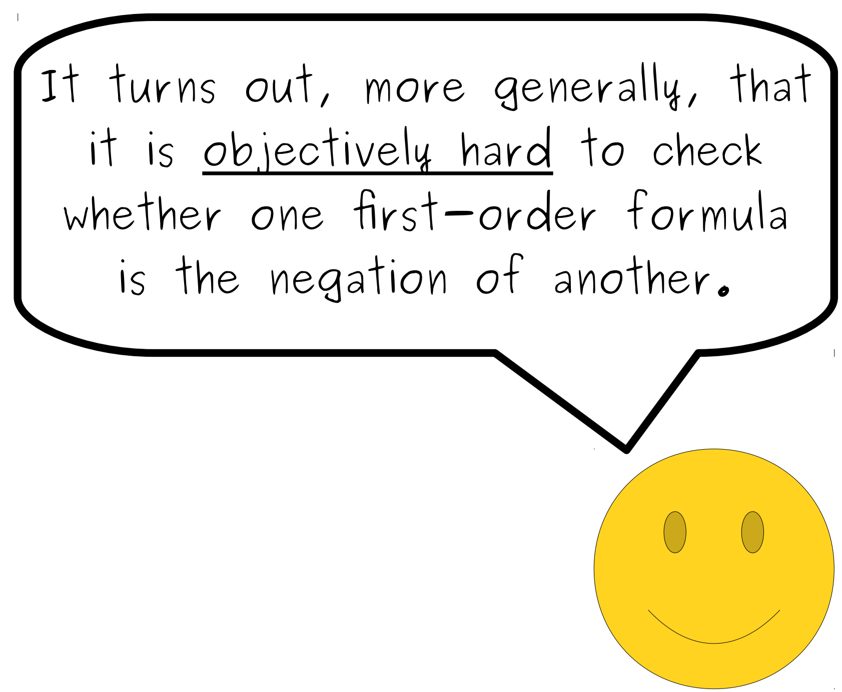

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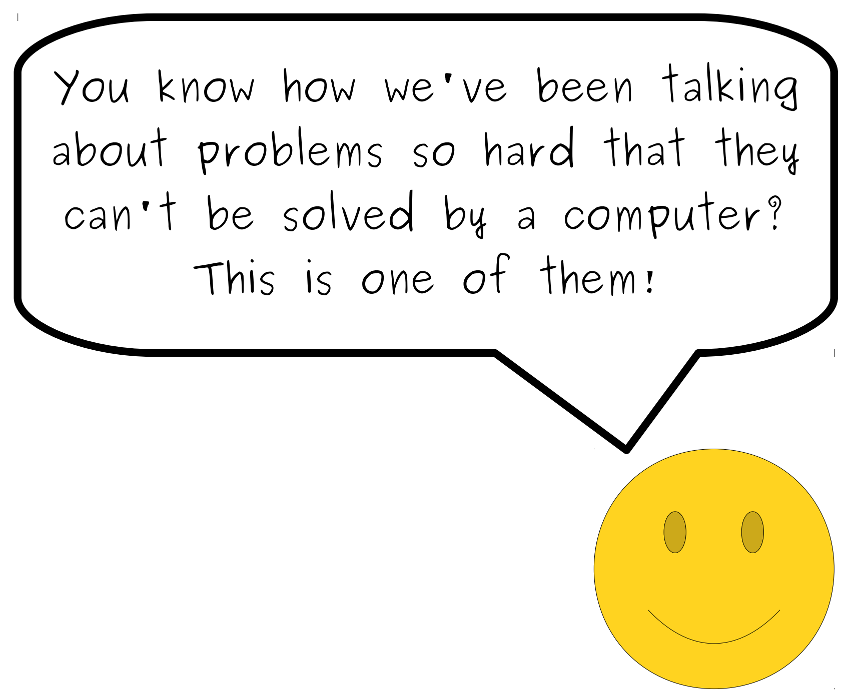

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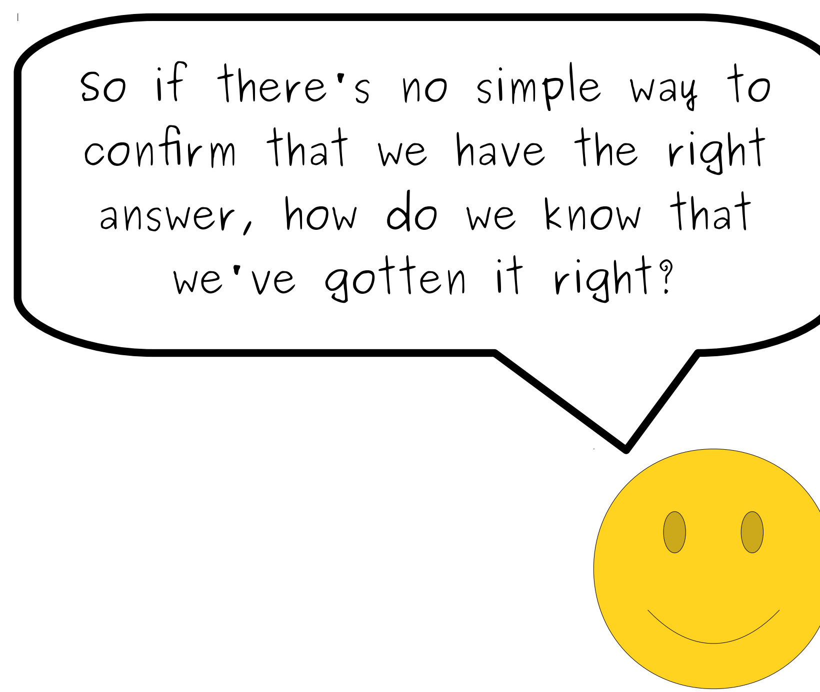

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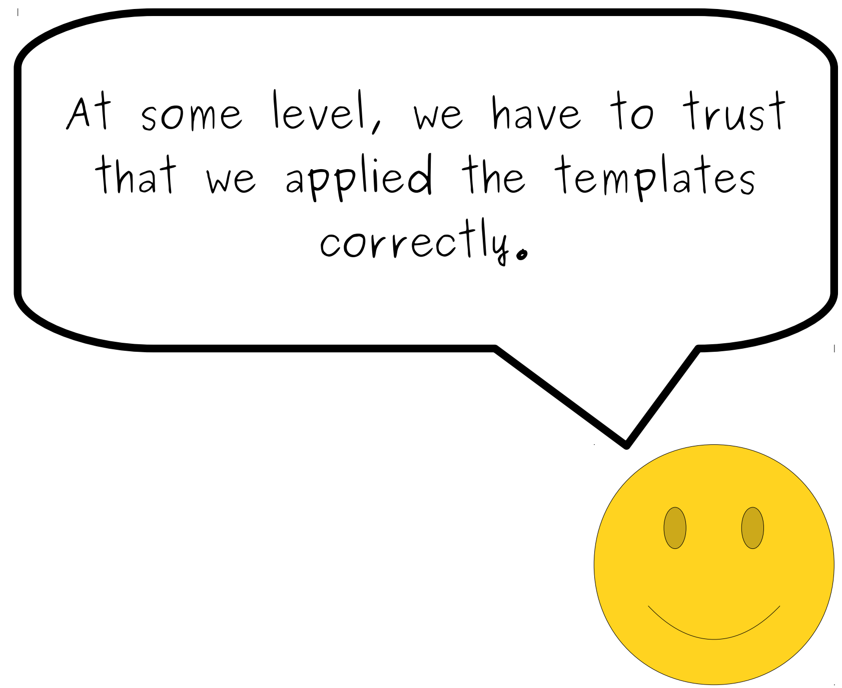

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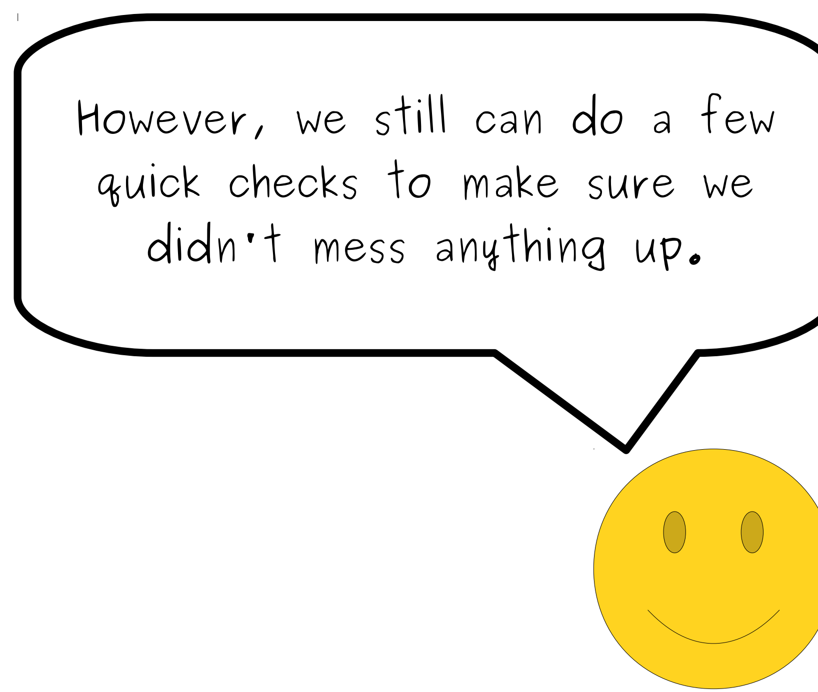

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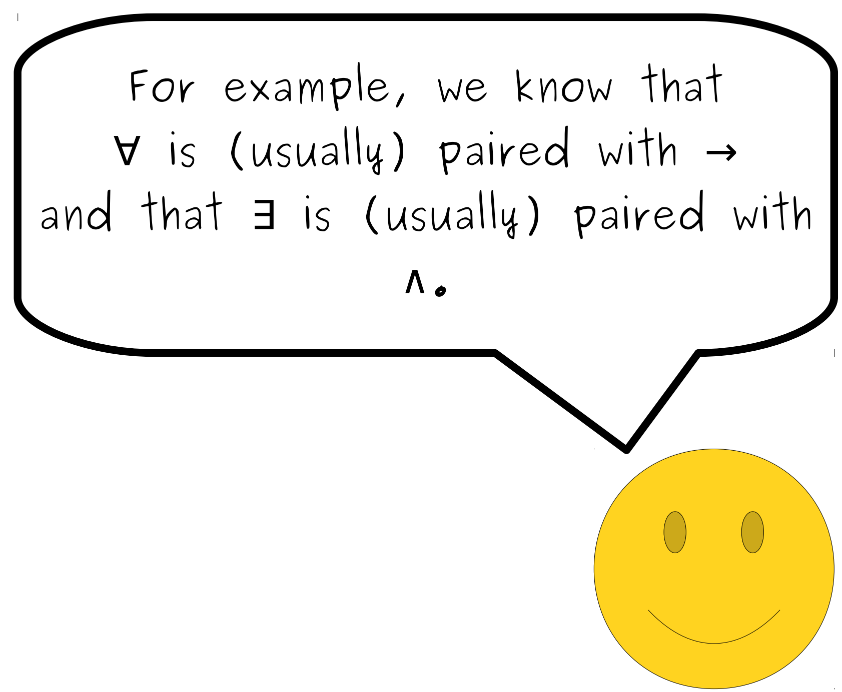

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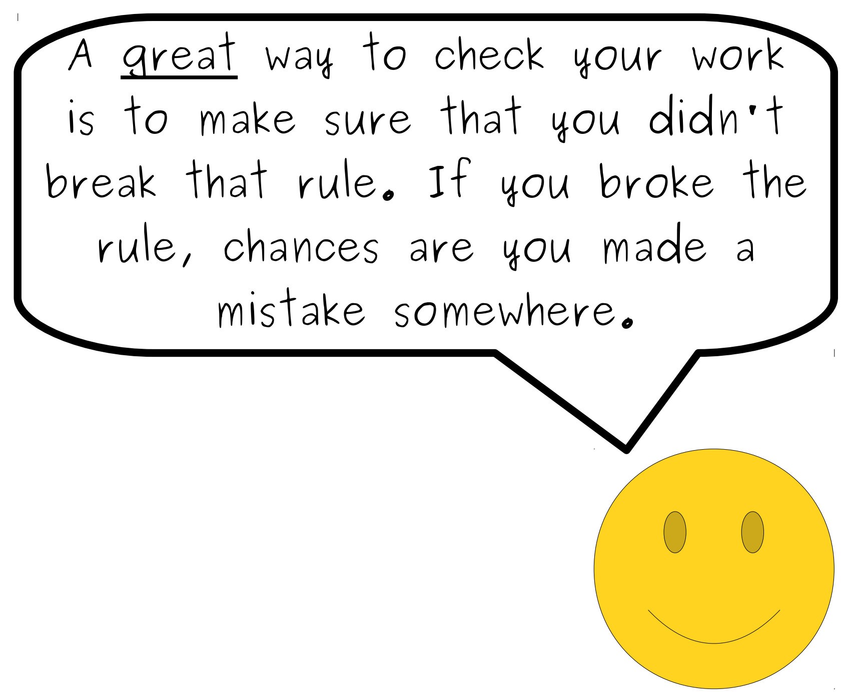

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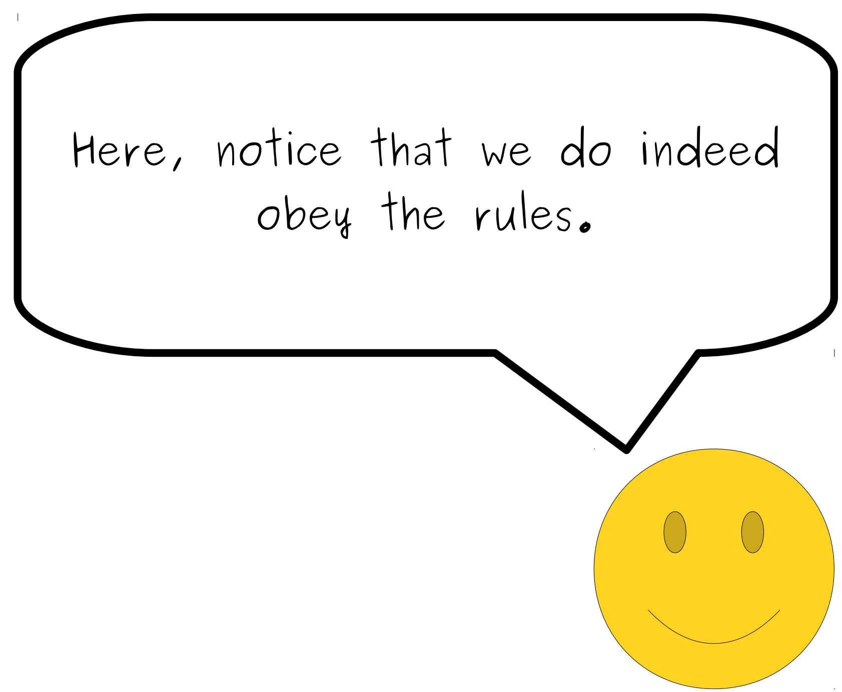

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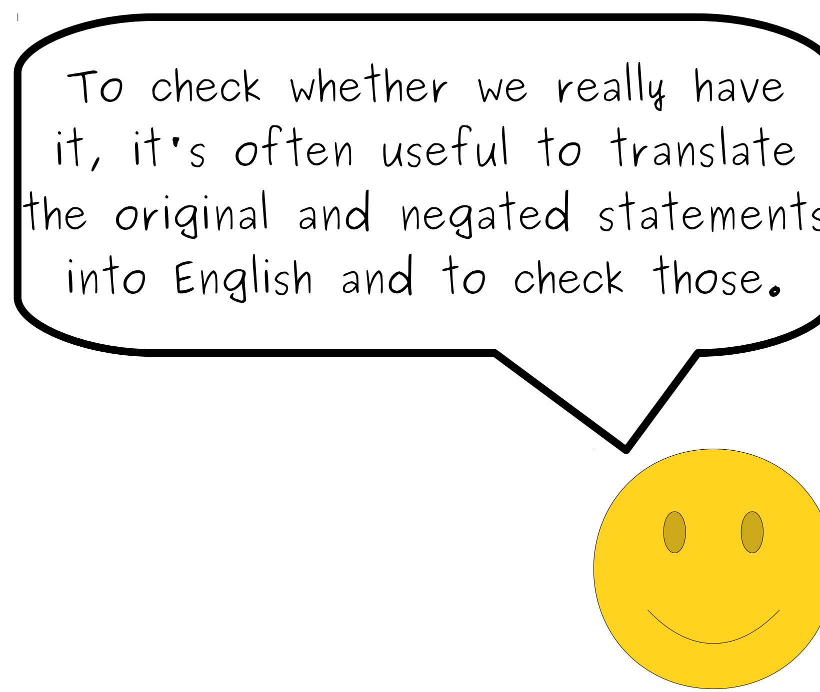

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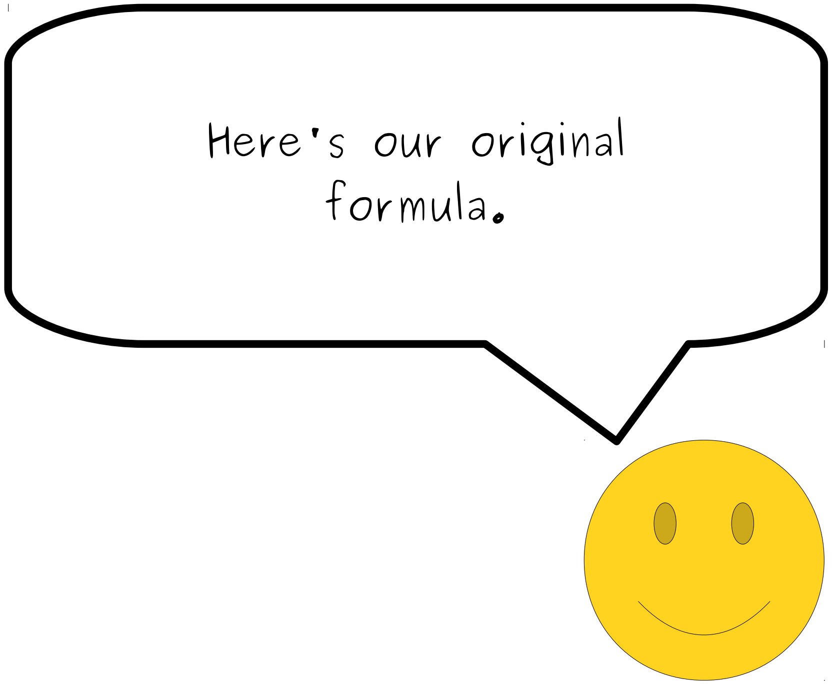

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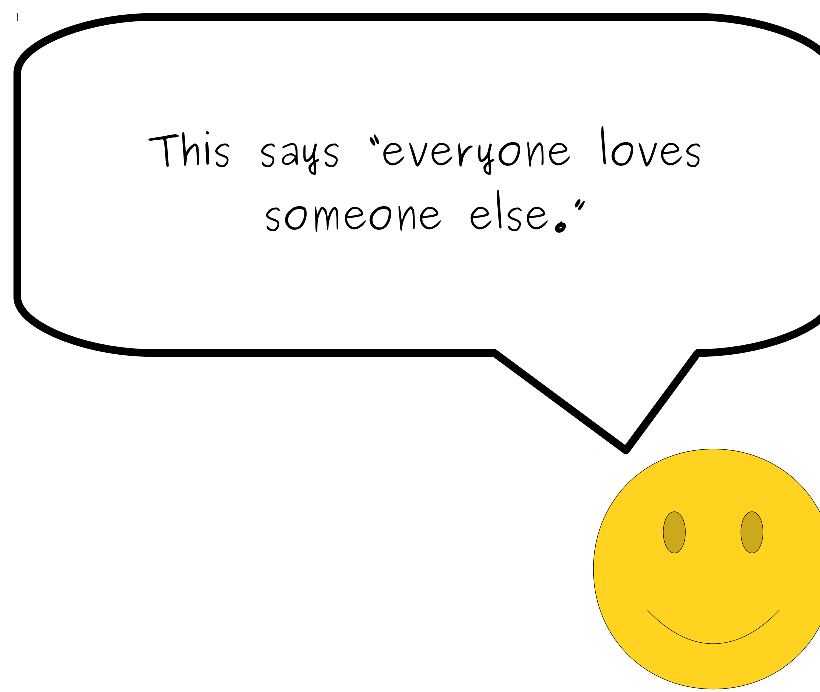

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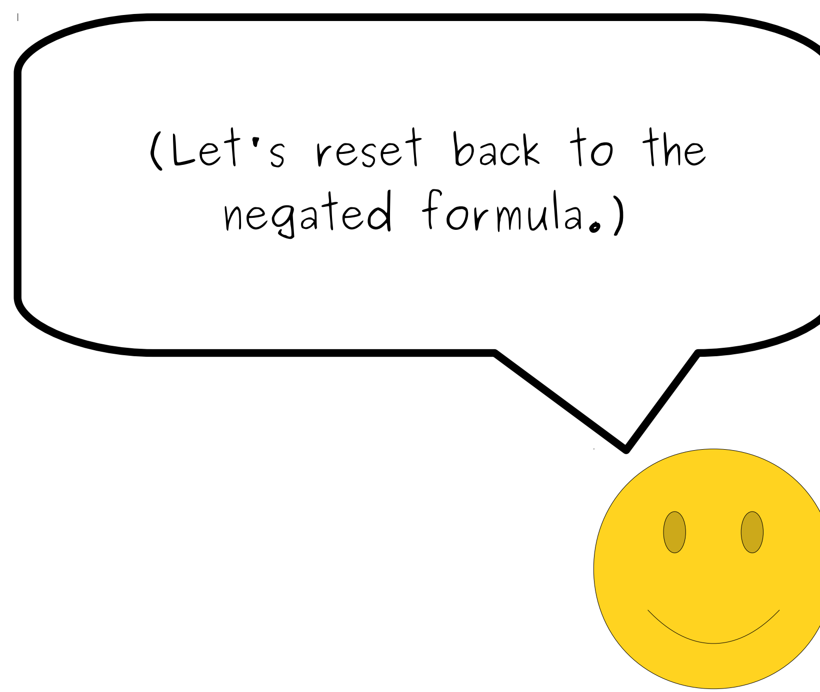

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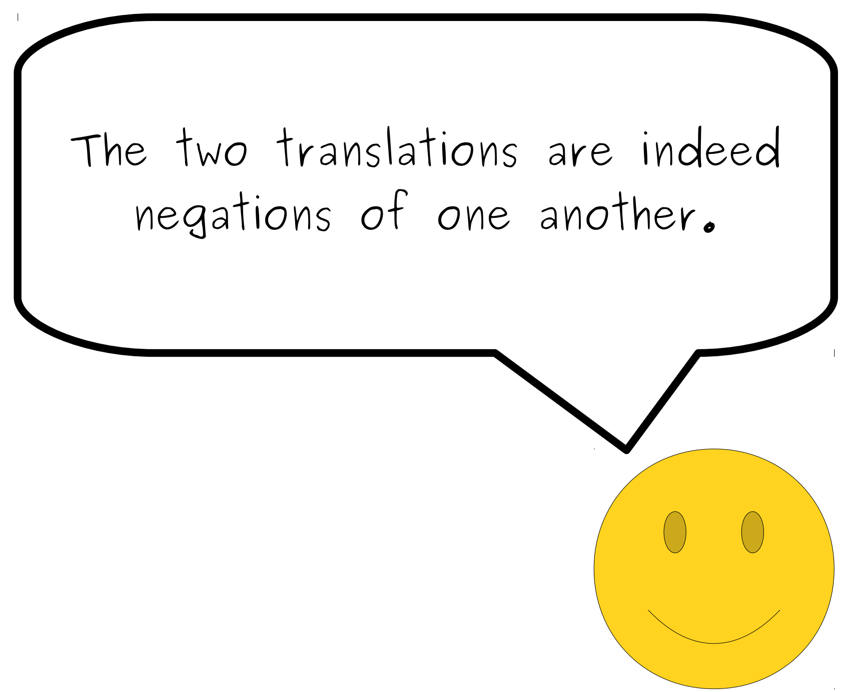
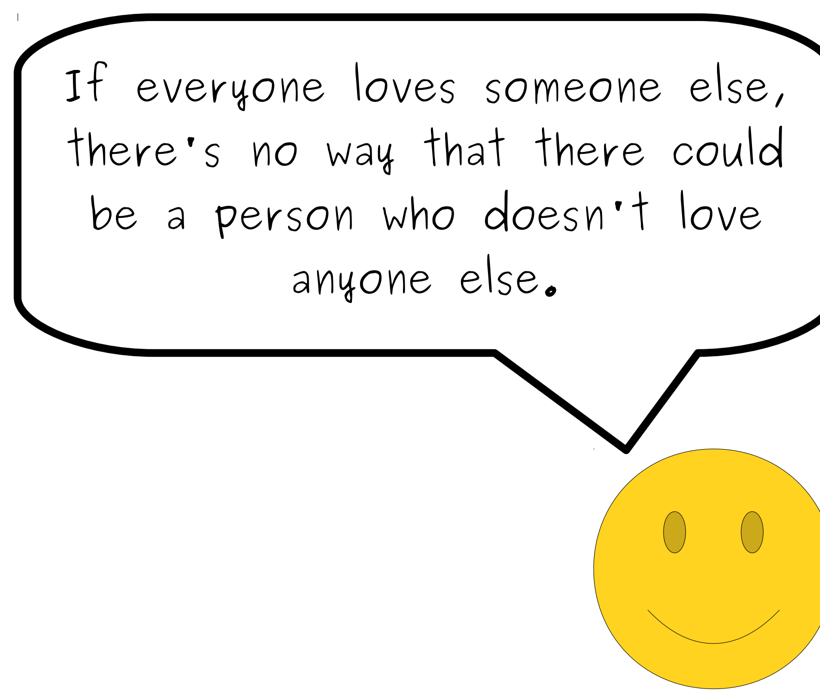


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

$$\frac{\neg(A \land B)}{A \to \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$

$$\frac{\neg\forall x.\ A}{\exists x.\ \neg A} \qquad \frac{\neg\exists x.\ A}{\forall x.\ \neg A}$$

$$\begin{array}{l}
\exists x.\ (Person(x) \land \\
\quad \forall y.\ (Person(y) \land y \neq x \to \\
\qquad \neg Loves(x, y) \\
\quad) \\
)
\end{array}$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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$$
\begin{aligned}
&\exists x.\ (Person(x)\ \land \\
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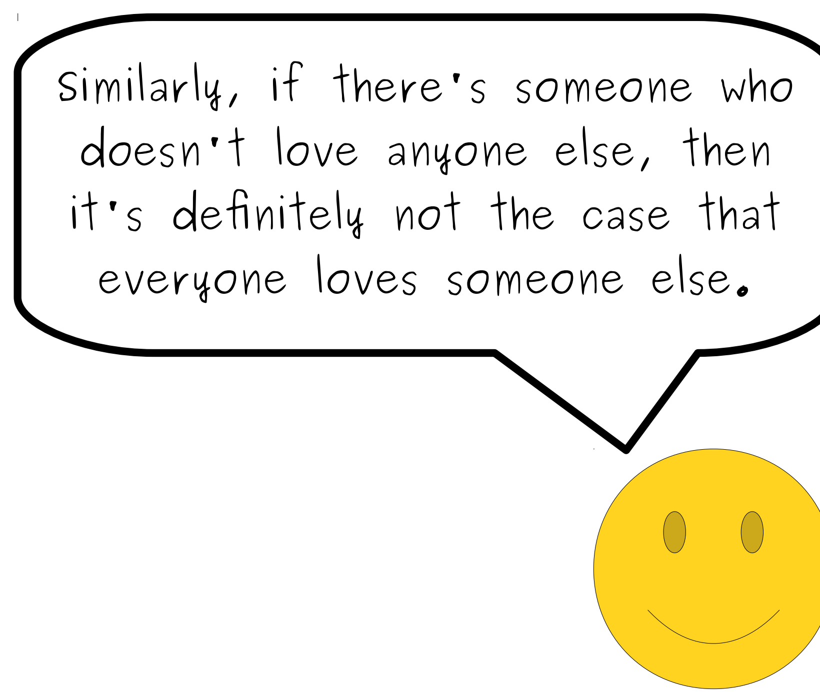
&\qquad \neg Loves(x, y) \\
&\quad) \\
&)
\end{aligned}
$$



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\begin{aligned}
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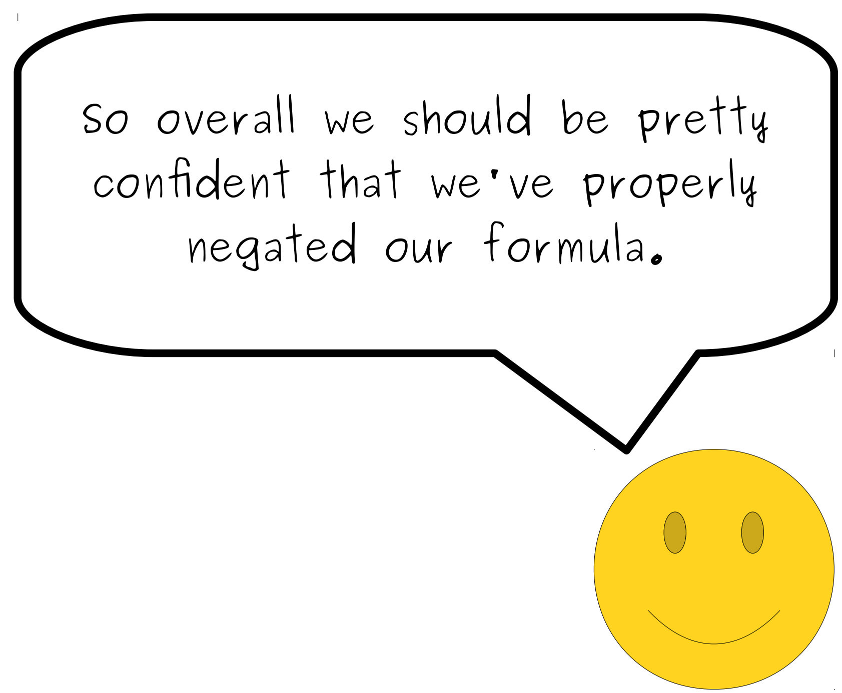
&\quad) \\
&)
\end{aligned}
$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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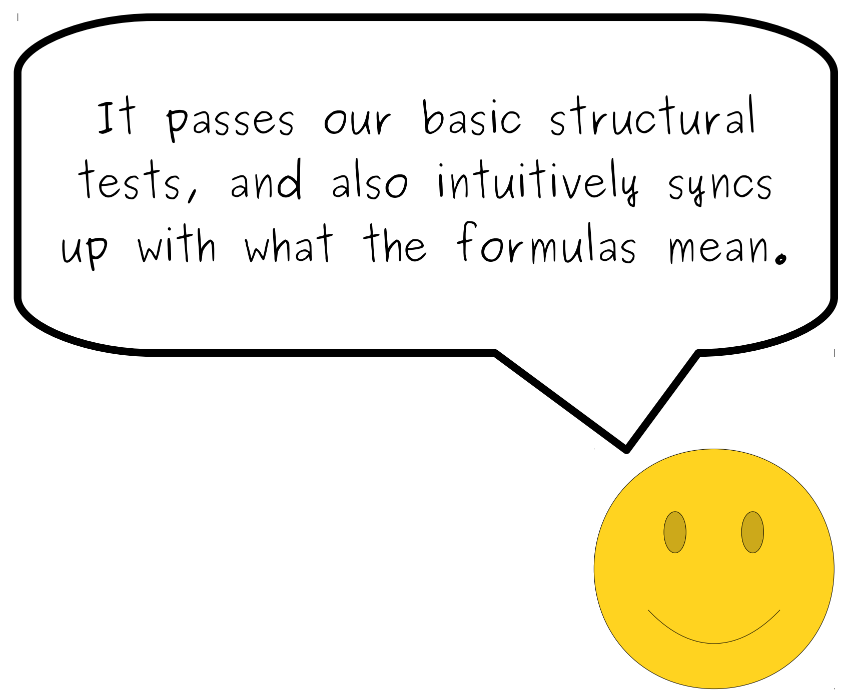
&\quad) \\
&)
\end{aligned}
$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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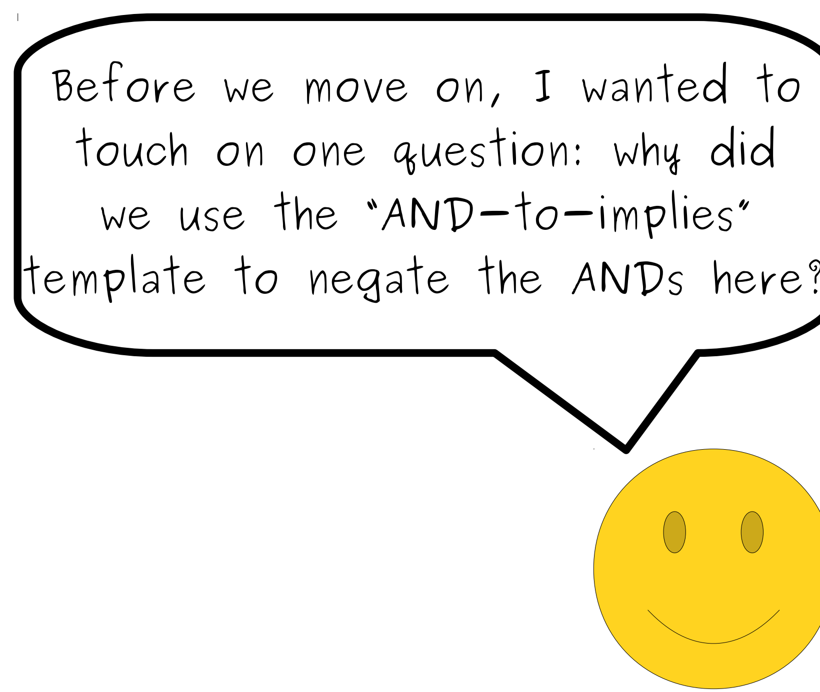
$$\frac{\neg \forall x.\ A}{\exists x.\ \neg A} \qquad \frac{\neg \exists x.\ A}{\forall x.\ \neg A}$$

$$\begin{array}{l} \exists x.\ (Person(x) \land \\ \quad \forall y.\ (Person(y) \land y \neq x \to \\ \quad\quad \neg Loves(x, y) \\ \quad) \\ ) \end{array}$$



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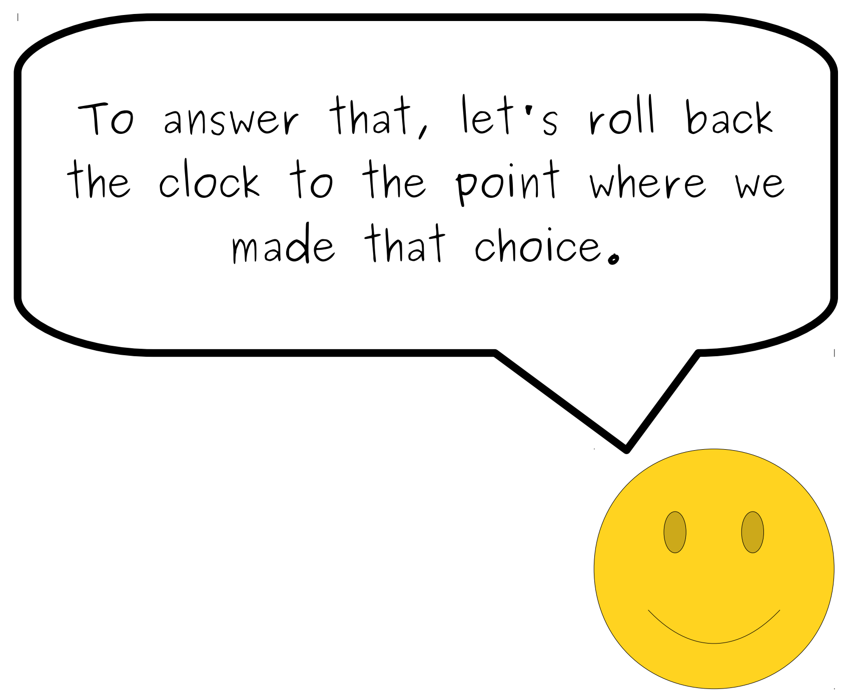
$$
\begin{array}{l}
\exists x.\ (Person(x) \land \\
\quad \forall y.\ (Person(y) \land y \neq x \to \\
\qquad \neg Loves(x, y) \\
\quad) \\
)
\end{array}
$$



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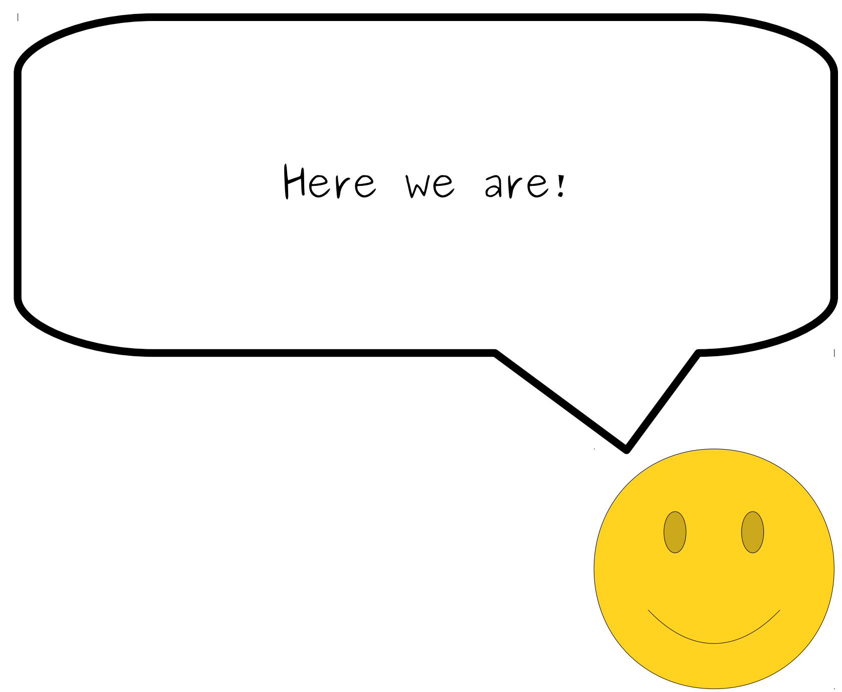
$$
\begin{array}{l}
\exists x.\ (Person(x)\ \land \\
\quad \forall y.\ \neg(Person(y)\ \land\ y \neq x\ \land \\
\qquad Loves(x, y) \\
\quad) \\
)
\end{array}
$$



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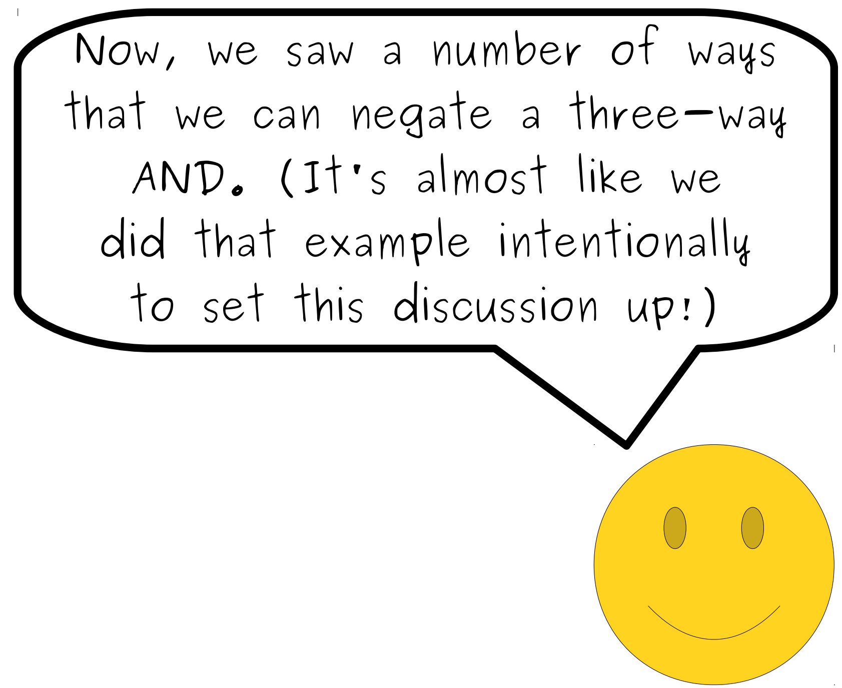
$$\frac{\neg(A \land B)}{A \to \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$

$$\frac{\neg\forall x.\ A}{\exists x.\ \neg A} \qquad \frac{\neg\exists x.\ A}{\forall x.\ \neg A}$$

$$\exists x.\ (Person(x) \land \forall y.\ \neg(Person(y) \land y \neq x \land Loves(x, y)))$$



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$$\frac{\neg\forall x.\ A}{\exists x.\ \neg A} \qquad \frac{\neg\exists x.\ A}{\forall x.\ \neg A}$$

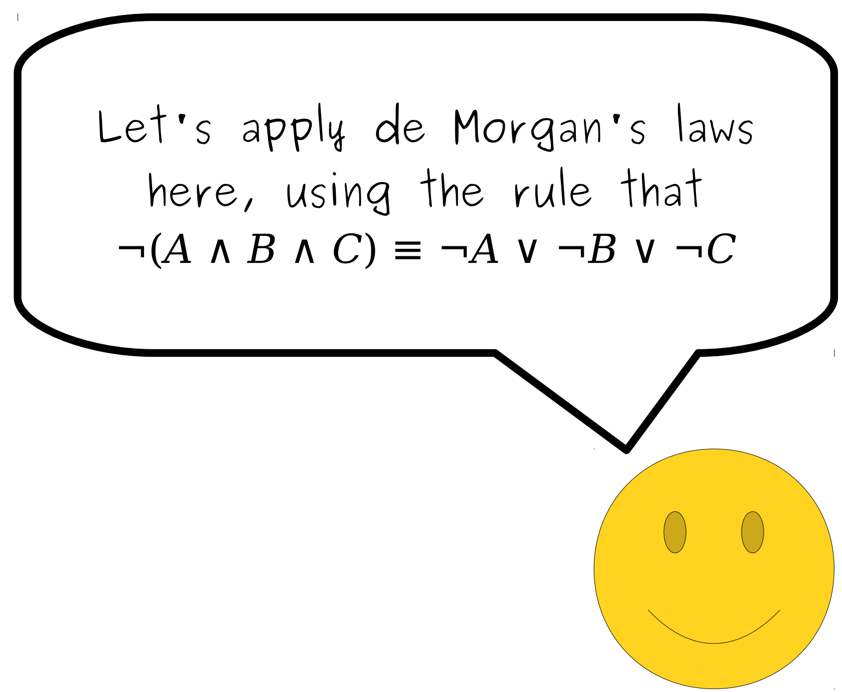
$\exists x.\ (Person(x)\ \land$

$\quad \forall y.\ \neg(Person(y) \land y \neq x \land$

$\qquad Loves(x, y)$

$\quad)$

$)$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

$$\frac{\neg(A \land B)}{A \to \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$

$$\frac{\neg \forall x.\ A}{\exists x.\ \neg A} \qquad \frac{\neg \exists x.\ A}{\forall x.\ \neg A}$$

$$
\begin{array}{l}
\exists x.\ (Person(x)\ \land \\
\quad \forall y.\ (\neg Person(y) \lor \neg y \neq x\ \lor \\
\qquad \neg Loves(x, y) \\
\quad) \\
)
\end{array}
$$

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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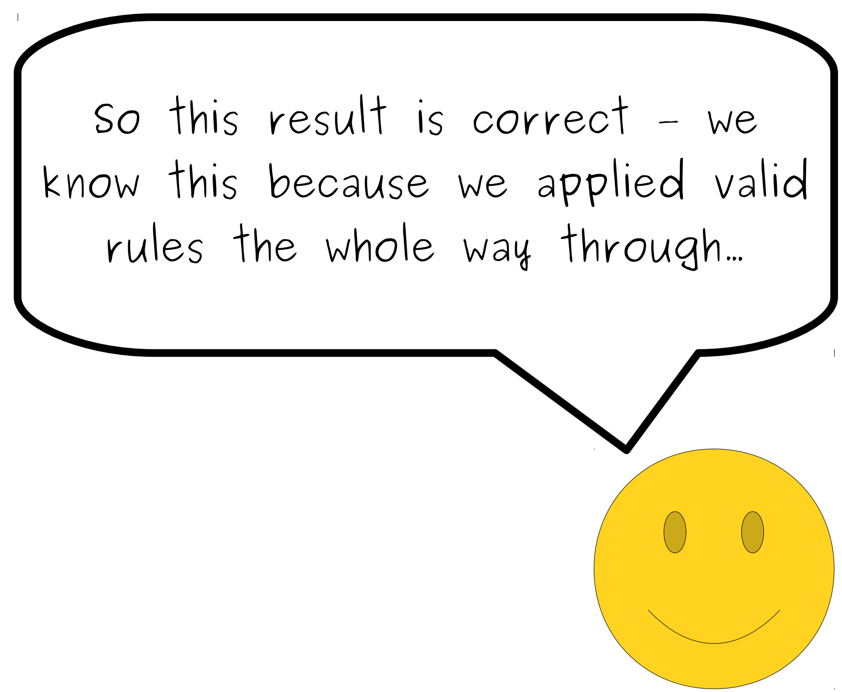
$$\begin{aligned}
&\exists x.\ (Person(x)\ \land \\
&\quad \forall y.\ (\neg Person(y) \lor \neg y \neq x \lor \\
&\qquad \neg Loves(x, y) \\
&\quad) \\
&)
\end{aligned}$$



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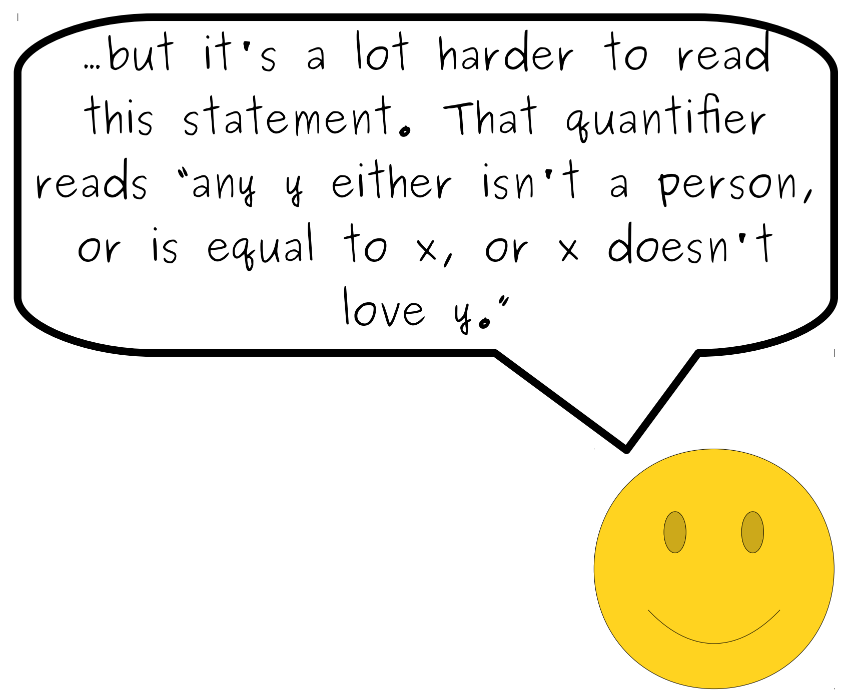
&)
\end{aligned}$$



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\quad) \\
)
\end{array}
$$

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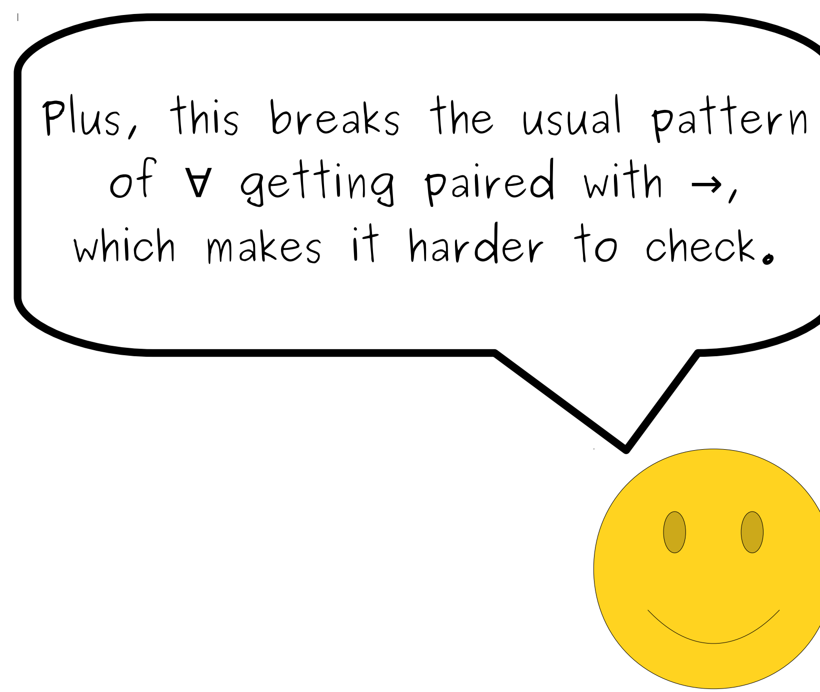
&\quad) \\
&)
\end{aligned}$$



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$$\begin{aligned}
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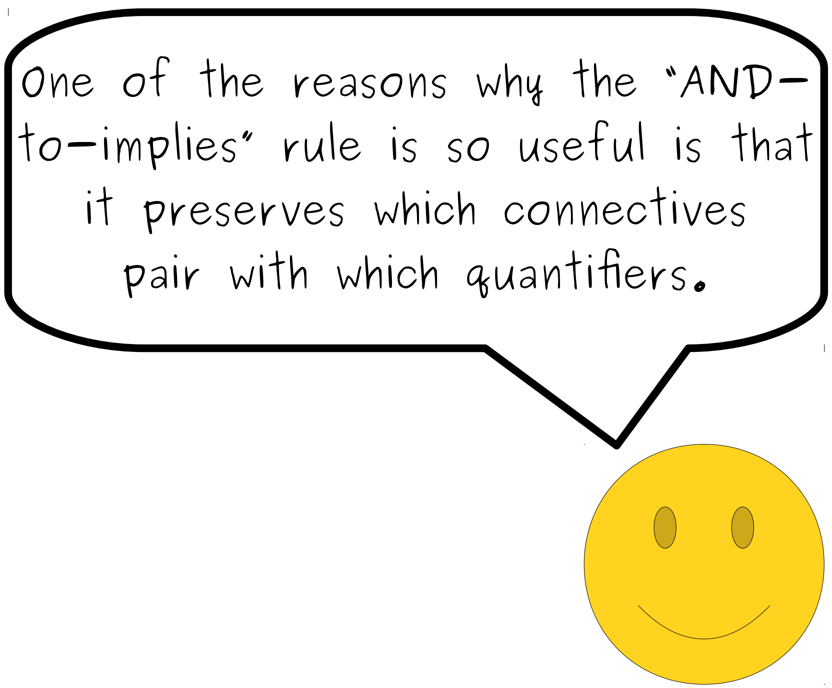

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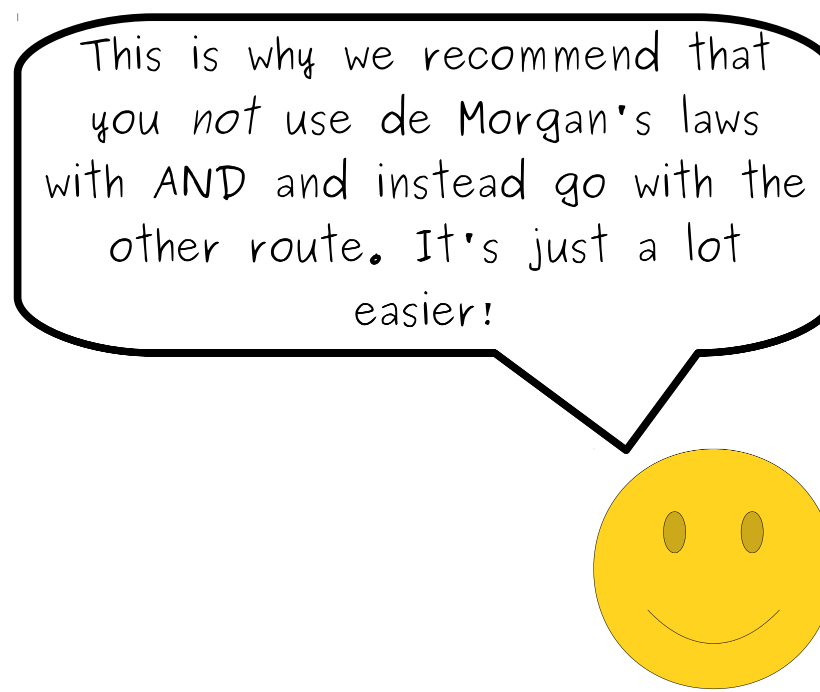
$$\begin{array}{l} \exists x.\ (Person(x) \land \\ \quad \forall y.\ (\neg Person(y) \lor \neg y \neq x \lor \\ \qquad \neg Loves(x, y) \\ \quad) \\ ) \end{array}$$



$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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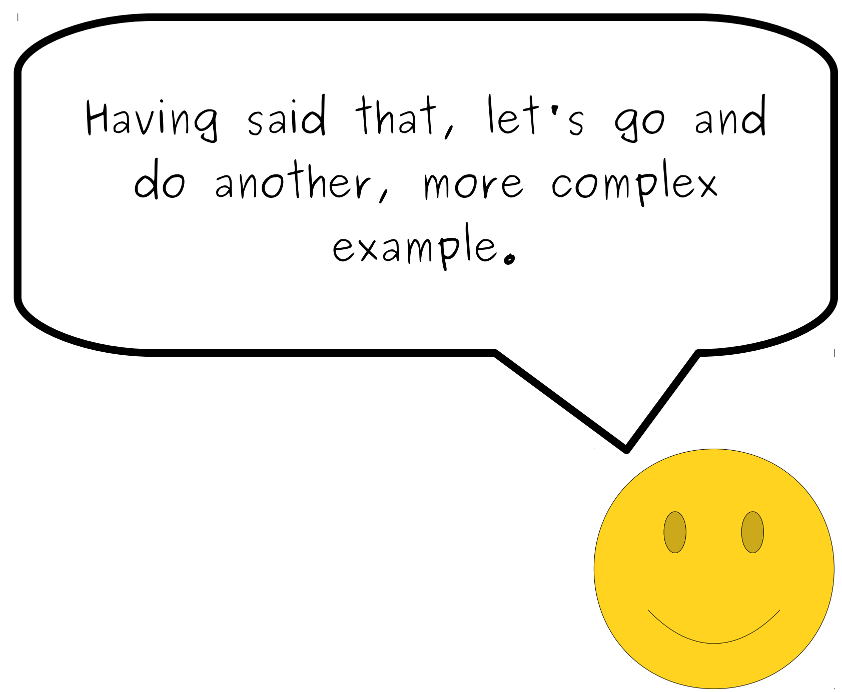

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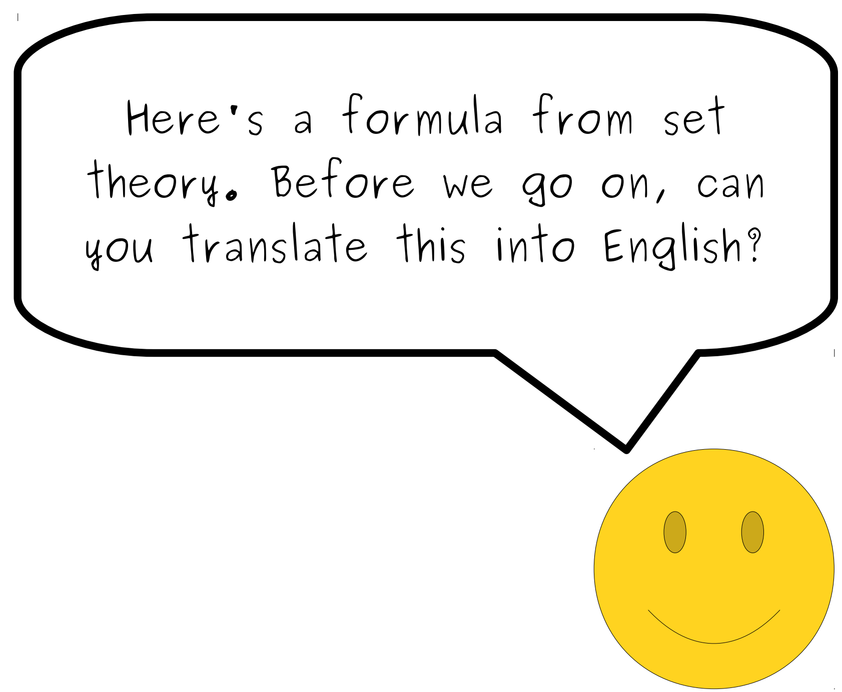
$$\frac{\neg\forall x.\ A}{\exists x.\ \neg A} \qquad \frac{\neg\exists x.\ A}{\forall x.\ \neg A}$$

$$\forall S.\ \forall T.\ (Set(S) \land Set(T) \to (S = T \leftrightarrow \forall x.\ (x \in S \leftrightarrow x \in T)))$$

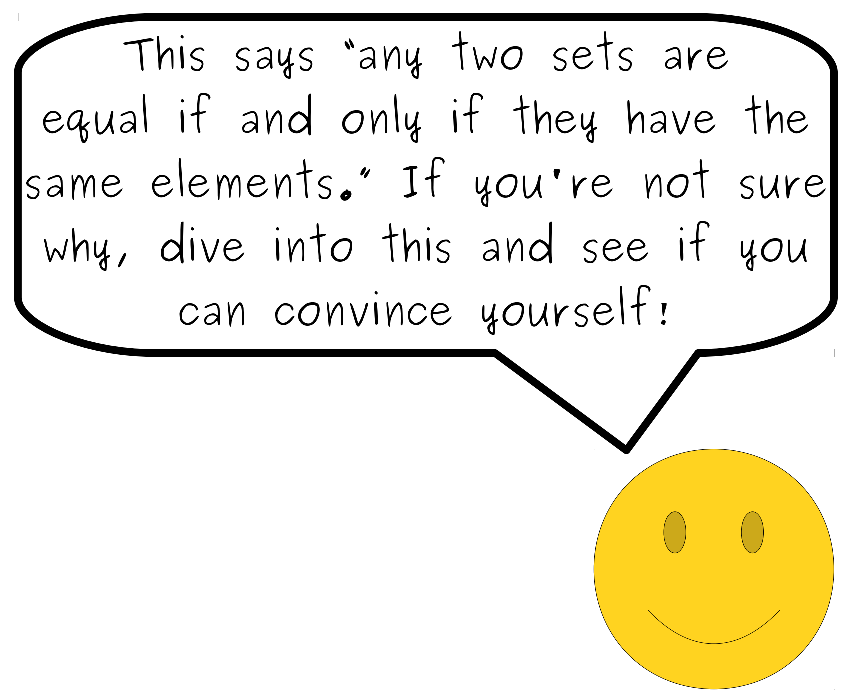


$$\frac{\neg\neg \mathbf{A}}{\mathbf{A}} \qquad \frac{\neg(\mathbf{A} \lor \mathbf{B})}{\neg \mathbf{A} \land \neg \mathbf{B}} \qquad \frac{\neg(\mathbf{A} \land \mathbf{B})}{\neg \mathbf{A} \lor \neg \mathbf{B}} \qquad \frac{\neg(\mathbf{A} \to \mathbf{B})}{\mathbf{A} \land \neg \mathbf{B}} \qquad \frac{\neg(\mathbf{A} \leftrightarrow \mathbf{B})}{\neg \mathbf{A} \leftrightarrow \mathbf{B}}$$

$$\frac{\neg(\mathbf{A} \land \mathbf{B})}{\mathbf{A} \to \neg \mathbf{B}} \qquad \frac{\neg(\mathbf{A} \leftrightarrow \mathbf{B})}{\mathbf{A} \leftrightarrow \neg \mathbf{B}}$$

$$\frac{\neg \forall x.\ \mathbf{A}}{\exists x.\ \neg \mathbf{A}} \qquad \frac{\neg \exists x.\ \mathbf{A}}{\forall x.\ \neg \mathbf{A}}$$

$$\begin{aligned} &\forall S.\ \forall T.\ (Set(S) \land Set(T) \to \\ &\quad (S = T \leftrightarrow \forall x.\ (x \in S \leftrightarrow x \in T)) \\ &) \end{aligned}$$



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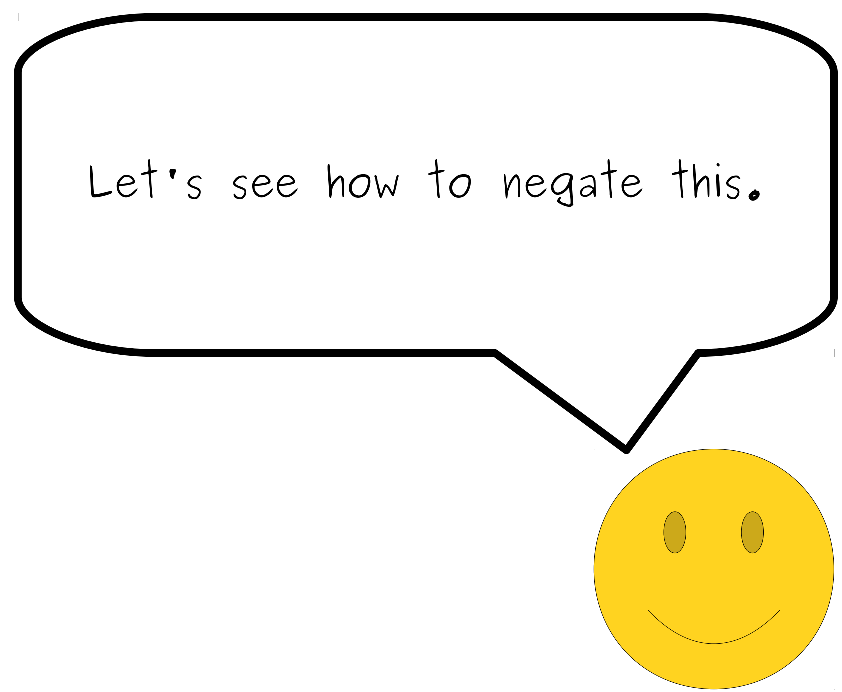

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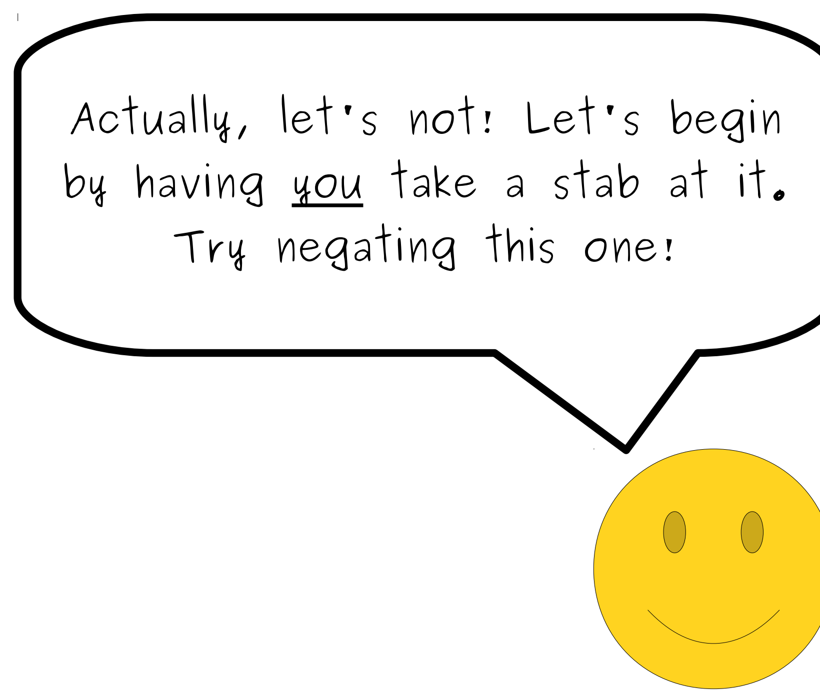

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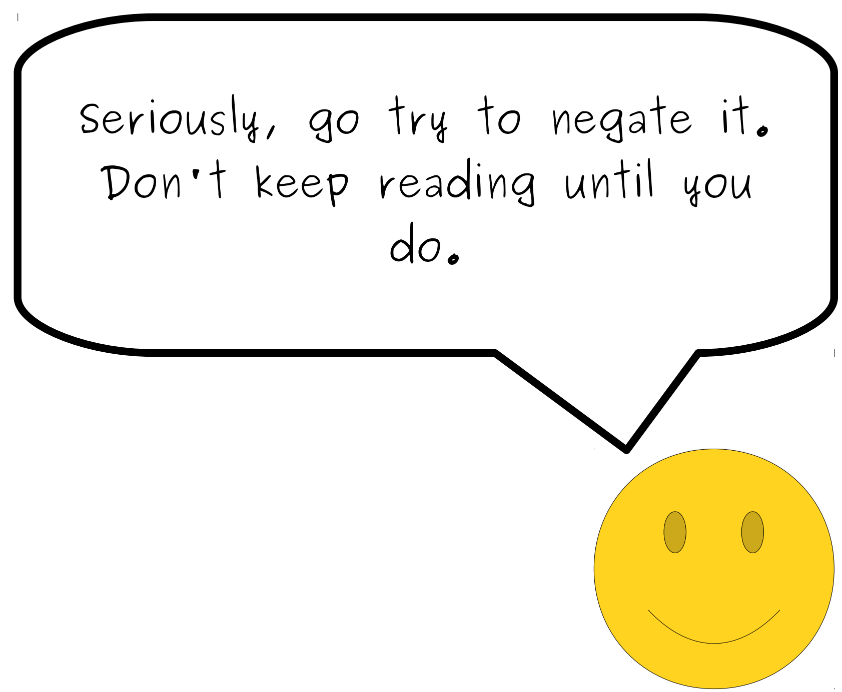

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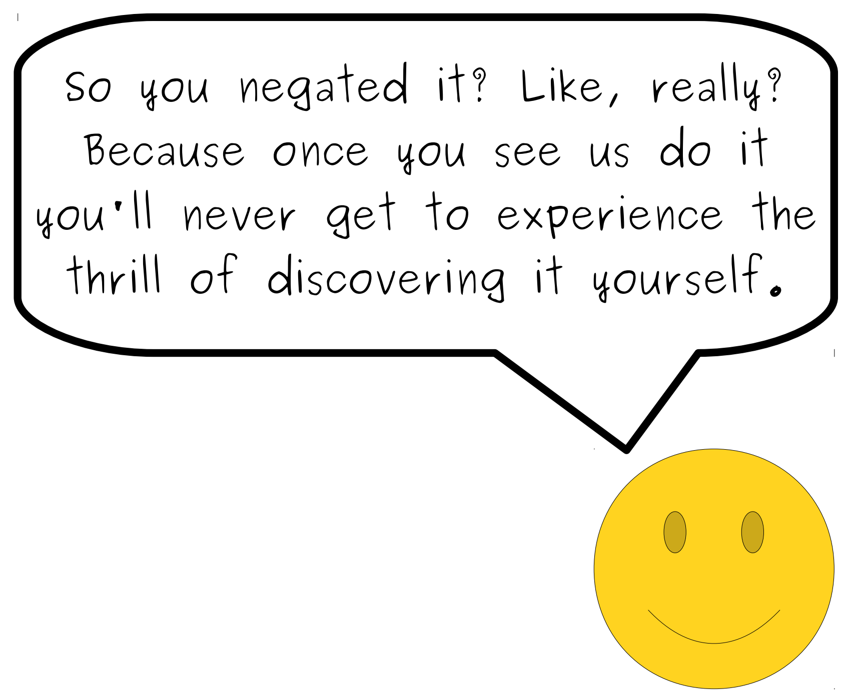

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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$$\forall S.\ \forall T.\ (Set(S) \land Set(T) \to$$
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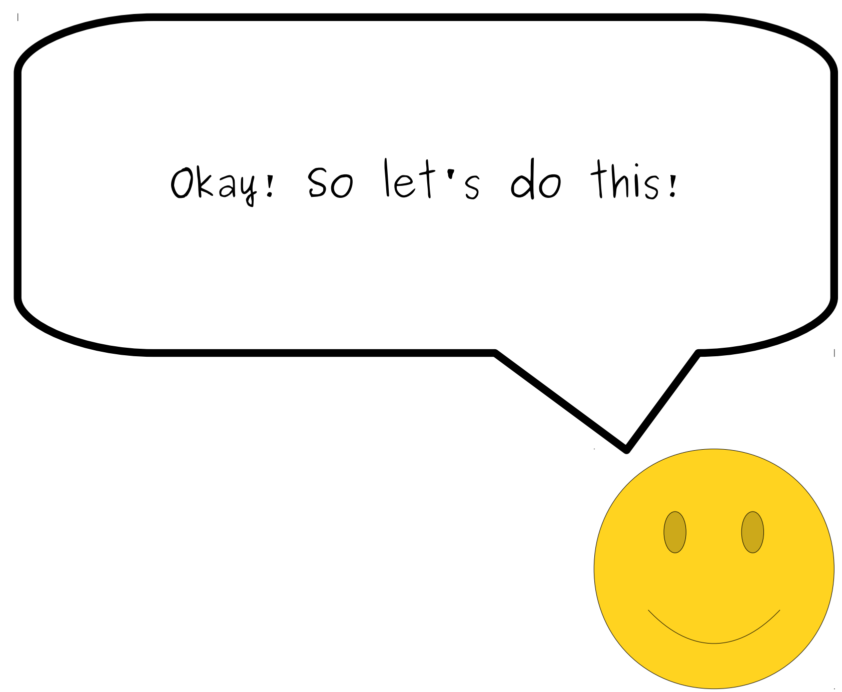
$$)$$

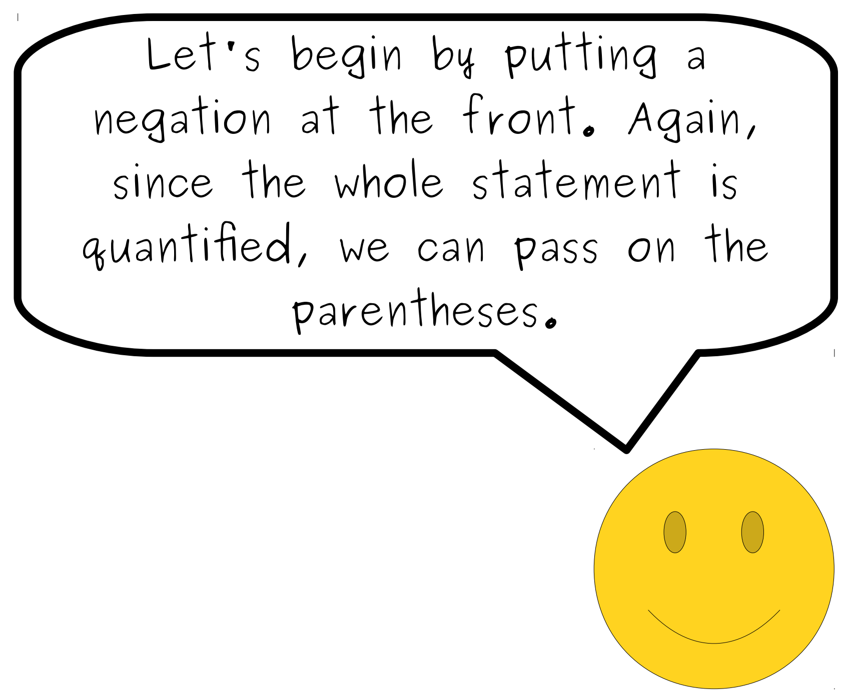


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

$$\frac{\neg(A \land B)}{A \to \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$

$$\frac{\neg\forall x.\ A}{\exists x.\ \neg A} \qquad \frac{\neg\exists x.\ A}{\forall x.\ \neg A}$$

$$\neg\forall S.\ \forall T.\ (Set(S) \land Set(T) \to$$
$$(S = T \leftrightarrow \forall x.\ (x \in S \leftrightarrow x \in T))$$
$$)$$



$$\frac{\neg\neg \mathbf{A}}{\mathbf{A}} \qquad \frac{\neg(\mathbf{A} \lor \mathbf{B})}{\neg\mathbf{A} \land \neg\mathbf{B}} \qquad \frac{\neg(\mathbf{A} \land \mathbf{B})}{\neg\mathbf{A} \lor \neg\mathbf{B}} \qquad \frac{\neg(\mathbf{A} \to \mathbf{B})}{\mathbf{A} \land \neg\mathbf{B}} \qquad \frac{\neg(\mathbf{A} \leftrightarrow \mathbf{B})}{\neg\mathbf{A} \leftrightarrow \mathbf{B}}$$

$$\frac{\neg(\mathbf{A} \land \mathbf{B})}{\mathbf{A} \to \neg\mathbf{B}} \qquad \frac{\neg(\mathbf{A} \leftrightarrow \mathbf{B})}{\mathbf{A} \leftrightarrow \neg\mathbf{B}}$$

$$\frac{\neg\forall x.\ \mathbf{A}}{\exists x.\ \neg\mathbf{A}} \qquad \frac{\neg\exists x.\ \mathbf{A}}{\forall x.\ \neg\mathbf{A}}$$

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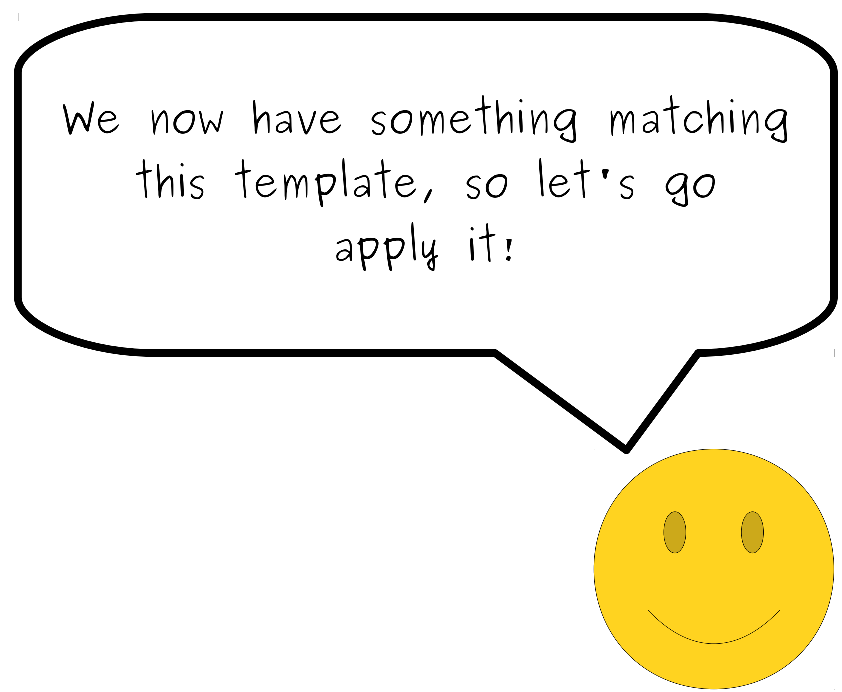

$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

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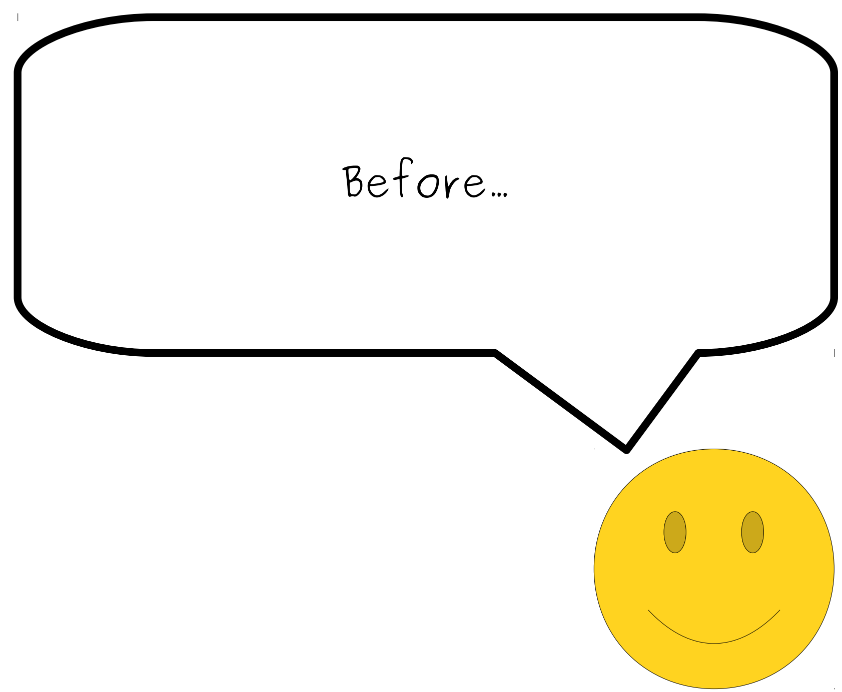

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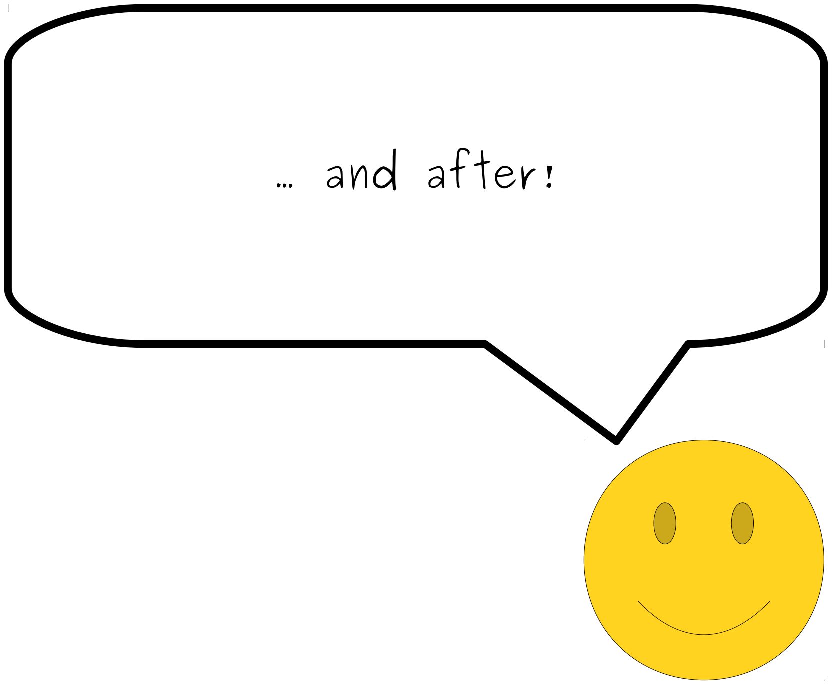

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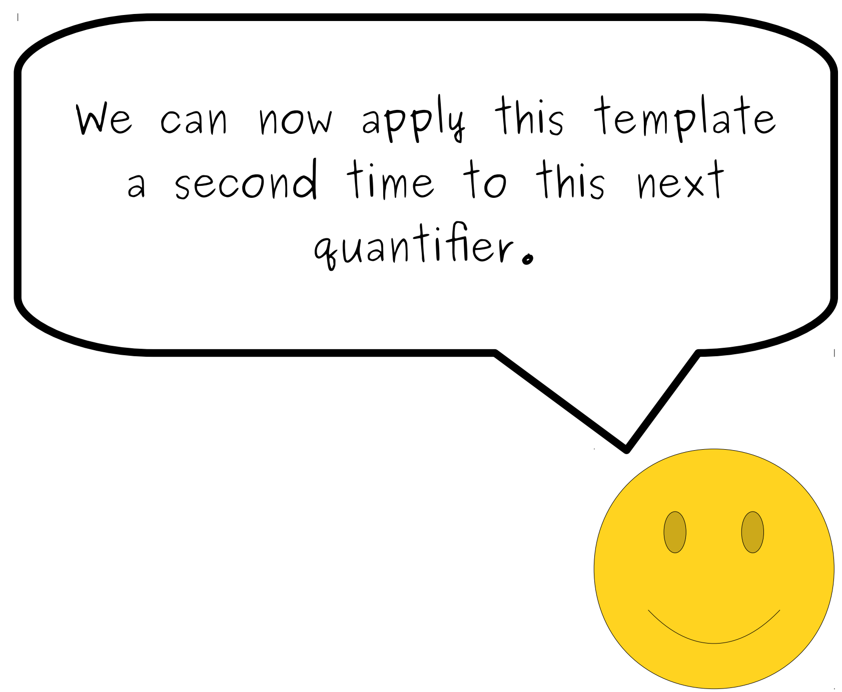

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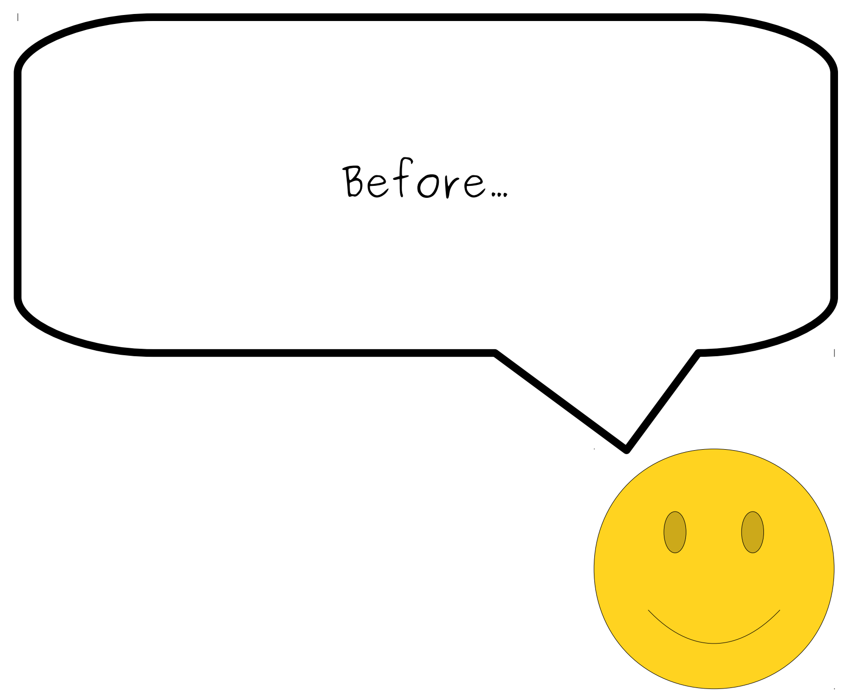

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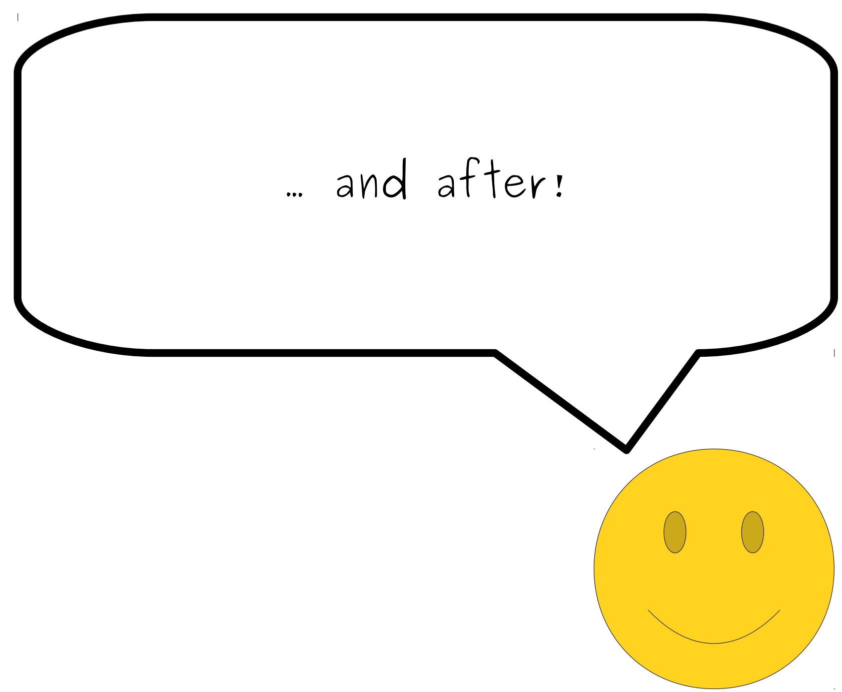

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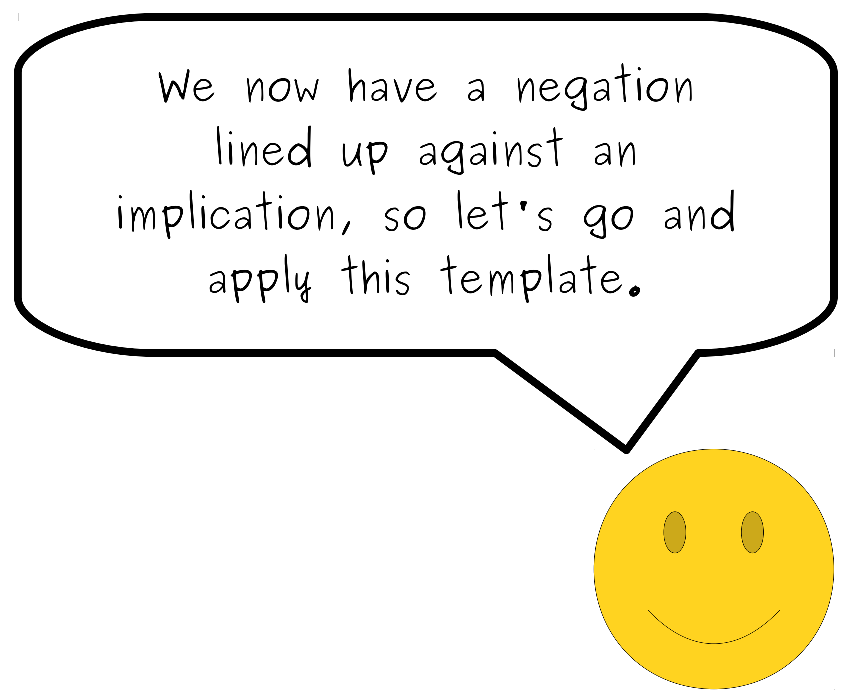

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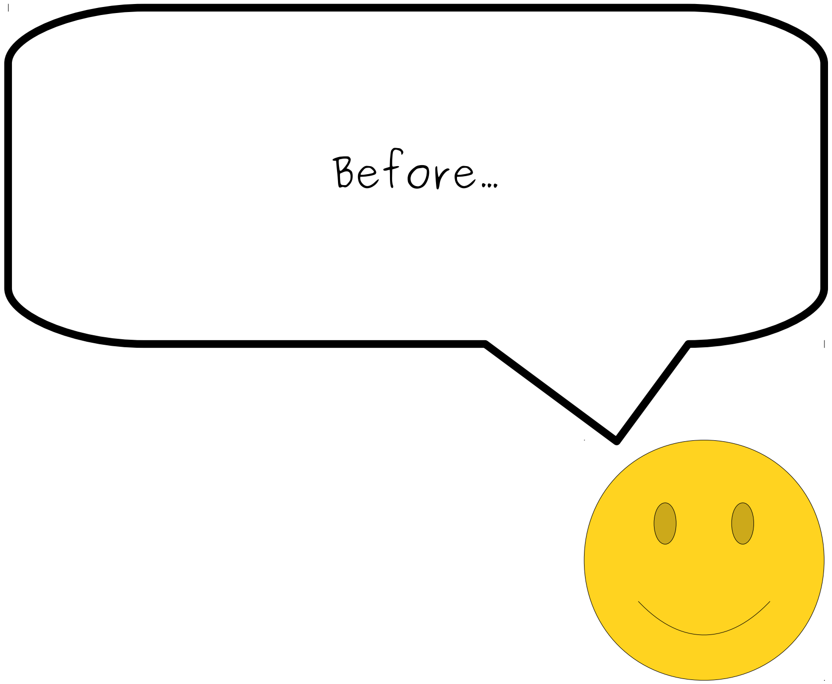

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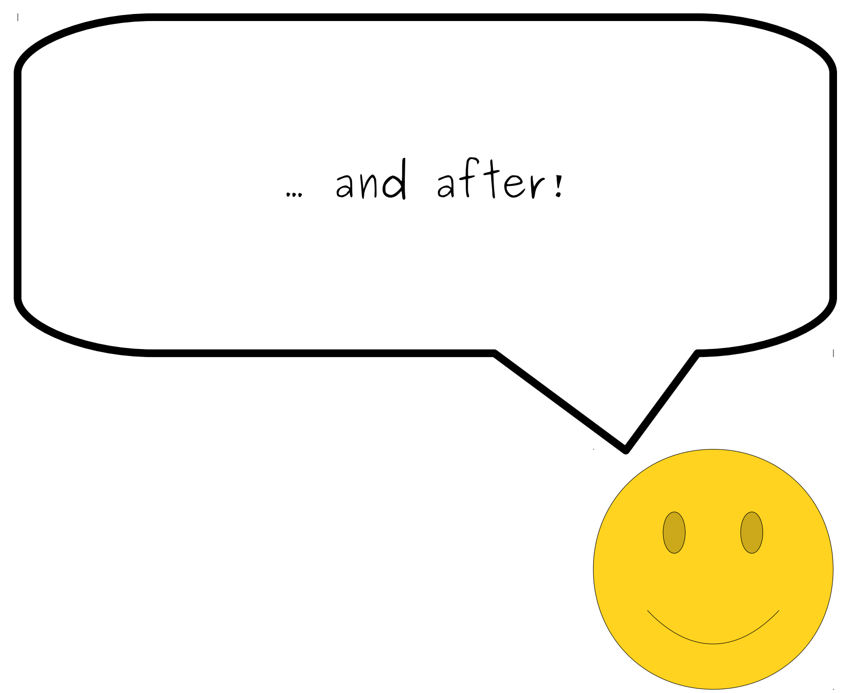
$)$



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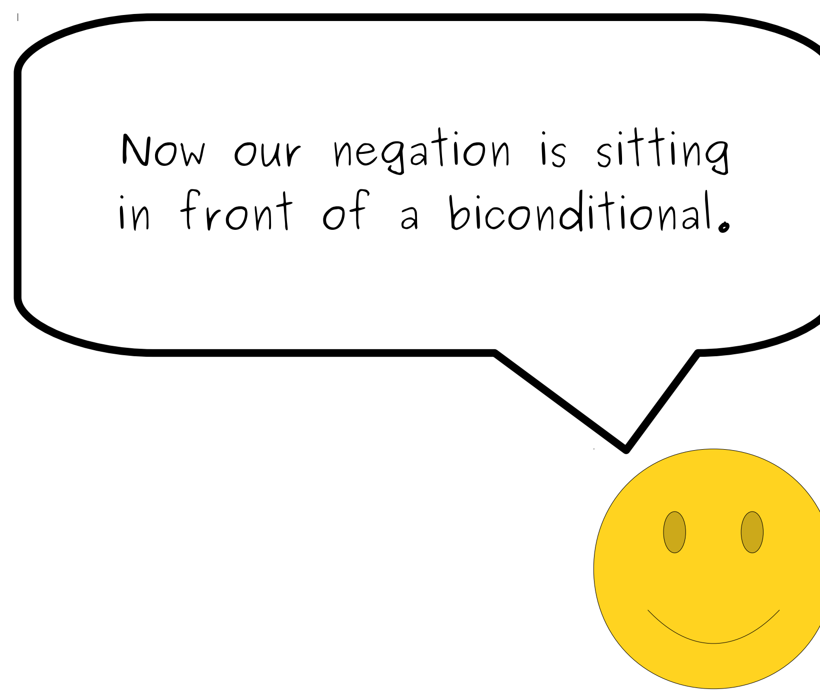

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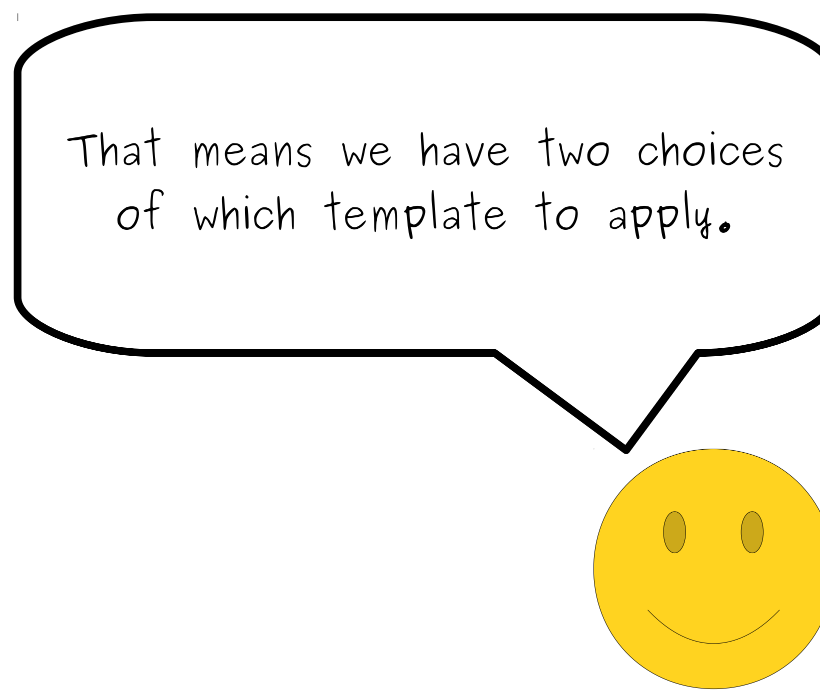

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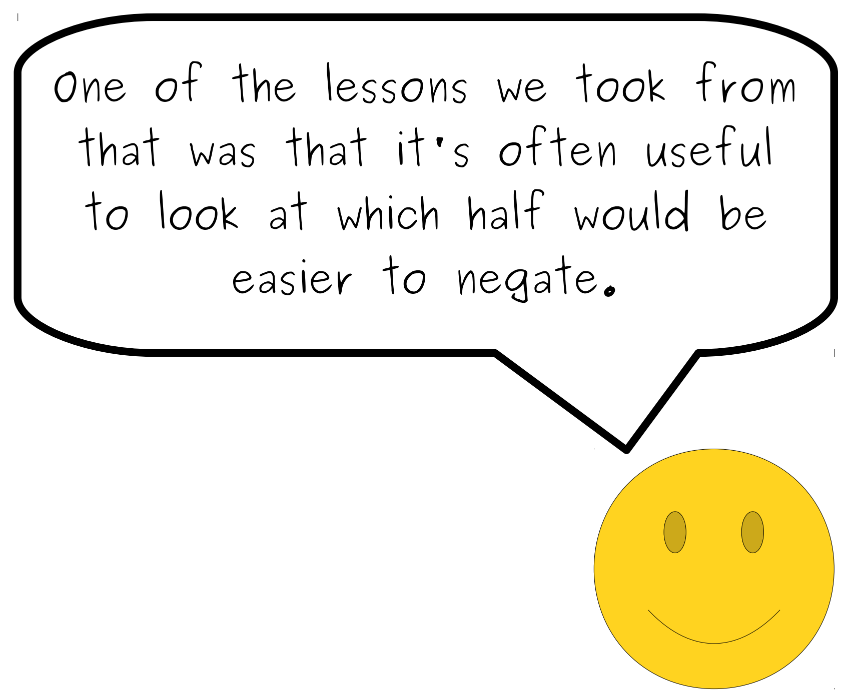

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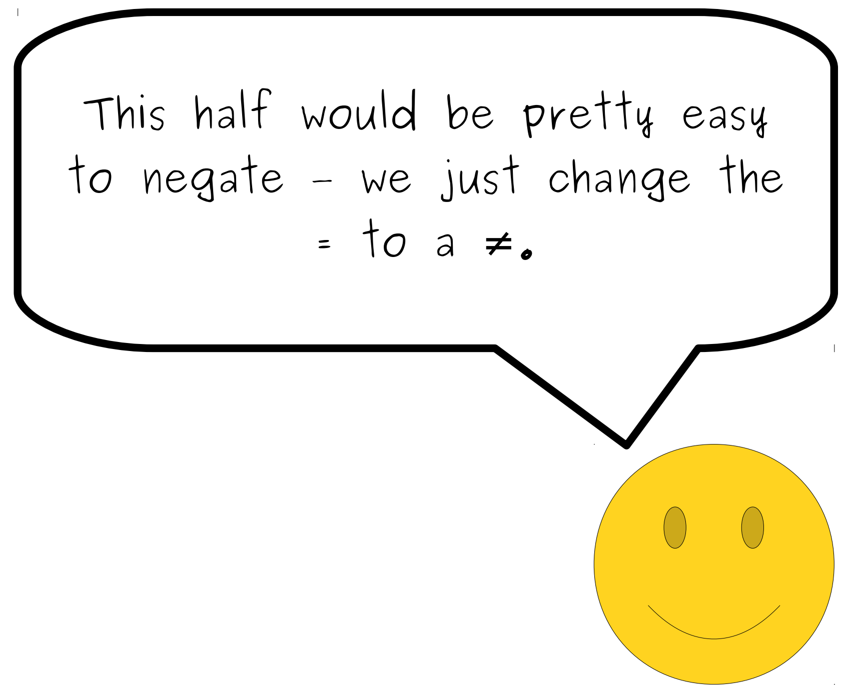

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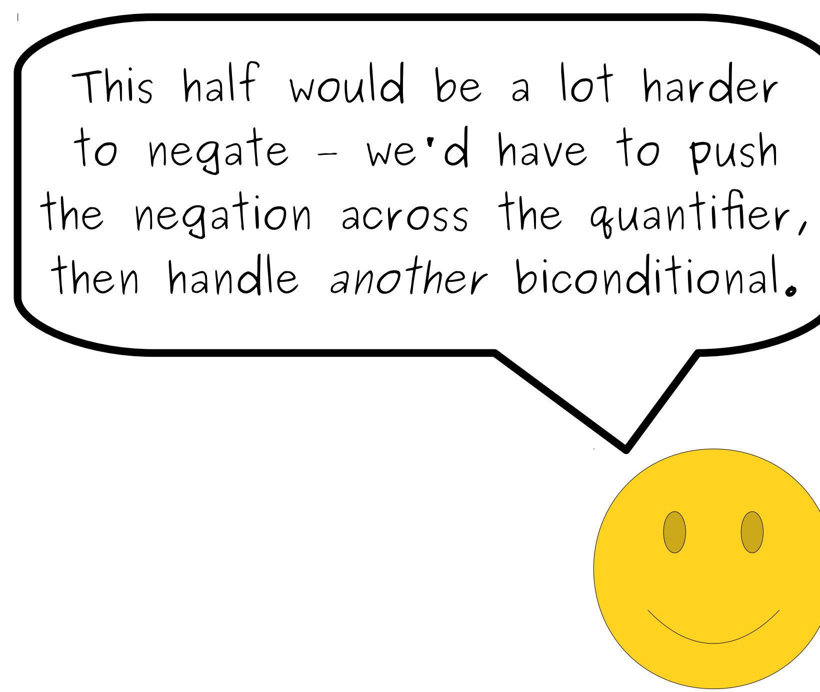

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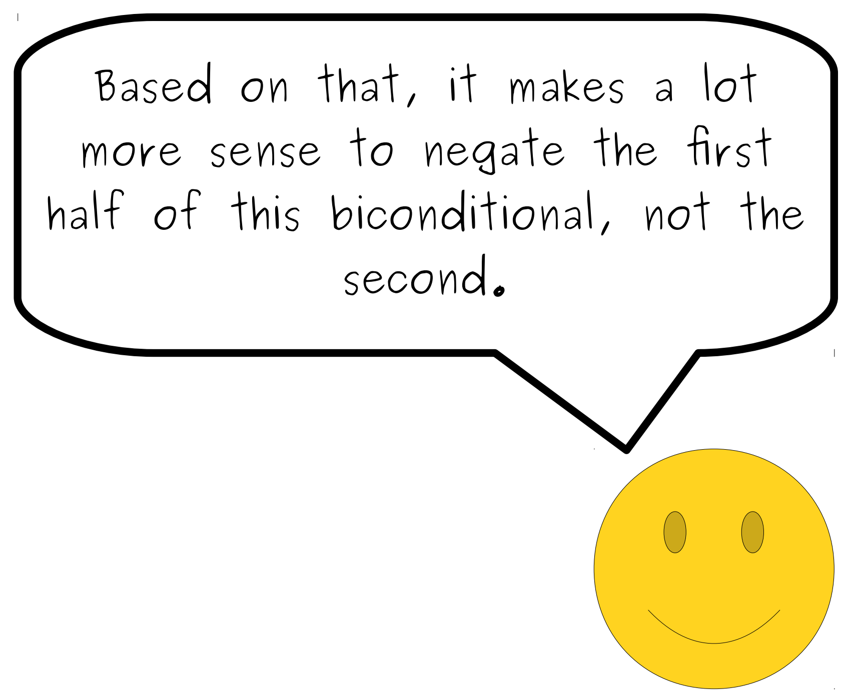

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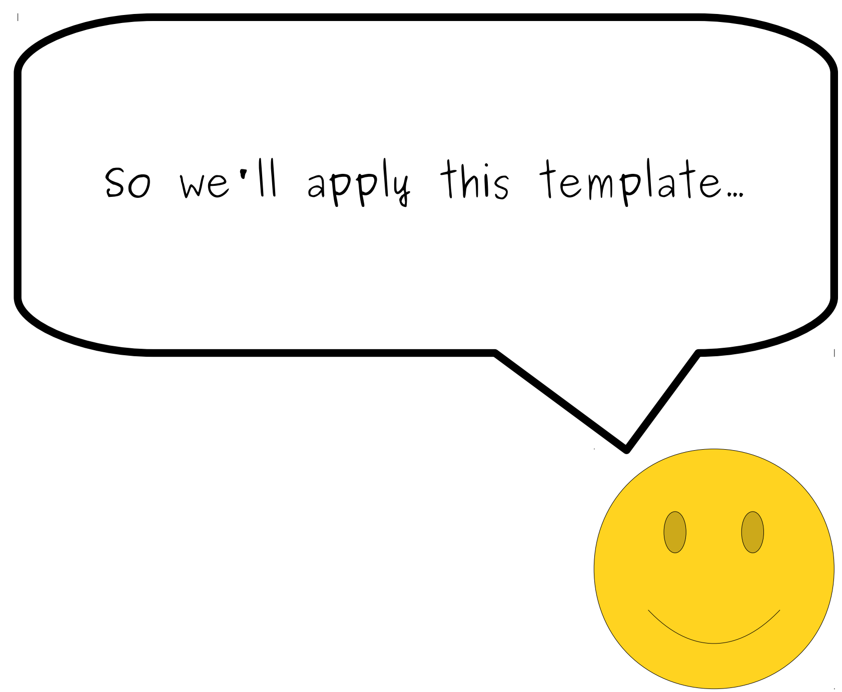
$$\begin{array}{l} \exists S.\ \exists T.\ (Set(S) \land Set(T) \land \\ \quad \neg(S = T \leftrightarrow \forall x.\ (x \in S \leftrightarrow x \in T)) \\ ) \end{array}$$



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$$\frac{\neg \forall x.\ A}{\exists x.\ \neg A} \qquad \frac{\neg \exists x.\ A}{\forall x.\ \neg A}$$

$$\exists S.\ \exists T.\ (Set(S) \land Set(T) \land$$
$$(S \neq T \leftrightarrow \forall x.\ (x \in S \leftrightarrow x \in T))$$
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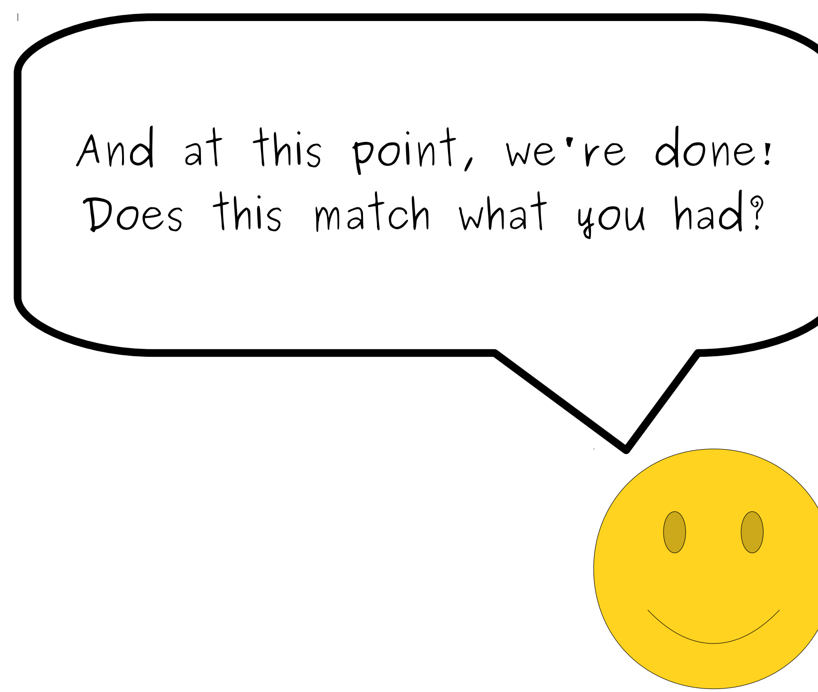

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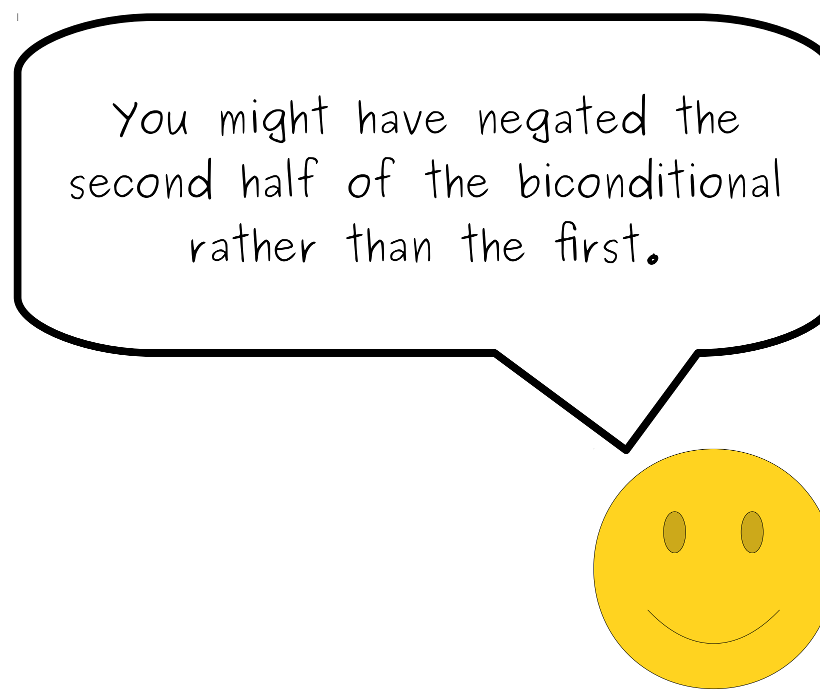

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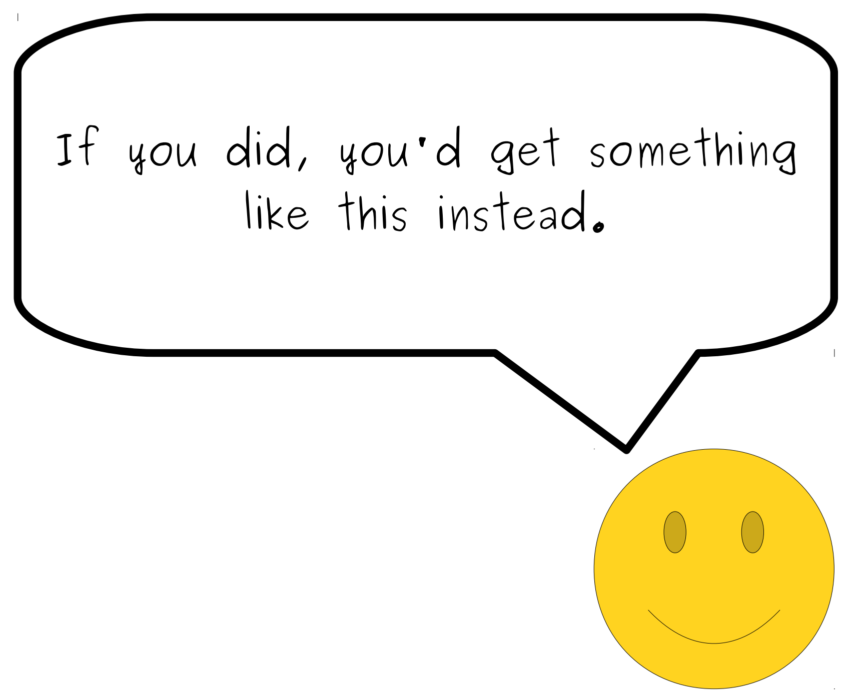

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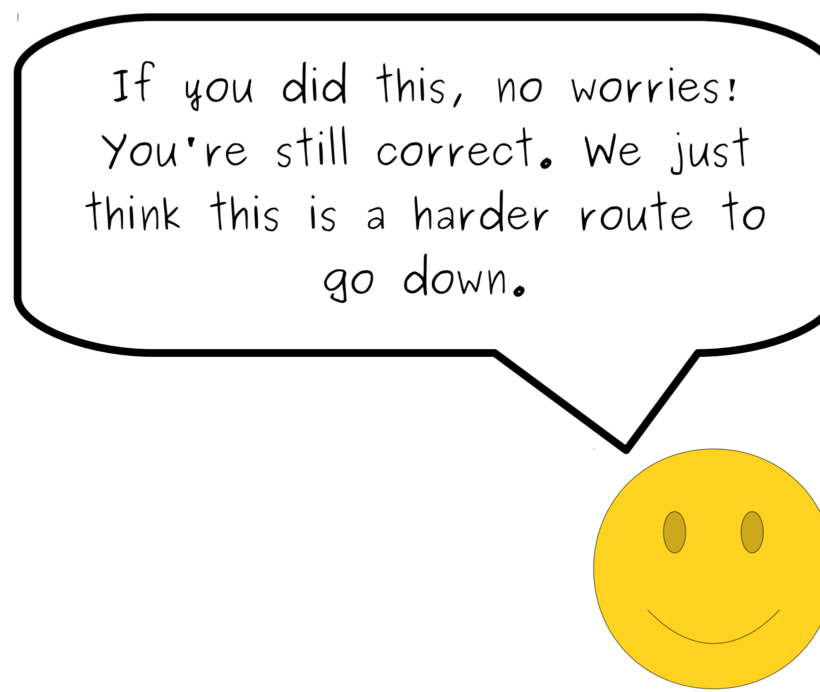

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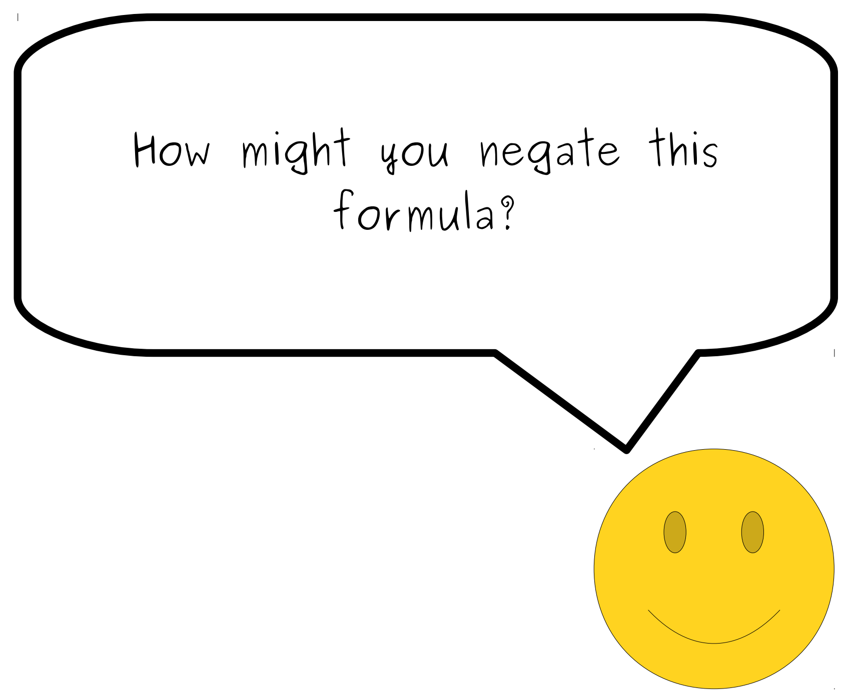
$$(\forall x.\ Happy(x)) \to (\exists y.\ Happy(y))$$



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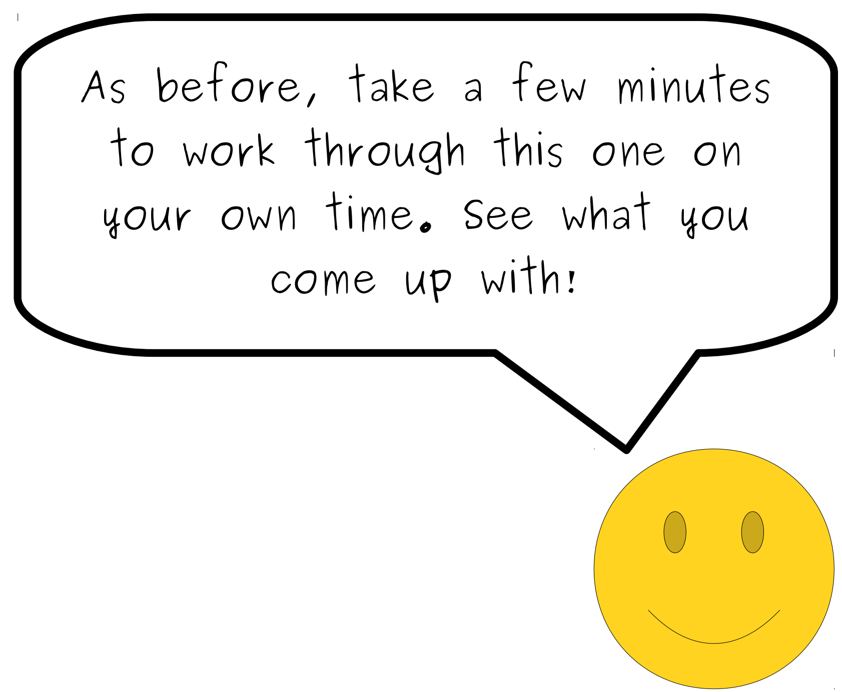
$$(\forall x.\ Happy(x)) \to (\exists y.\ Happy(y))$$



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$$\frac{\neg \forall x.\ A}{\exists x.\ \neg A} \qquad \frac{\neg \exists x.\ A}{\forall x.\ \neg A}$$

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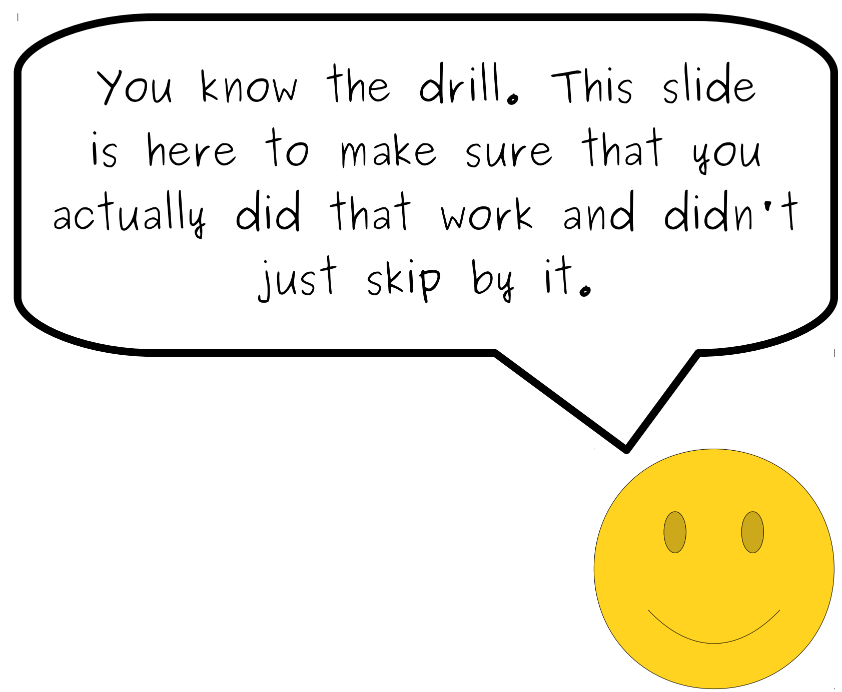

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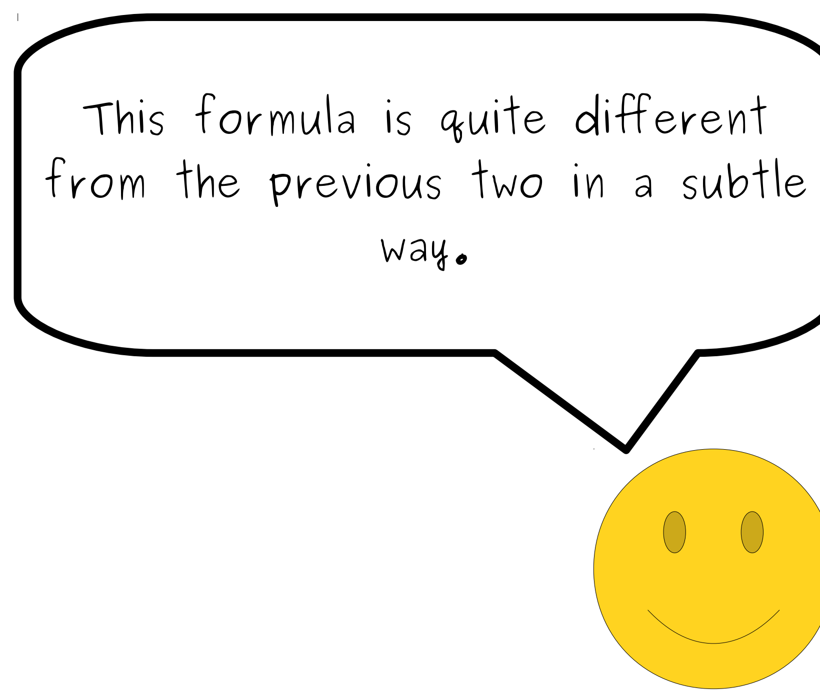

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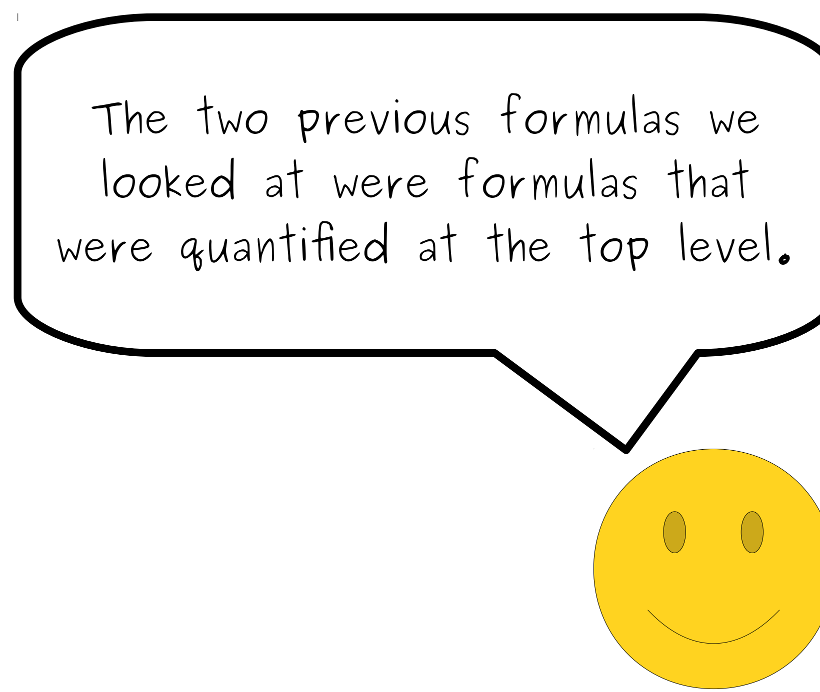

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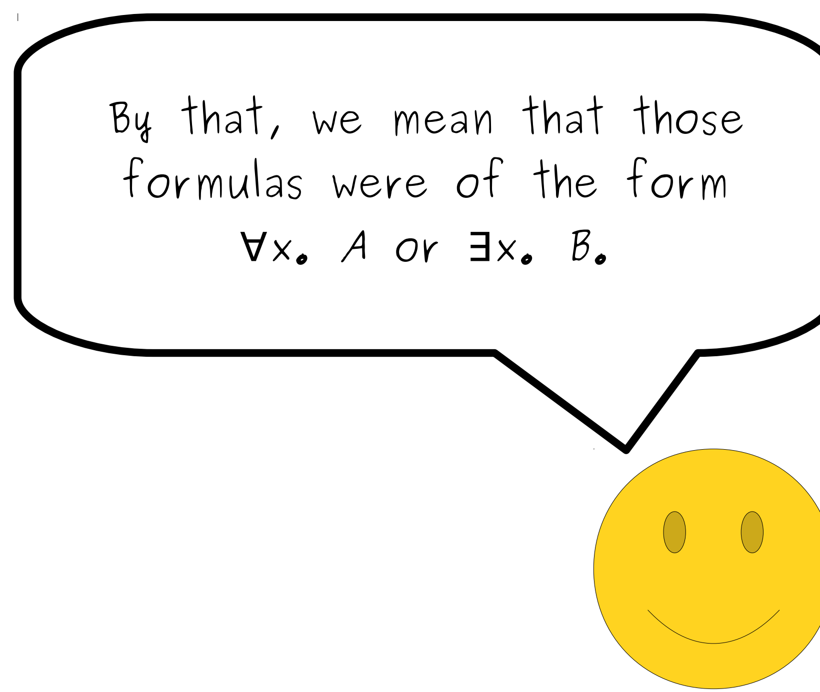

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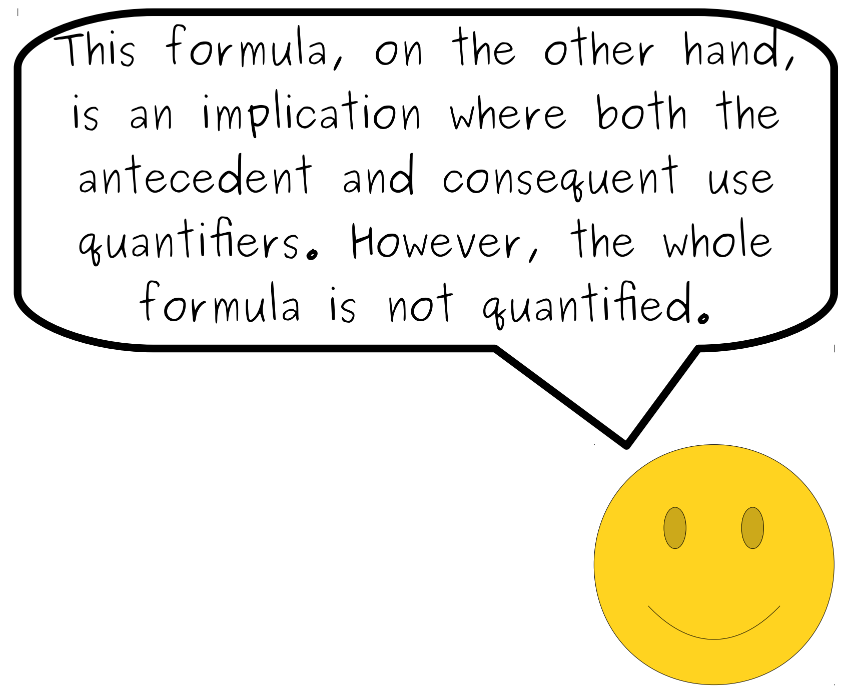

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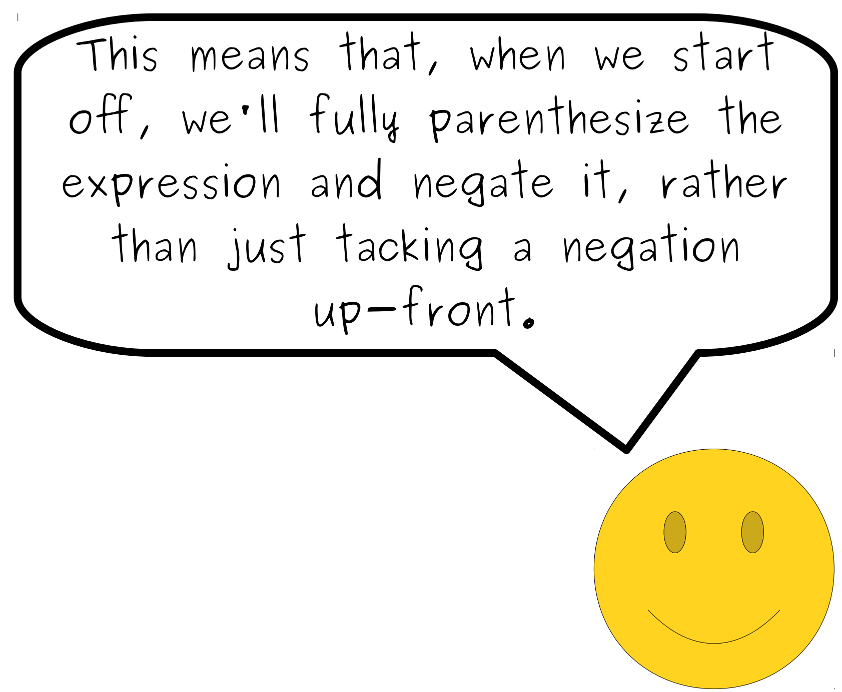
$$(\forall x.\ Happy(x)) \to (\exists y.\ Happy(y))$$



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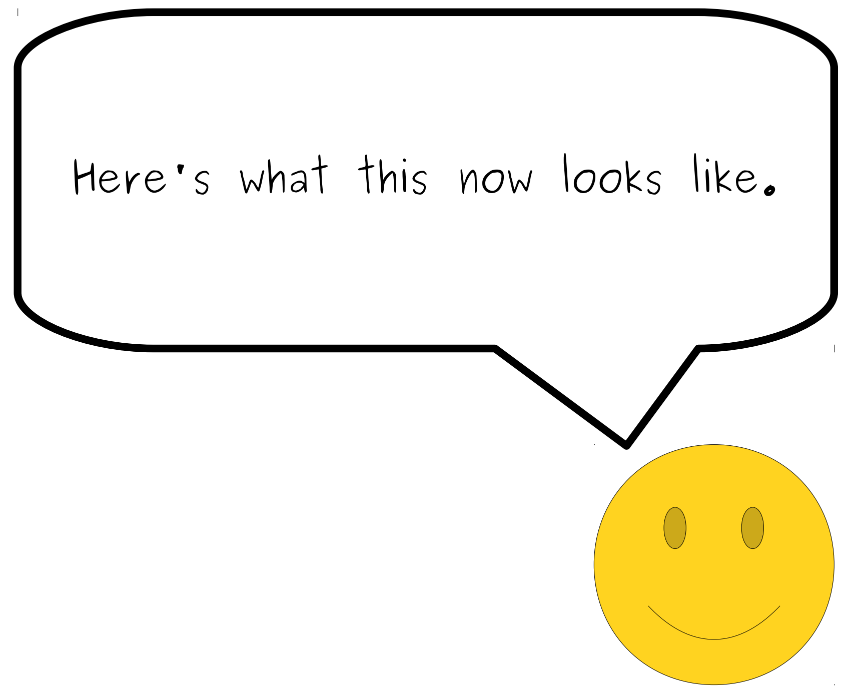

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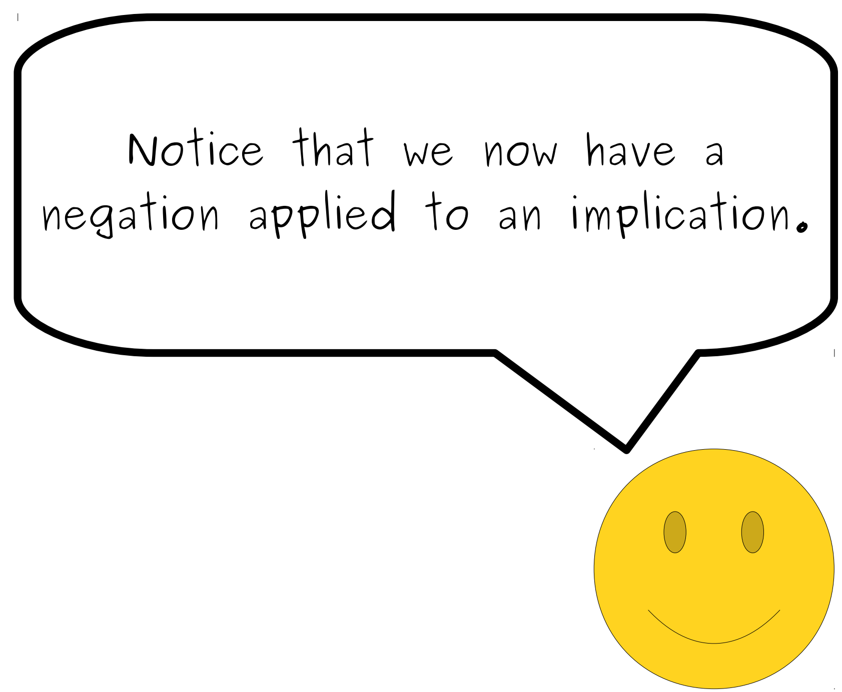

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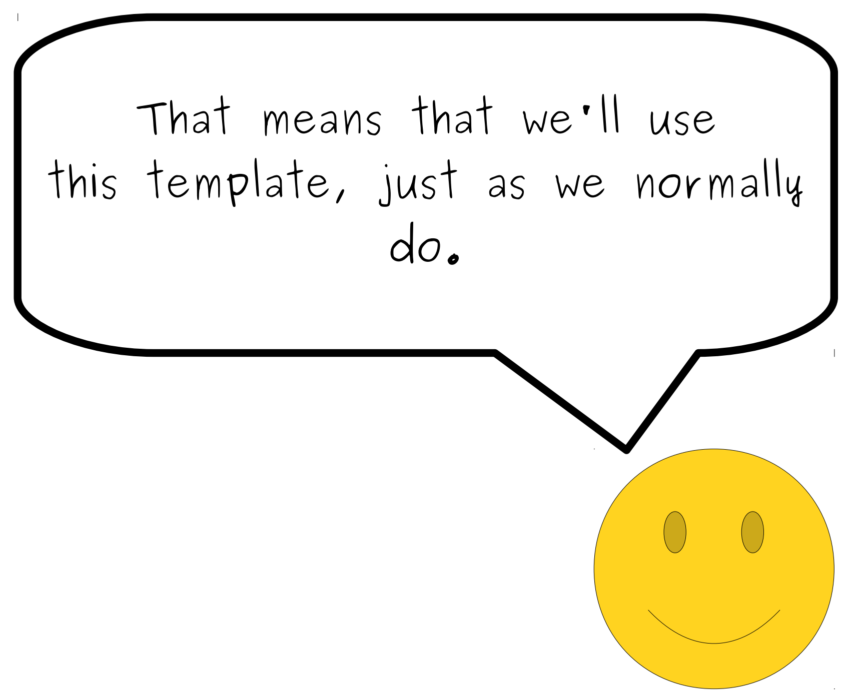

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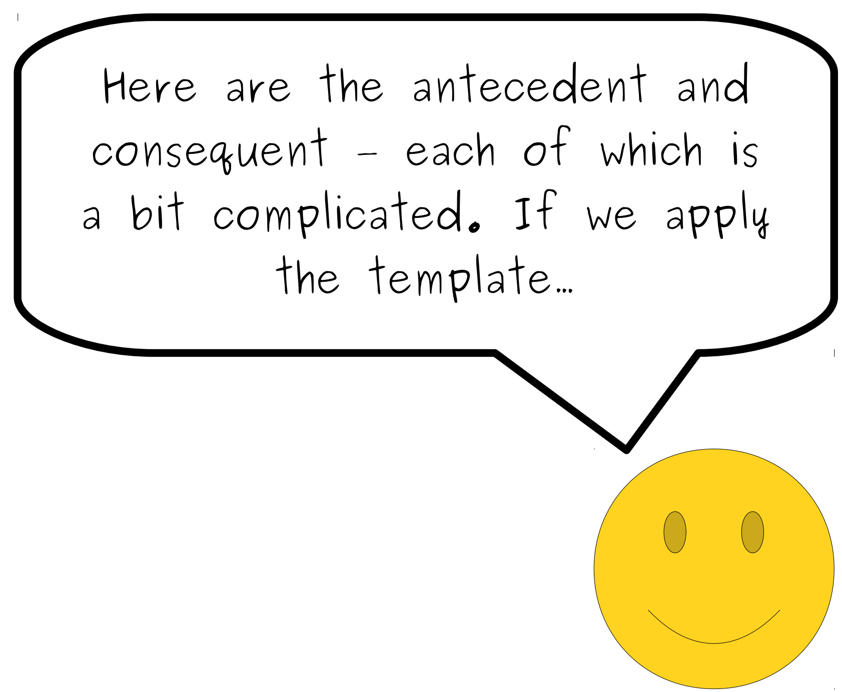

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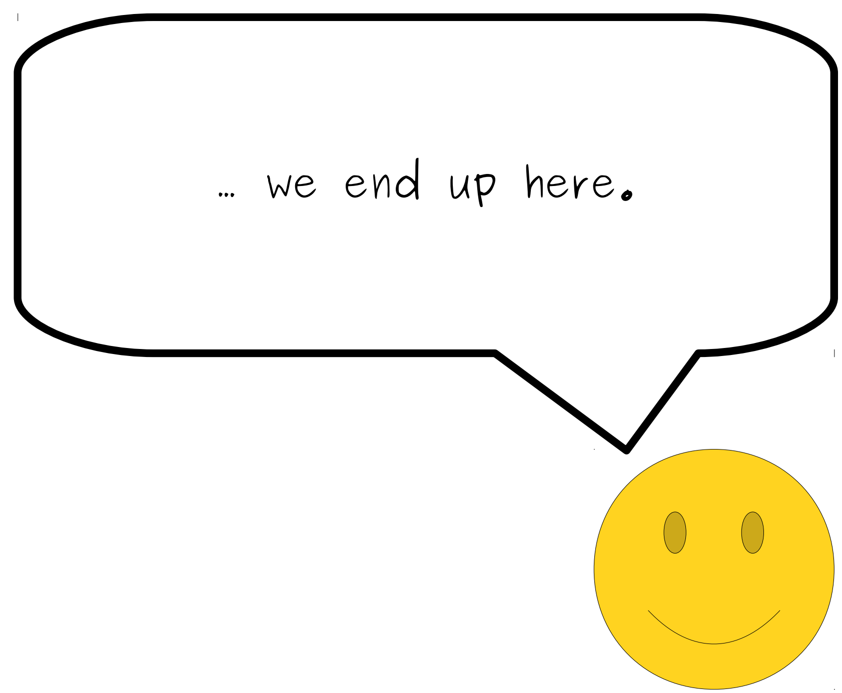
$$(\forall x.\ Happy(x)) \land \neg(\exists y.\ Happy(y))$$



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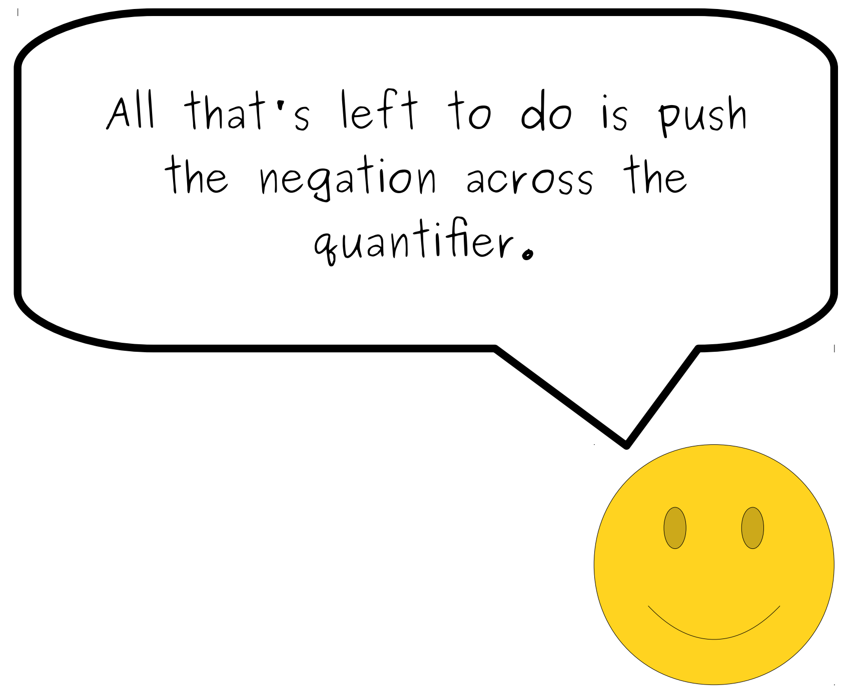
$$(\forall x.\ Happy(x)) \land \neg(\exists y.\ Happy(y))$$



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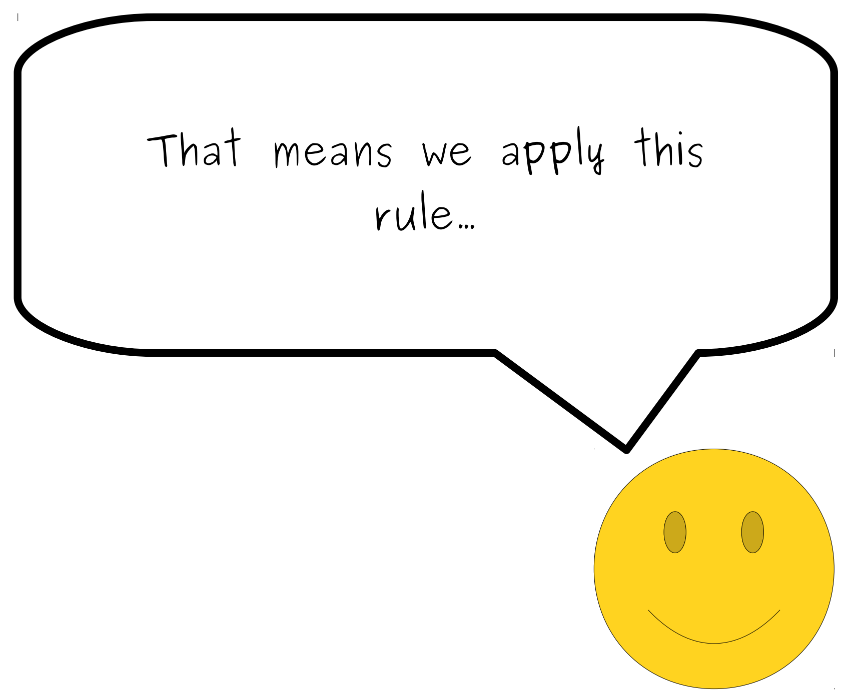

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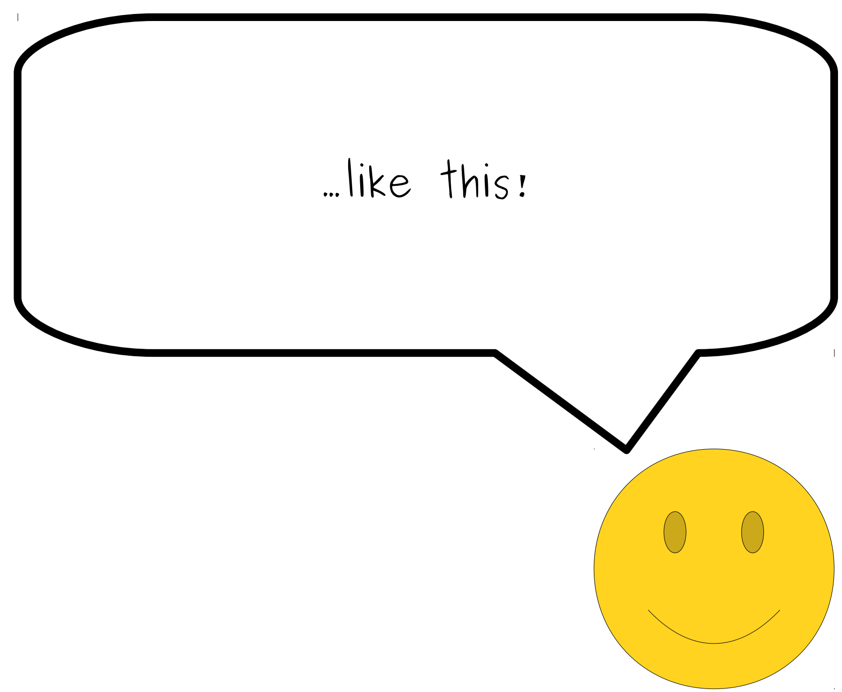
$$\frac{\neg(A \land B)}{A \to \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$

$$\frac{\neg\forall x.\ A}{\exists x.\ \neg A} \qquad \boxed{\frac{\neg\exists x.\ A}{\forall x.\ \neg A}}$$

$$(\forall x.\ Happy(x)) \land (\forall y.\ \neg Happy(y))$$



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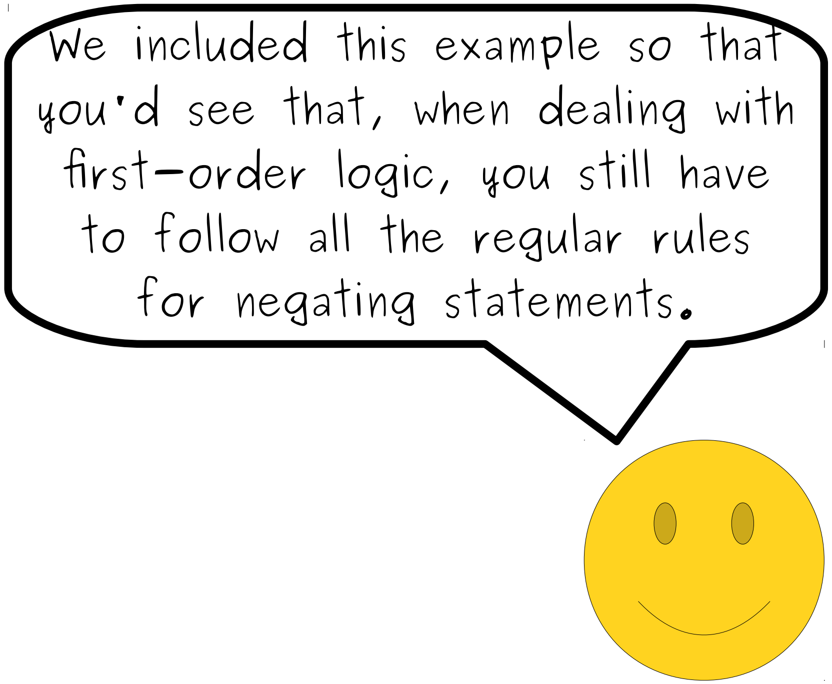

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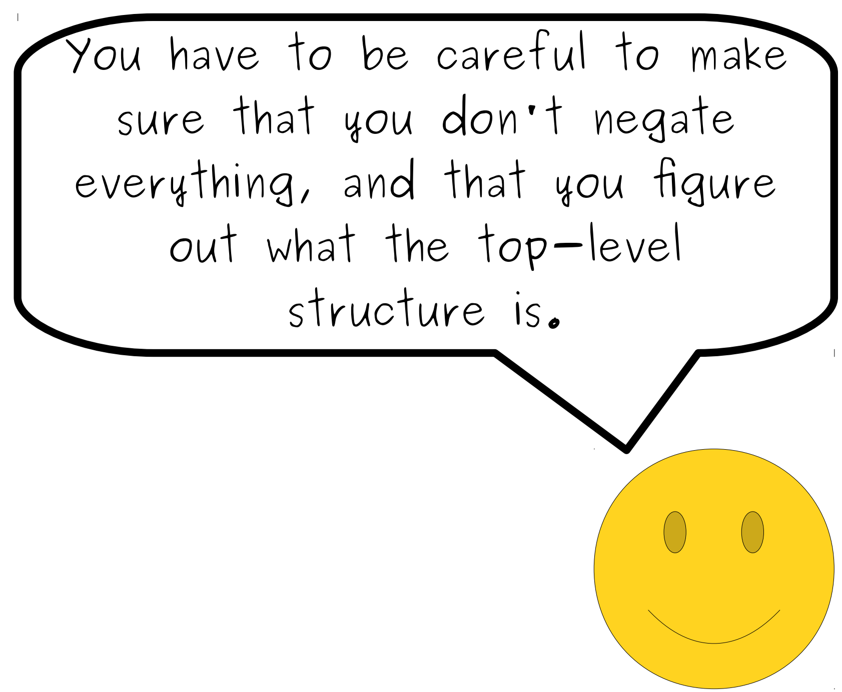
$$(\forall x.\ Happy(x)) \land (\forall y.\ \neg Happy(y))$$



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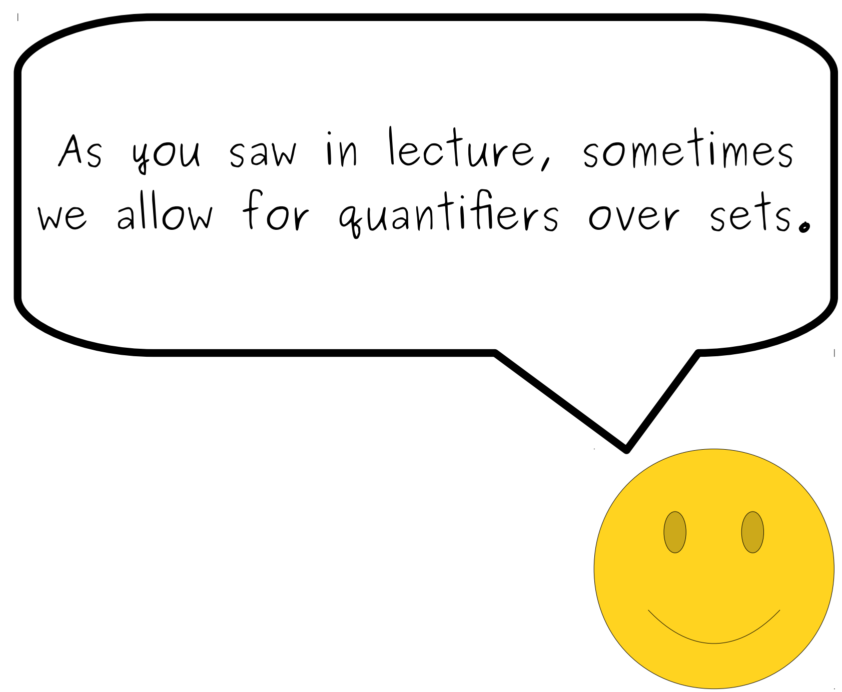

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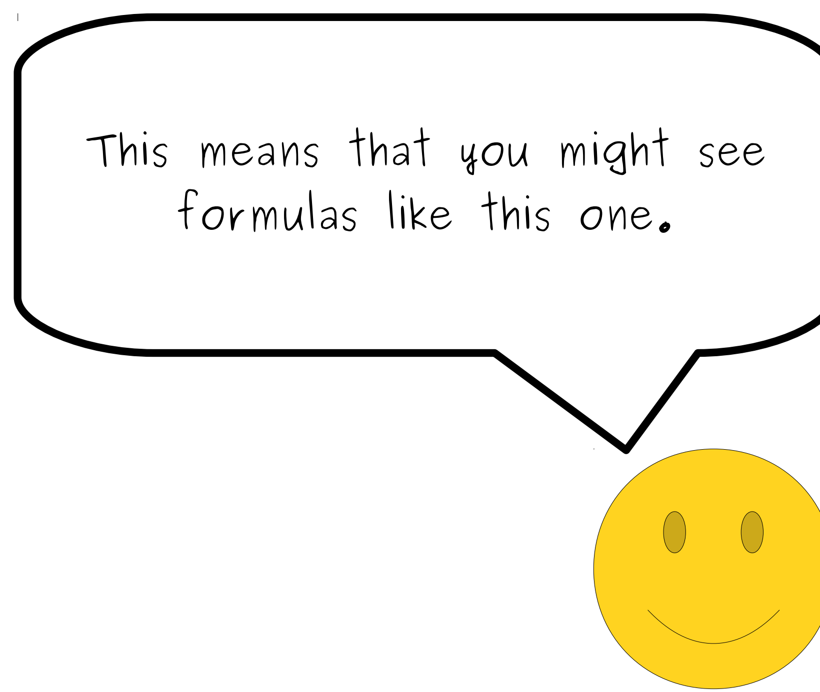
$$\forall x \in S.\ \exists y \in S.\ x \neq y$$



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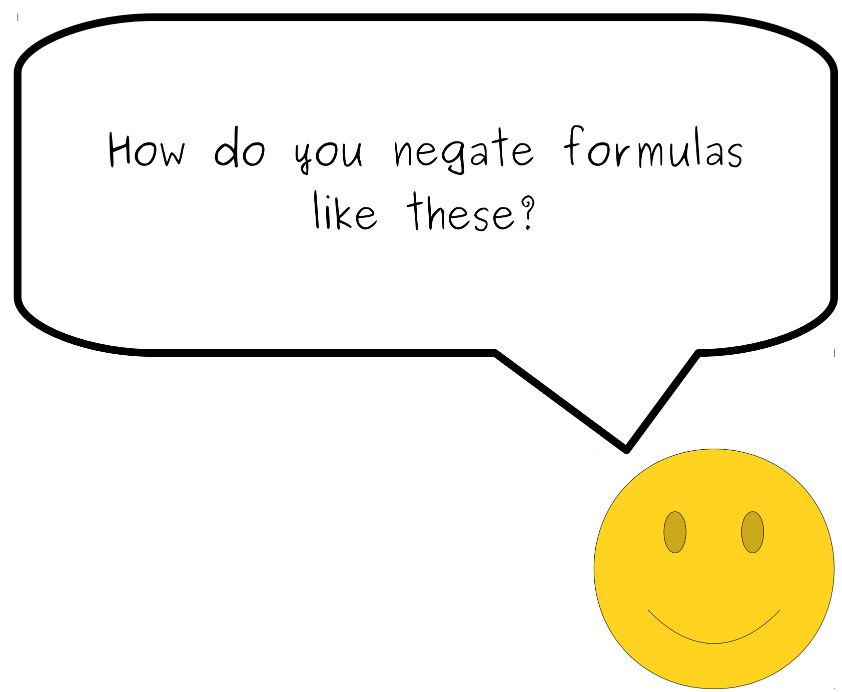

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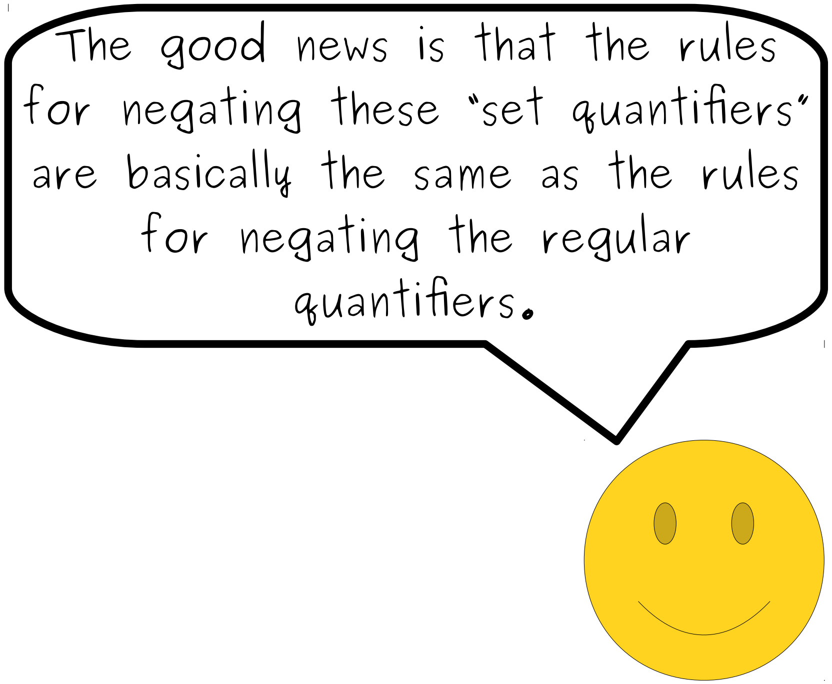

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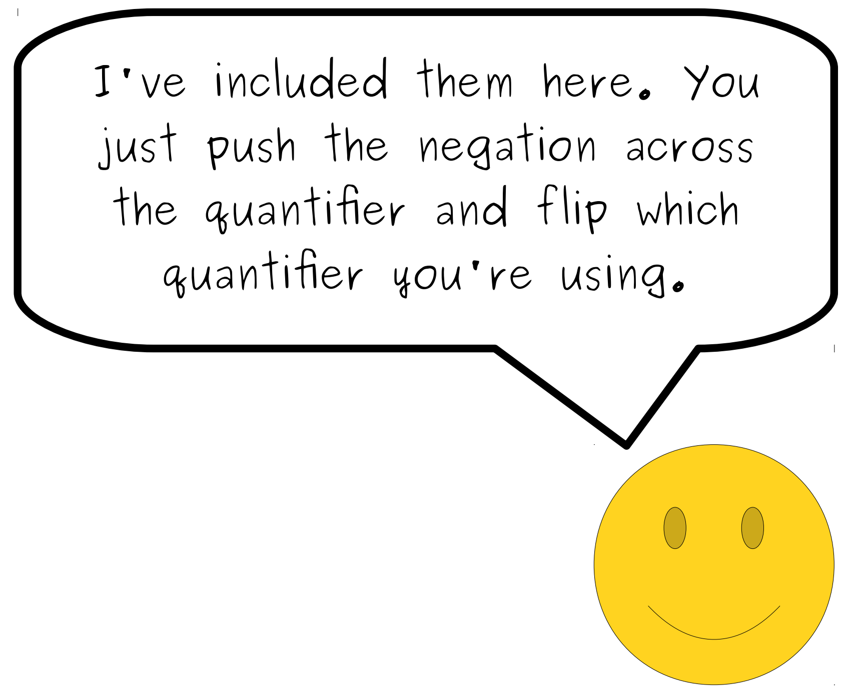

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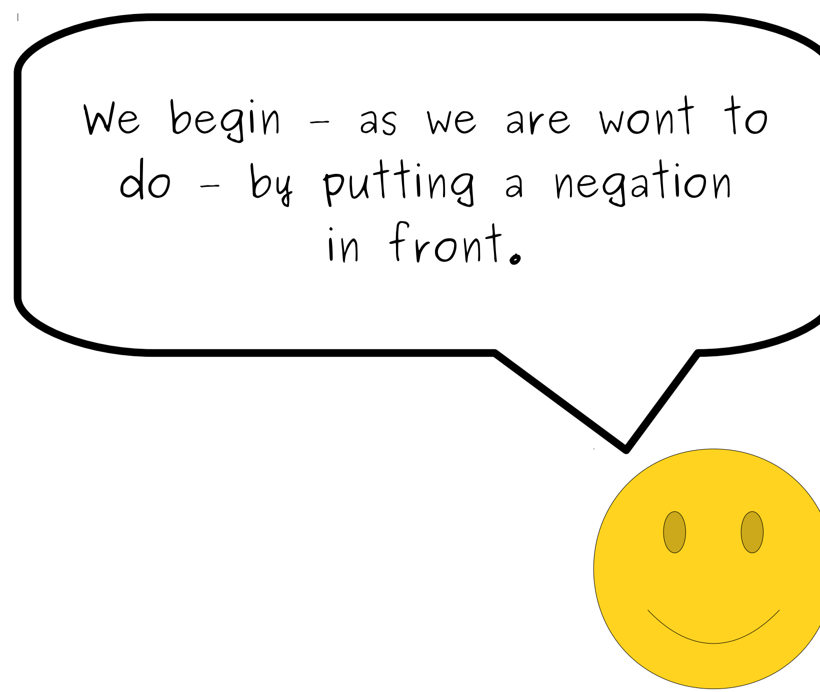

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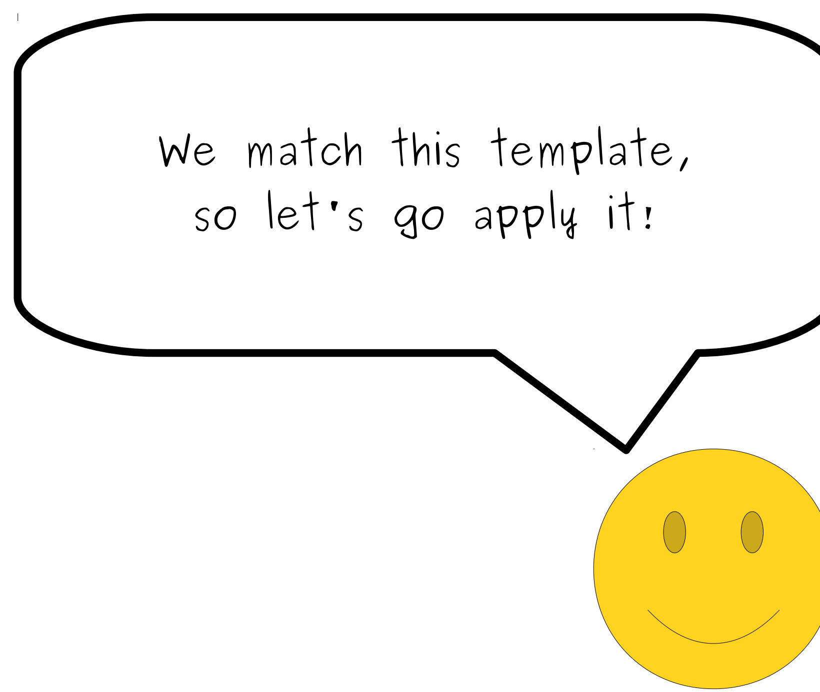

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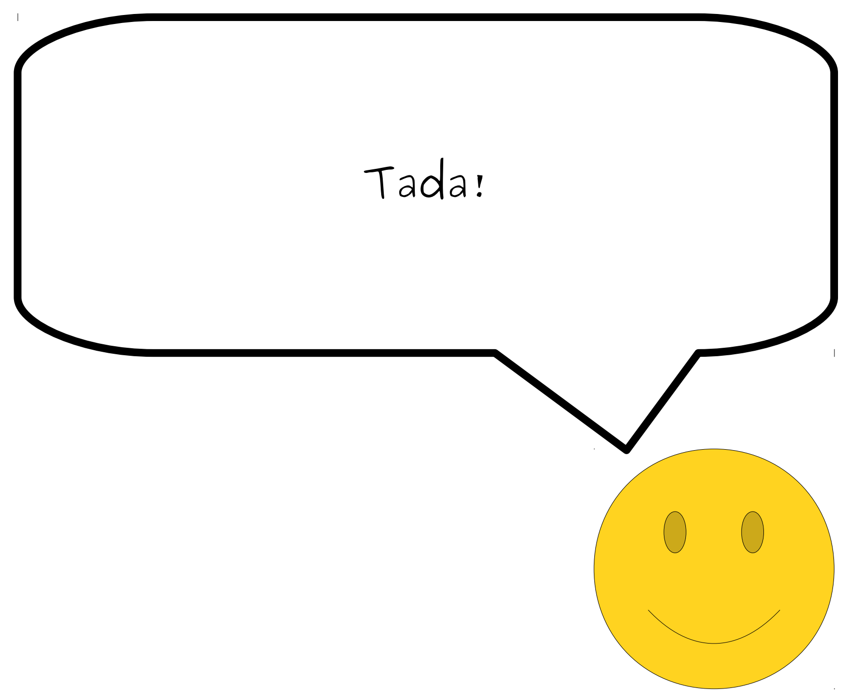
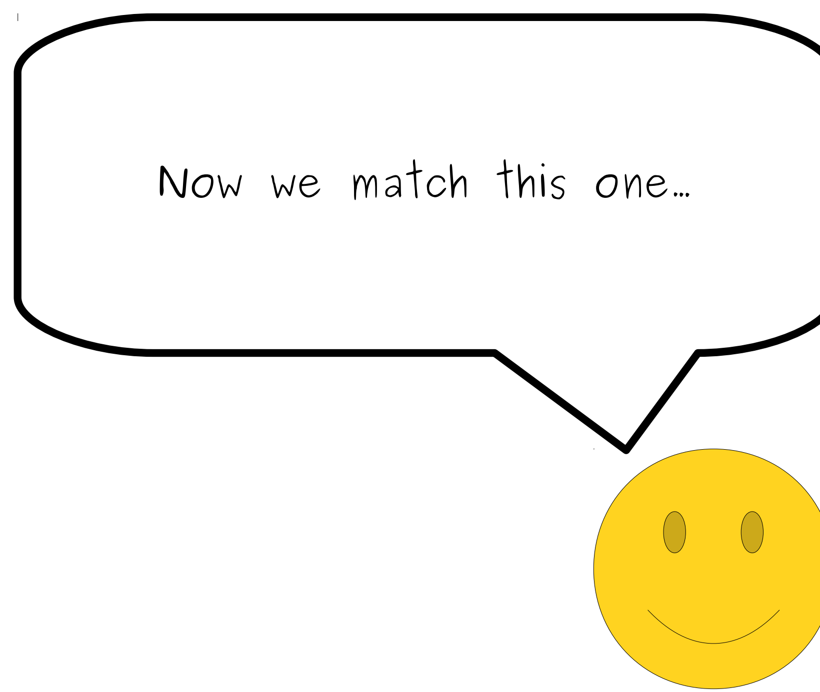


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

$$\frac{\neg(A \land B)}{A \to \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$

$$\frac{\neg\forall x.\ A}{\exists x.\ \neg A} \qquad \frac{\neg\exists x.\ A}{\forall x.\ \neg A} \qquad \frac{\neg\forall x \in S.\ A}{\exists x \in S.\ \neg A} \qquad \frac{\neg\exists x \in S.\ A}{\forall x \in S.\ \neg A}$$

$$\exists x \in S.\ \neg\exists y \in S.\ x \neq y$$

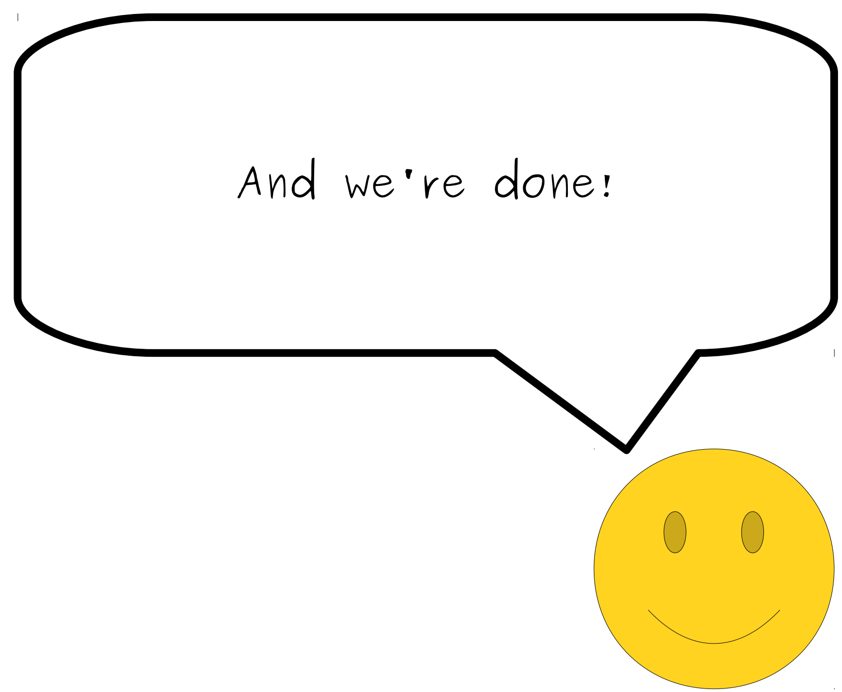


$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

$$\frac{\neg(A \land B)}{A \to \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$

$$\frac{\neg\forall x.\ A}{\exists x.\ \neg A} \qquad \frac{\neg\exists x.\ A}{\forall x.\ \neg A} \qquad \frac{\neg\forall x \in S.\ A}{\exists x \in S.\ \neg A} \qquad \frac{\neg\exists x \in S.\ A}{\forall x \in S.\ \neg A}$$

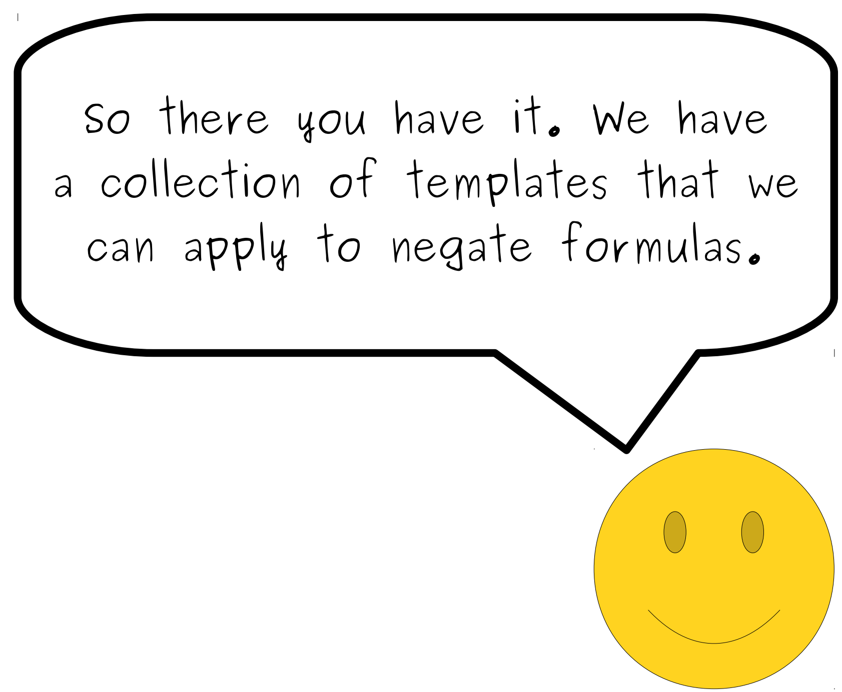
$$\exists x \in S.\ \forall y \in S.\ x = y$$



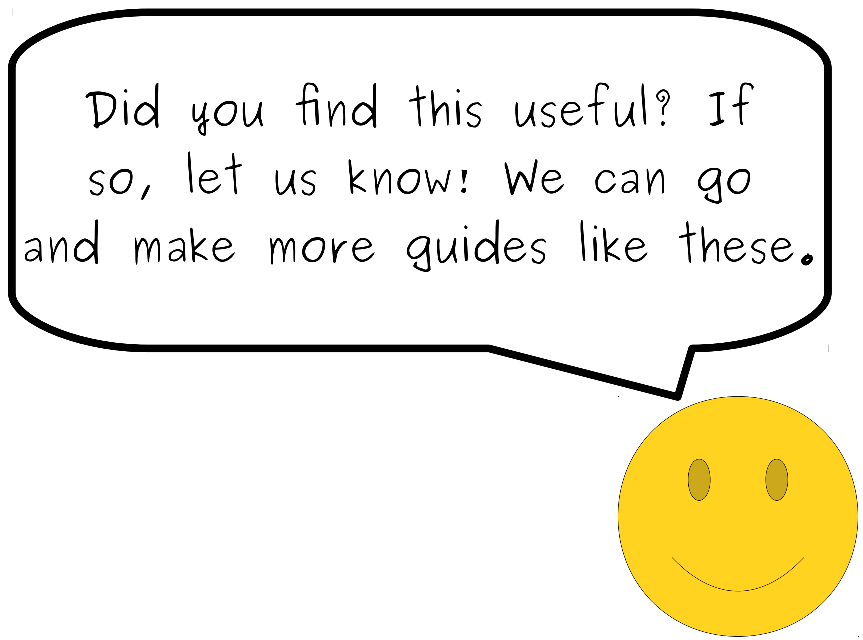
$$\frac{\neg\neg A}{A} \qquad \frac{\neg(A \lor B)}{\neg A \land \neg B} \qquad \frac{\neg(A \land B)}{\neg A \lor \neg B} \qquad \frac{\neg(A \to B)}{A \land \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{\neg A \leftrightarrow B}$$

$$\frac{\neg(A \land B)}{A \to \neg B} \qquad \frac{\neg(A \leftrightarrow B)}{A \leftrightarrow \neg B}$$

$$\frac{\neg \forall x.\ A}{\exists x.\ \neg A} \qquad \frac{\neg \exists x.\ A}{\forall x.\ \neg A} \qquad \frac{\neg \forall x \in S.\ A}{\exists x \in S.\ \neg A} \qquad \frac{\neg \exists x \in S.\ A}{\forall x \in S.\ \neg A}$$




Hope this helps!
Please feel free to ask questions if you have them.


Did you find this useful? If so, let us know! We can go and make more guides like these.